

COMPUTER PROGRAM FOR THE CONTROL CHART FOR MEANS AND STANDARD DEVIATIONS

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Abstract: *The chart for means and standard deviations represents one of the most important control charts by variables that can be used for controlling the processes. The paper presents a method of application for this type of control chart, based on a computer program. Using the measurements of the samples, the program automatically plots the control chart, performs the process stability analysis and the process capability analysis. The presented program can be used in plant practice for controlling the processes.*

Key words: *statistical process control, computer aided quality, control chart for means and standard deviations.*

1. Introduction

The control charts for variables are used for maintaining control over both the process mean and the process variability [2].

One of the most important control charts for variables is the *xbar and s* chart, which uses the means of the samples \bar{x}_{i} for controlling the process mean, and the standard deviations of the samples s_i for controlling the process variability.

In order to plot an *xbar and s* control chart, m samples are taken from the process.

The center lines on the control chart are calculated by means of the formulas (1) and (2) [2], [3]:

$$CL_{\bar{x}} = \bar{\bar{x}} = \frac{\sum_{i=1}^m \bar{x}_i}{m}, \quad (1)$$

$$CL_s = \bar{s} = \frac{\sum_{i=1}^m s_i}{m}. \quad (2)$$

The control limits for the *xbar* chart are given by the formulas (3) and (4) [2], [3]:

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_3 \bar{s}, \quad (3)$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_3 \bar{s}. \quad (4)$$

The control limits for the *s* chart are given by the formulas (5) and (6) [2], [3]:

$$UCL_s = B_4 \bar{s}, \quad (5)$$

$$LCL_s = B_3 \bar{s}. \quad (6)$$

In order to estimate the process standard deviation, it is used the formula (7) [2]:

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$$\hat{\sigma} = \frac{\bar{s}}{c_4} \tag{7}$$

The constants A_3, B_3, B_4 and c_4 depend on the sample size and their values are tabulated [2].

Let's consider that the quality characteristic is normally distributed and the upper and lower specification limits are U and L . In this case, the process capability indices can be calculated by means of formulas (8) and (9) [2], [3]:

$$\hat{C}_p = \frac{U - L}{6\hat{\sigma}} \tag{8}$$

$$\hat{C}_{pk} = \min \left\{ \frac{U - \bar{\bar{x}}}{3\hat{\sigma}}, \frac{\bar{\bar{x}} - L}{3\hat{\sigma}} \right\} \tag{9}$$

2. Objectives

The application of the *xbar and s* control chart requires many calculations and graphical representations.

The objective of this paper is to develop a computer program that makes easier the application of this type of control chart.

Using the input data, the program performs the following operations:

- calculates the mean and the standard deviation for each sample;
- calculates the center lines and the control limits;
- automatically plots the control chart;
- interprets the chart for process control, using the Western Electric rules;
- tests the normality of the measurements distribution and performs the process capability analysis.

3. Material and Methods

The computer program for the application of the *xbar and s* control chart was developed using MS Excel 2007 and Visual Basic for Applications for Excel 2007.

The functions and procedures of the program are created according to the syntaxes presented in [4].

The paper presents the operation of the program by means of an example.

The input data for the presented example are normally distributed, with the parameters $\mu = 80$ and $\sigma = 0.01$.

They are obtained by means of the Excel's random numbers generator.

The program has an input data area that is used by the user of the program for recording the measurements.

Each sample can contain up to 10 measurements (Figure 1).

In the presented example, because the size of the sample is equal to seven, there are used for each sample only seven cells of the range that allows the data recording.

Sample	Measurements								
1	79.997	79.987	80.002	80.013	80.012	80.017	79.978		
2	79.998	80.011	79.989	79.993	79.982	79.990			
3	79.992	79.979	79.994	79.996	80.001	79.996	79.990		
4	79.996	80.013	79.999	79.998	79.995	80.020	80.009		
.....									
24	80.017	80.002	80.001	80.019	80.000	79.993	80.009		
25	80.000	80.010	80.004	80.015	79.999	79.990	79.985		

Fig. 1. The input data area

Using the measurements, the program calculates the points and the center lines for the *xbar and s* chart (Figure 2).

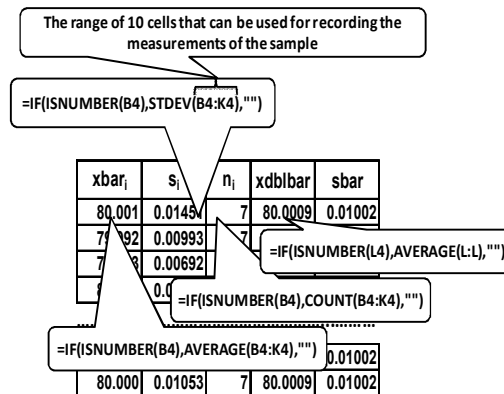


Fig. 2. The points and the center lines

The program uses the *AVERAGE* function for calculating the mean value, and the *STDEV* function for calculating the standard deviation, for each sample.

The *ISNUMBER* function is used for testing if there is a value recorded in the first cell of the range that is allocated for each sample.

The *COUNT* function is used for counting the measurements that are recorded in the same range.

The center lines are calculated by means of the *AVERAGE* function.

Then, the program calculates the control limits of the control chart (Figure 3).

A ₃	B ₄	B ₃	UCLxbar	LCLxbar	UCLs	LCLs
1.182	1.882	0.118	80.012749	79.989057	0.018861	0.00118
1.182	1.882	0.118	80.012749	79.989057	0.018861	0.00118
1.182	1.882	0.118	=IF(ISNUMBER(\$L4), \$O4+\$Q4*\$P4, "")			
1.182	1.882	0.118	80.012749	79.989057	0.018861	0.00118

=IF(ISNUMBER(L4), HLOOKUP(N4, Factors!\$B\$1:\$J\$5, 2, FALSE), "")						
---	--	--	--	--	--	--

Fig. 3. The calculation of the control limits

The values of the constants A_3 , B_3 and B_4 are selected by the program from the worksheet “Factors” by means of a *HLOOKUP* function.

The program also calculates the intermediate limits (Figure 4).

UALxbar	UBLxbar	LBLxbar	LALxbar	UALs	UBLs	LBLs	LALs	
80.0088001	80.0048515	79.9969542	79.9930056	0.01591484	0.01296839	0.00707549	0.00412904	
80.0088001	80.0048515	79.9969542	79.9930056	0.01591484	0.01296839	0.00707549	0.00412904	
80.0088001	80.0048515	79.9969542	=IF(ISNUMBER(\$L4), \$O4+1/3*\$Q4*\$P4, "")				0.01296839	0.00707549
80.0088001	80.0048515	79.9969542	79.9930056	0.01591484	0.01296839	0.00707549	0.00412904	

=IF(ISNUMBER(\$L4), \$O4+2/3*\$Q4*\$P4, "")							
---	--	--	--	--	--	--	--

Fig. 4. The calculation of the intermediate limits

The intermediate limits (Figures 5 and 6) are used by the program for interpreting the chart by means of the Western Electric rules.

These limits are defined as follows:

- *UALxbar* and *LALxbar* are the two-sigma

warning limits on the *xbar* chart;

- *UBLxbar* and *LBLxbar* are intermediate limits on the *xbar* chart that are situated at a distance of one-sigma from the center line;

- *UALs* and *LALs* are the two-sigma warning limits on the *s* chart;

- *UBLs* and *LBLs* are intermediate limits on the *s* chart that are situated at a distance of one-sigma from the center line.

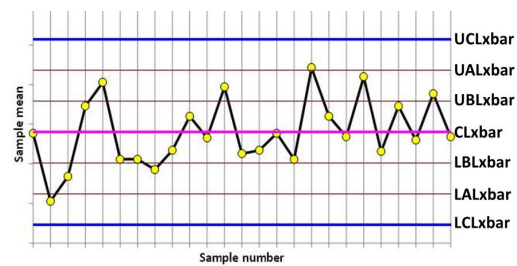


Fig. 5. The intermediate limits for the *xbar* chart

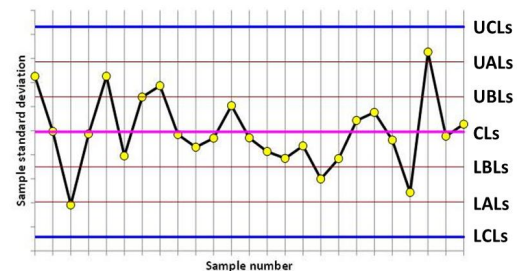


Fig. 6. The intermediate limits for the *s* chart

According to the Western Electric decision rules, a process is influenced by a special cause of variation in the following cases [2]:

- One point is situated beyond the three sigma control limits;

- Two out of three consecutive points are situated outside the two-sigma warning limits, on the same side of the center line;

- Four out of five consecutive points are situated at a distance that is greater than one-sigma from the center line, on the same side of the center line;

- Eight consecutive points are situated on the same side of the center line.

4. Results and Discussions

In order to plot the chart for the sample standard deviations, the user of the program must click the control button labelled “*s Chart*”.

When this button is clicked, the *s* chart (Figure 7) is automatically plotted by the procedure *Chart_s*, which was written in Visual Basic for Applications for Excel 2007.

The procedure *Chart_s* performs the following operations:

- creates an empty chart on the current worksheet;
- adds to chart the data series for the sample standard deviations, center line, intermediate limits and control limits;
- formats the data series;
- marks on the chart the points that correspond to special causes of variation. The chart is analysed by means of the Western Electric rules.

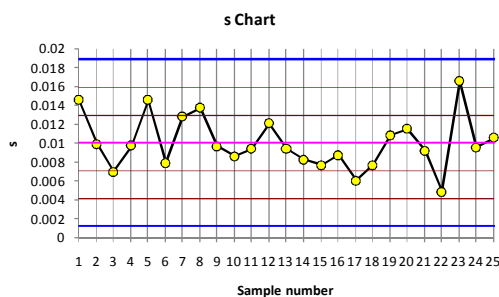


Fig. 7. *The s chart*

For the presented example, it can be observed that there are no points marked by the program on the chart.

In this case, it can be concluded that the process variability is under control, and the process stability analysis can continue with the *xbar* chart.

In order to plot the chart for the sample means, the user must click the control button labelled “*xbar Chart*”.

When the user clicks this button, the *xbar* chart (Figure 8) is automatically plotted by

the procedure *Chart_xbar*, which was developed in the same manner as the procedure *Chart_s*.

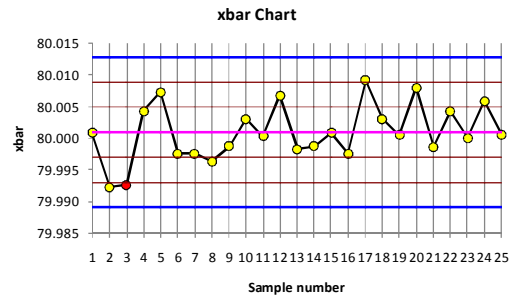


Fig. 8. *The xbar chart*

In the presented example, the point 3, which is the second of three consecutive points situated beyond the lower two-sigma warning limit, is automatically marked by the program.

This is a signal that a special cause of variation changed the process mean.

In the case when a point is marked by the program, the special cause of variation must be identified and removed from the process.

Then, the user of the program will click the control button labelled “*Copy Worksheet*”.

This action will run the procedure *CopyWorkSheet*, which will create a copy of the current worksheet.

The user of the program will exclude from the chart the point that corresponds to the special cause of variation that has been removed from the process.

The center lines and the control limits will be automatically recalculated.

Then, the revised *s* chart (Figure 9) and the revised *xbar* chart (Figure 10) will be plotted.

The charts will be revised as many times as needed, until the process will be in a state of statistical control.

For the presented example, it can be observed that there are no more points marked by the program, so it can be concluded that the process is in statistical control.

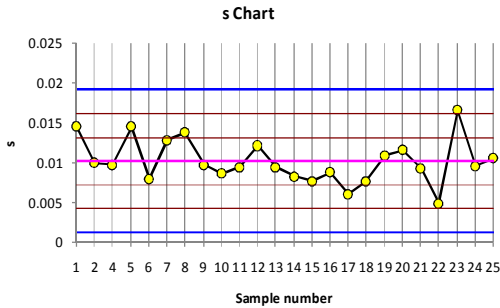


Fig. 9. The revised s chart

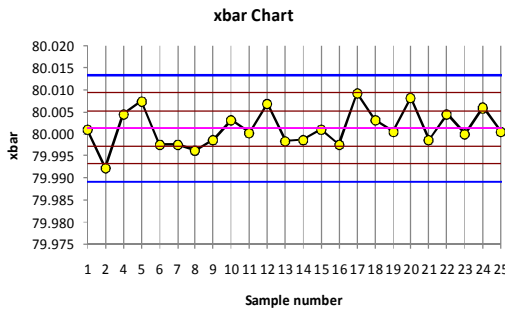


Fig. 10. The revised xbar chart

Before the achievement of the process capability study, the program tests the normality of the distribution of measurements.

At the activation of the worksheet "TestNorm", which is used for this purpose, the procedure *WorkSheetActivate* will be executed.

This procedure performs the following operations:

- collects from the worksheet that corresponds to the last revised chart the values of n , $sbar$ and $xdoublebar$;
- selects from the worksheet "Factors" the value of c_4 that corresponds to the sample size, by means of a *HLOOKUP* function;
- records these values in the corresponding cells on the worksheet "TestNorm".

The program uses the formula (7) for estimating the process standard deviation.

When the user of the program clicks the control button labelled "Get Data", it will be executed the procedure *GetData()*, that

will collect all the measurements from the worksheet that corresponds to the last revised $xbar$ and s chart and will record them into an array.

The listing of the procedure is presented in Figure 11.

```

Sub GetData()
    Dim m As Integer, n As Integer, i As Integer, j As Integer, k As Integer, el As Integer
    Dim x() As Double
    Dim aux As Double
    Dim Number_Of_Sheets As Integer
    Number_Of_Sheets = Worksheets.Count
    el = 0
    Sheets(Number_Of_Sheets - 3).Select
    m = WorksheetFunction.Count(Range("$L:$L"))
    n = WorksheetFunction.Count(Range("B4:K4"))
    For i = 1 To m
        For k = 2 To n + 1
            el = el + 1
            ReDim Preserve x(el)
            x(el) = Cells(i + 3, k)
        Next k
    Next i
    For i = 1 To el - 1
        For j = i + 1 To el
            If x(i) > x(j) Then
                aux = x(i)
                x(i) = x(j)
                x(j) = aux
            End If
        Next j
    Next i
    Sheets(Number_Of_Sheets - 2).Select
    Range("B2:B1001").ClearContents
    For i = 1 To el
        Cells(i + 1, 2) = x(i)
    Next i
End Sub
    
```

Fig. 11. The procedure *GetData()*

The array will be ordered and then, recorded in the column labelled x_i (Figure 12).

In order to test the normality of the distribution of measurements, the program uses the modified Kolmogorov-Smirnov test of normality.

The procedure of application for this test is presented in [1].

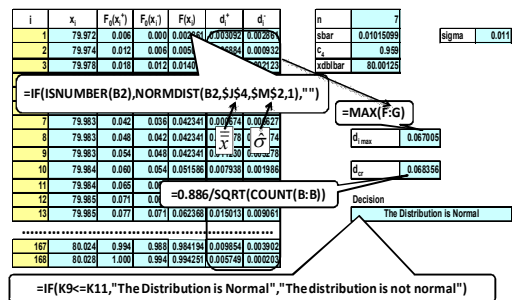


Fig. 12. The test of normality

If the distribution of the measurements is normal, the process capability study is performed (Figure 13).

This study uses the values of the process mean and process standard deviation that were calculated by the activation procedure of the worksheet "TestNorm".

The program calculates the capability indices using the upper specification limit U and the lower specification limit L .

These values must be recorded by the user of the program.

The decision about the process capability is automatically made by the program by means of an *IF* function.

The program also calculates the proportions of nonconforming products, using the *NORMSDIST* function.

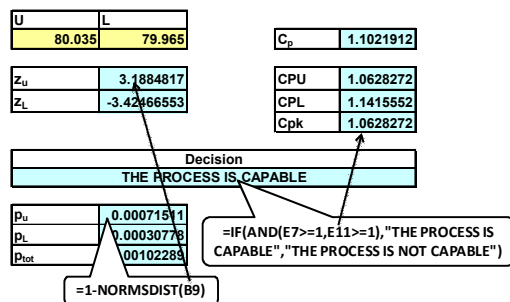


Fig. 13. *The process capability study*

For the presented example, it can be observed that the process is capable.

5. Conclusions

The presented program can be used in plant practice for controlling the processes by means of the *xbar and s* control chart.

The program is developed so that users do not have to possess computer programming knowledge.

They only must record the measurements of the samples and the specification limits, and all the calculations and graphical representations are performed by the program.

The points that are signals for special causes of variation according to the Western Electric rules are automatically marked on the chart.

When the process is in statistical control, the program tests the normality of the distribution of the measurements and, in the case when the distribution is normal, performs the process capability study.

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