

# THE VERTICAL LOADS VARIATIONS STUDY AND THE GUIDANCE CAPACITY OF SIX AXLE LOCOMOTIVES AT CURVES CIRCULATION

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**Abstract:** *The current trend of increasing power diesel locomotives require more efficient use of their weight especially during the startup, when does the hazard happen slip axles to be unloaded. Providing guidance in curves in complete safety and with minimum wear of wheels and rails is a basic requirement of railway vehicles. In this paper is made an analysis of the circulation conditions in curve of a bogie with elastic driven wheelsets, type Y 32, used by railways in Romania. The system of wheelsets elastic driving allows their quasi radial position in curves, leading to the reduction of friction between wheels and rails and to lower wear. The presented mathematical model is original, taking into account the wheel loads transfer and the creep coefficients evaluated according to Kalker theory. It is found that high elasticity causes a reduction of the hunting critical speed. Therefore the paper presents also an original study model of the hunting movement of a high speed bogie.*

**Key words:** *wheel slip, starting, adherence, gallop, non pitching, stick-slip.*

## 1. Introduction

The format of the bulletin will be A4. The article, inclusively the tables and the figures, should be of 6-8 pages, an even number of pages being compulsorily. The last page will be filled at least 70%. The individual locomotive drive force of the drive motor for each axle can be greater than the force of adhesion to the axle at more than discharged as excess adherence to one of axles would cause slippage, and

the traction power needed would be broken down other axles that will slip and they. The study in curve movement of a railway vehicle aims to establish conditions which ensure safe vehicle guidance. From the point of view of the guidance, the vehicle is an assembly consisting of a number of axles connected rigidly or resiliently on a frame. In the presence paper is presented the case of vehicles (bogie) with steerable axles that besides transverse displacement

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is possible to rotate the axle to the chassis due to their elastic binding, longitudinal and transverse chassis. This rotational movement allows the axle to orient themselves quasi radial in curves, which has the effect of reducing contact forces and also tracks the wear of tread and wheel guide rails.

As the elasticity of the longitudinal axle guidance system is higher, also the driving axle of the vehicle will be closer to the radial position thus creating conditions for runs "pure" if the wheels wear profile.

A provides high elasticity in the longitudinal direction can lead to a movement of the axle hunting unstable at a speed of movement of the vehicle less than the stipulated.

Taking into account the size of the elasticity of adversarial proceedings, the proper design of the vehicle must be chosen "middle way" that would be acceptable from both points of view were presented previously.

## 2. The loads variation static nature and the elasticities influence of conduction system of the settlement axle bogie geometric in curves

The loads variation static nature occurs due to rotation box locomotive bogie rotation and traction motors action. In equation (1) is shown the actual amount  $Q$  of load on the axle, where components are static relationship  $Q_0$  per axle, of the axle load variation  $\Delta Q_s$  due to factors such as static of the axle load variation  $\Delta Q_d$  due to factors such as dynamically. This depends on various factors such as mechanical, mainly on the type of connection between the box and the bogie of the locomotive slurry, suspension mode of the traction motor as well as any devices "non pitching". The influence of these

factors are considered a type of locomotive Co - Co, considering the bearing and alignment line, coupled bogie vertical tractive force equal to all axles.

Considering released locomotive box links with external forces and moments acting on it (figure 1), the conditions of equilibrium of moments, in relation to A and B support box bogies, bogie vertical reactions are obtained, presented in relation (2) where  $2l$  is the wheelbase of locomotive,  $H$  is the height above the rail the draw hook,  $h$  is the height of point the traction force transmission from the locomotives box and the bogie  $M'$  and  $M''$  are moments reaction forces on the box due to the device "non pitching".

Considering their longitudinal axes represented bogies with low torque forces and moments at the points  $A'$  and  $B'$  (that is, centres of rotation of the bogie, as schematically shown in figure 2), the conditions of static equilibrium and deformation will result reactions  $\Delta P_i (i = 1, \dots, 6)$  of the suspension of bogie, as shown in fact in the relationship (3) in which  $c_{a_1}, c_{a_2}, c_{a_3}$  are the suspension of the axles are rigidities (1,6), (2,5) and (3,4) while  $(F', M')$  and  $(F'', M'')$  are the torsion points reduction  $A'$  and  $B'$ . Likewise,  $c$  it represents the distance between the middle axle of the bogie and the centre of rotation of the bogie.

Given the forces and moments acting on the bogie [5] will results the relations (4) where  $h_1$  is the height of the point of transmission of the drive force from the bogie to the box while  $e$  is the distance of point of application of the vertical reaction. Also,  $\lambda_0 F_0$  is the reaction engine to the bogie ( $\lambda_0$  being a coefficient which depends on the traction motor suspension [5]) while  $M'_1, M''_1$  represent the reaction times on the bogies of the "non pitching" device.

Between the loads variations on the springs  $\Delta P_i (i = 1, \dots, 6)$  given by the system of equations (3) and the axle load variations  $\Delta Q_{is} (i = 1, \dots, 6)$  is the relationship (5) where the positive sign (+) corresponding positioning electric engine the traction before the front axle of the running and the negative sign (-) is used when the engine is positioned after the axle. To note is the fact that variations in axle loads given by the relation (5) after solving the system of equations (6) leading to the system of equations (6) wherein the variables  $N'$  and  $N''$  are defined as mathematical expressions by the form (7). The individual training axle reduces the possibilities to use the full weight of the adherence, so the use of appropriate means to minimize download axles (by the "non pitching" phenomenon), because of high traction effects have now become a necessity in modern locomotives construction type [5], [6]. For an elastic driving bogie axle shall be analyzed assuming inclusion curve established a regime of movement stationary quasi-static.

Under the action of external  $F_n$  and contact forces between wheel also rail bogie sits curve position in Figure 1 axles by the normal to the curve angles (attack)  $\alpha_1$  and that respectively  $\alpha_2$ .

They also assume that there are no large sliding wheels (with profile wear) but pseudo slidings proportionate to contact forces and forces balance can ensure contactless running axles lips (of wheelsets). To the axis of the track, the axle centers are offset towards the outside with  $y_{c1}$  and, respectively,  $y_{c2}$  and the bogie chassis lowered to its longitudinal axis, is offset with the right axle  $y_1$  and, respectively,  $y_2$ . During the movement, the forces of friction forces also the balance of guidance system, of each axle must be balanced.

Moreover, the forces acting on the steering system of the chassis of the bogie must be in balance with the external force applied to the bolster (Fig. 2). Towards chassis axles are rotated  $\Psi_{1,2}$  (equations no. (1)). The longitudinal forces of the suspension springs will be  $F_{1,2}$  (equations no. (2)) could be reduced to the moments  $M_{1,2}$  (equations no. (3)). The overhung transverse forces of the suspension springs are  $F_{1y}$  and, respectively  $F_{2y}$  and are also explained in relations (4). If it is noted in this case  $f_x = \chi_x \cdot Q$  and  $f_y = \chi_y \cdot Q$  the pseudo-slip coefficients (units of force), the pseudo-sliding (dimensionless),  $Q$  being the load on the wheel then the forces of pseudo-slip will be  $T_{1x}, T_{1y}, T_{2x}, T_{2y}$  which are explained in relations (5). The forces of balance system of the two axles  $C_{1,2}$  are shown in the form of equations (6), which, as noted with kg the elastic constant balance this with the expression (7), wherein the center position of the axle,  $Q$  is the wheel load;  $\gamma$  - the effective conicity of the wheel profile;  $\gamma_0$  - the flank angle of the wing tread;  $c_x$  and  $c_y$  are the rays of curvature of the profile of the wheel and, respectively, of the rail. With such the forces established equilibrium equations can be written for the axle and bogie frame, which takes into account the small values of the angles involved (equations no. (8)), which, after substitution, becomes to the (9) form.

The equations no. (9) are enable for determining the position of a vehicle on a curve on two axles in the general case when the axles are connected to the chassis by resilient longitudinal and lateral.

Taking the view in this case that the  $k_g = 0$ , from the system of equations (6), we obtain equations (10).

The first term in equation (10) highlights the deviation from the track centre line and

the second is the radial displacement of the axle due to lateral force  $F_n$ .

If  $2k_x \cdot b^2 \langle f_x \cdot a$  the slurry suspension elasticity of the curve does not improve the vehicle registration. If  $2k_x \cdot b^2 \langle f_x \cdot a$ , the deviation from the track centre line and will drop to a springy suspension deviation will be close to the minimum (it possible for free axles)  $k_{c1} k_{c0}$ .

The deviation from the track centre line when  $2k_x \cdot b^2 \langle f_x \cdot a$  is being reduced by increasing the value of the term  $(e\gamma/r)[1 + 4c_x \cdot b^2 / (k_y \cdot a^2)]$  that is by the effective conicity  $\gamma$  high transverse rigidity and decrease  $c_y$ . The displacement of the rear axle to axle path  $k_{c2}$  is given by the relation (11). For  $F_n = 0$ , the angles that they make axles with normal path will be  $\alpha_{1,2}$  (explained in equation (12)). The maximum force of pseudo-slip will occur on the two front wheels. At this, for  $F_n = 0$ , the pseudo-slips are given in the equation (13) and the pseudo-slips force  $T$ , considering that  $f_x = f_y = f$ , is given by the equation (14).

It follows therefore that an elastic driving bogie axle will slide on every curve whose radius is  $R$  (see equation (15)). As the suspension is elastic, and the radius of the curve  $R$  is a lower axle aiming at a radial position.

Comparing the displacements of lateral force  $F_n$ , is observed that the rear axle is shifted more than the front axle, this movement being independent of the radius of the curve.

The displacement under the effect of force  $F_n$  is also independent of the deviation from the track centre line, which occurs even if the bogie side not exercise any power and actually indicate the inherent ability to be self-guided bogie in way pseudo slidings forces between wheels and rails.

### 3. The loads variation dynamically nature and the elasticities influence from conduction system of the axles on the bogie hunting stability

Variations dynamic axle loads occur due to fluctuations locomotive during the startup. Out of these the most influential have oscillations “gallop” of the locomotives box due to longitudinal forces [5]. Considering negligible oscillations of electric engines bogies and traction differential equation of oscillations will be of the form (8) while  $\Psi$  is the angle of rotation of the vertical of the locomotives box,  $I_c$  is the moment of inertia of the locomotives box to the center of gravity,  $\Delta V'_d$  and  $\Delta V''_d$  are the vertical reactions of the bogies to the locomotives box,  $F_{ds}$  the force on the coupling hook locomotive,  $F_{bd}$  is horizontal reaction locomotive bogie over the box while  $M'_d$  and  $M''_d$  are the moments given by the “non pitching” phenomenon devices.

To note is the fact that, in the equation (8) were only considered dynamically nature forces and moments, their expressions are given in relations (9), (10) and (11), where  $dV/dt$  is the acceleration of railway vehicle,  $\gamma$  is a coefficient that takes into account the mass inertia in rotation,  $m_L$  is the mass of the locomotive,  $m_b$  is the mass of the bogie,  $R_L$  locomotive is the resistance to progress while  $c_c$  is the stiffness of the suspension locomotives box (on bogies). Also, taking into account the relationship (9), then the equation (8) can be written in the form (12) whose solution can be explicit as (13) expression of the factor can be deduced that  $\Psi_0$  defining in the form (14), The equilibrium position about which the oscillation takes place “gallop” whose own pulsation  $\omega$  is given in equation form (15).

The variations in the maximum dynamic axle load are obtained by replacing the factor  $\Psi_{\max}$  from the relationship (13) in the equation no. (9) where the negative sign (-) take the first three axles of the locomotive while the positive sign (+) for the next three. Because in general the locomotive drive systems with “shaft torsion” [5], [6] we have  $\omega \ll p$ , where  $p$  is the angular frequency of the oscillations of stick - slip, we can neglect the influence of the “gallop” oscillations of the locomotives box over the stick - slip oscillations. The study of hunting motion for stability of an elastic driving bogie axle is based on relationships obtained after linearization phenomenon of hunting. Linearization of the phenomenon of hunting is realized: considering that the contact forces vary linearly with lateral movement of the axle; neglecting friction and games of various elements of the bearing structure of the vehicle; neglecting tread irregularities and discontinuities; considering equivalent conicity wheel profile as constant and proportional to the tangential force pseudo slidings point of contact of the wheel with the rail.

This study aims to determine the velocity at which the hunting stable movement of a vehicle equipped with elastic driving axle bogies will turn into an unstable motion, namely the establishment of critical speed, which when exceeded will result in a rapid deterioration walking. In other words, we aimed to determine the maximum speed that can be reached safely by vehicle. Consider the case of general motion of the bogie in hunting which the suspension consists of axle springs having spring constants  $k_x$ ,  $k_y$ , and the linear characteristic of the shock absorbers (non-viscous), which damping constants  $c_x$  and  $c_y$  (Fig.3). Center of mass of the bogie is considered located in the axle axles. Determination of critical speed and critical

pulse respectively when the sprung mass of the bogie neglect and depreciation can be based on the equations of motion of the bogie frame, respectively, axles, axle balance obtained neglecting the effect of spin and gyroscopic effect (equations (15)) where noted:  $k_x$  and  $k_y$  - the elastic constants in the longitudinal and transverse,  $m_0$  - the wheelset (axle) mass,  $Q$  - the wheel load,  $a$  - the wheelbase bogie,  $b$  - the transverse cross midway between the suspension springs,  $e$  - the midway between the nominal rolling circles,  $r$  - the wheel ray (radius),  $v$  - the velocity running speed,  $\gamma$  - equivalent conicity,  $I_{oz}$  - the moment of inertia of the sprung mass above vertical axis which passing through the center of mass of the bogie,  $\nu$  - pseudo sliding coefficient and  $k_y^* k_y \cdot k_x b^2 / (k_y \cdot a^2 + c_x b^2)$  which signifies an equivalent elastic constant cross. With the change of variables in equations (16) are the equations of motion of the form (17). Neglecting the mass of the axle axis, mean while considering  $I_{oz} = m_0 e^2$  and using the notation  $A, B, C, D$  from the equations (18) yields equation own pulsation (19). Considering that the stability limit was reached when  $v = v_c$ , and making the substitution in the equation (19)  $p = j \cdot \omega_c$ , resulting final form of the equation own pulsations (20). Shall be defined the functions  $f(\omega_c)$  and  $g(\omega_c)$  made explicit relations (21) and (22) allowing the calculation of critical speed  $v_c$  and critical pulsation  $\omega_c$ . Thus stroke critical pulsation value results as a root of the equation  $f(\omega_c) = 0$  and critical velocity resulting from equation (21). For the coefficients of friction wheel - rail can be considered the work of P. van Bommel [2] which had recommends some approximate values of the coefficients of pseudo slip. Thus based on the results of

Kalker [3] it is found that (for  $Q$  expressed in tons), the parameter  $\gamma_x$  has approximately the same value with  $\gamma_y$ , as shown in equation no. (23).

#### 4. Numerical Application - The establishing the variation of tasks to starting and drive axle locomotives class 060 EA

As noted above, slip axle locomotive tasks depends not and does not remain constant during walking. Knowing the variation of static and dynamic tasks axle is absolutely necessary because they depend only on the mechanical construction of the locomotive. For this case study was taken as an example such as electric locomotive type 060 EA, which have been carried out some experiments with the train and the power of the diesel type 060 Carpatia, electric traction with engines into alternating current - alternating, who performed a test sample train in October 2010 to the distance between Berlin East and Dresden. The parameters of these kind of diesel electrical type locomotive, are:  $l = 5.15$  m;  $a = 2.25$  m;  $b = 2.1$  m;  $c = 0.05$  m;  $e = 0.438$  m;  $H = 1.05$  m;  $h = 0.59$  m;  $h_l = 0.484$  m;  $h_c = 2.3$  m;  $r_0 = 0.625$  m;  $c_{a1} = c_{a3} = 228 \cdot 10^4$  N/m;  $c_{a2} = 134 \cdot 10^4$  N/m;  $c_c = 320 \cdot 10^4$  N/m;  $I_c = 1.4 \cdot 10^6$  kg.m<sup>2</sup>;  $m_L = 126 \cdot 10^3$  Kg;  $m_b = 24.5 \cdot 10^3$  Kg;  $\gamma = 0.135$ ;  $\lambda_0 = 1.427$ . Because locomotive traction is low moments due to the mode of transmission of the thrust will be to form the system of equations (17), where  $d = 3.23$  m represents the points of articulation of drawbar on the locomotives box,  $d_1 = 2$  m is the distance between of hinge points on the bogie drawbars while  $\alpha = 10^\circ$  is the angle of inclination from the horizontal drawbars.

The values of static variations axle loads calculated with relations (6) and (17) depending on tractive force  $F_0$  are summarized in Table 1 wherein positive sign (+) corresponding axle load while the negative sign (-) corresponds to its unloading. From this table it can be seen easily that the download of the locomotive axle is axle 1, it having so therefore the first tendency to skate.

The pulsation own oscillations "gallop" of the box locomotive, calculated with equation (15) will have the value  $\omega = 11,011 \text{ rad.s}^{-1}$ , this value is much lower pulsation due to the phenomenon of stick - slip which is generally the value  $p = 180, \dots, 375 \text{ rad.s}^{-1}$ . It thus follows therefore that the axle load at the time of slip can be considered constant.

The unloading one axle maximum dynamic nature because of tasks will be given by (16) where  $\Psi_0$  depends on the starting of Vehicle acceleration  $dv/dt$  who are made explained in the relations (10), (11) and (14) which is determined by the equation of the train motions (18), where  $\varphi = g/(1 + \gamma)$ ;  $R$  is the total resistance to the train progress;  $G_L, G_V$  are respectively the weight of the locomotive and the weight wagons. It also will consider and locomotive towing a train consisting of freight cars in alignment and landing tier. In this case,  $R = r_L \cdot G_L + r_V \cdot G_V$ , where  $r_L, r_V$  represents the specific resistance of the locomotive forward wagons respectively, are determined by the following relations respectively

$$r_V = 1,6 + V^2 / 2700 \left[ \text{daN} / 10^3 \text{ daN} \right] \quad \text{and} \\ r_L \cdot G_L = R_L = 296 + 7,068(V/10)^2 \left[ \text{daN} \right],$$

wherein  $V$  is expressed in km/h.

The force of the axle 1 such are limited by the adherence, most unloaded will be given by the relation no. (19), where



$\mu_a = \mu_a(V)$  represents for varying the adherence coefficient depending on speed  $V$  [5]. It may reveal the influence of constructive parameters of the locomotive, and resistance to progress  $R_L; R_V$  of the locomotive and wagon respectively adhesion coefficient  $\mu_a(V)$  over dynamic load  $Q_i$  the axle downloaded if you take into account the dynamically nature of the loads variation of. For this system to be solved formed by equations (16), (18) and (19), thereby achieving the equation (20) in the canonical form.

Because the phenomenon of stick - slip axle occurs with the most discharged after passing this axle and as the adhesion force of the drive motor for each axle can be greater than the adherence force to the axle at more than unload, the load variations length was calculated for  $F_0 = F_a$ , where  $F_a(V)$  is given by the relation (19) for  $\mu_a(V)$  determined by the Curtius - Kniffner relations and  $G_v = 1500 \cdot 10^3$  daN. Comparing calculated and limited adherence strength, taking into account only the variations of static axle loads ( $\Delta Q_{id} = 0$ ) and the results were shown in the table no. 2.

The locomotive force will be limited by adherence and became  $F_{aL} = 6F_a$  and the weight adherent will be  $G_a = 6Q$ , both of which are functions of the speed  $V$  by running of the train. To be able to observe the influence of train velocity on the emergence of the phenomenon of stick - slip, were represented (in figure no. 2 and in figure no. 3), the variation curves of forces  $F_{aL}; F'_{aL}; R$  and respectively  $Q_L, -\Delta Q_s$  and  $-\Delta Q_d$  to overcoming the adhesion for speeds between 0 and 50,4 km/h.

Considering also that the locomotive speed control is constant tensile force

during the startup and since the adherence force decreases as walking speed, axle slippage will occur at the speed corresponding to the thrust intersection with adherence forces. The locomotive traction force value during the startup, determine the size of which will depend on vehicle acceleration directly proportional to the speed of movement of the train speed. This can be seen from the diagram shown in Figure no. 4, in which, have been presented the curves  $C_i (i = 2, \dots, 8)$  the variation of acceleration with walking speed  $V$  of the train and that have been calculated using the relationship (18) for different values of constant traction force.

The boundary points  $A_1; A_2; \dots; A_8$  of acceleration correspond to the velocity at which the train the locomotive slip curve  $C$  which linking them actually representing acceleration variation for adjustment after the limited adherence force whose experimentally determined values were summarized in table no. 3. In order to highlight the influence of the coefficient of adhesion  $\mu_a$  (to  $V=0$ ) over the variation of the loads on the axles are calculated the forces  $F_a$  and the loads, for values of the coefficient of adhesion between 0,340 and 0.486 such us the apparent fact of table no. 4. With these values, in Figure no. 4 were represented the variation curves of functions  $F_a(\mu_a)$  and  $Q_i(\mu_a)$ . For example we considered a passenger car equipped with bogies Y 32 R type at which  $m_0 = 2000$  kg,  $Q = 59.65$  kN and geometrical characteristics:  $a = 1.28$  m,  $b = 1$  m,  $e = 0.75$  m,  $r = 0.46$  m. The pseudo slip coefficients calculated with (23) have the values  $\chi = 76,8$ . A special importance for the stability of the bogie cross has elastic characteristics of the axle driving system. R. Joly [4] indicates the speed bogies with elastic driving axles values  $k_x = 107$  N/m and  $k_y = 5.107$  N/m . The

equivalent transverse stiffness is set  $k_y^* = 1,481 \text{ kN/mm}$ .

The profile of the wheel, the effective conicity  $\gamma$  its influence on the stability of the vehicle. A reduced taper contributes generally to speed up critical observing that influence effective taper the critical speed is dependent on the values of rigidities  $k_x$  and  $k_y$ . For values of  $k_x$  and  $k_y$  greater than  $107 \text{ N/m}$ , the optimal effective conicity is between  $0.10$  and  $0.15$ . It adopts effective conicity  $\gamma = 0.15$ . Critical and critical speed pulsation calculation, was based on equations (19) and (20). The positive real square roots of the equation (20) and critical pulsations are:  $\omega_c = 20.79836 \text{ rad/s}$  and  $\omega_c = 139.40631 \text{ rad/s}$ . From equation (12) that the vehicle is travelling without slipping in curves of radius  $R = 1552 \text{ m}$ .

Considering the example calculation  $R = 1600 \text{ m}$  is obtained  $y_{c0} = 1,438 \cdot 10^{-3} \text{ m}$  and also  $A = 1,3667$ ;  $B = 0,0439$ ;  $C = 1,0179$ . For  $F_n = 0$ , according to the equation (7) gives  $y_{c1} = 7,061 \cdot 10^{-3} \text{ m}$ ,  $y_{c2} = 4,044 \cdot 10^{-3} \text{ m}$ . The wheelsets angles according to equation (9) are  $\alpha_1 = -\alpha_2 = -0,786 \cdot 10^{-3} \text{ rad}$ . The pseudo front wheel slippage, calculated with (10), is:  $v_{1x} = 183,325 \cdot 10^{-3}$ ;  $v_{1y} = 0,786 \cdot 10^{-3}$ .

For comparison we studied the movement curve and a fixed axle bogie. Thus, considering  $k_x = k_y = \infty$  and  $k_g = 0$  it obtain:  $y_1 = y_{c1}$ ;  $y_2 = y_{c2}$  and for  $F_n = 0$  it result:  
 $\alpha_1 = \alpha_2 = -a/R = -0,8 \cdot 10^{-3} \text{ rad}$ ;  
 $v_{1y} = v_{2y} = v_y = a/R = 0,8 \cdot 10^{-3}$   
 $y_1 = y_{c2} = y_{c0} \cdot [1 + (f_y/f_x) \cdot (a^2/e^2)] = 5,626 \cdot 10^{-3} \text{ m}$ ;  
 $v_{1x} = v_{2x} = v_x = (f_y/f_x) \cdot [a^2/(e \cdot R)] = 1,365 \cdot 10^{-3}$

There is a decrease in the angle of attack to the elastic drive axles fixed. The difference is small because the comparison was made for a relatively large radius curve. The movement of small radius curves becomes apparent advantage elastic management but also increases the risk of landslides unacceptable. The pulsations critical values determined above can be highlighted and a graphical representation of the function  $f(\omega_c)$ , by the form of Fig. 4. Since whereas the first value of transition from stability to instability hunting movement of the axle, it will be taken into account in calculating the critical speed. Therefore, according to this pulse is critical to obtain the  $v_c = 69.4 \text{ km/h}$ .

## 5. Conclusions

By analyzing the characteristics shown schematically in the figures above it can be concluded that the limited force adherence  $F_{al}$  is less than the adherence force  $F'_{al}$  calculation which was not taken into account the variation of the dynamic axle loads (shown schematically in Figure 3). Likewise, given the dependence of dynamic load train acceleration will result in a worsening of the locomotive the traction feature with increasing the train acceleration. Finally it should be mentioned the fact that slip axle and consequently, the emergence and manifestation of the phenomenon of stick-slip will occur especially in the case of "strength avulsion / pull-out" in place of locomotive train that occurs when starting with a jolt powerful practice that can be amplified by linking locomotive first railway vehicle car of the train without proper tightening torque (of the hook) the traction, allowing a wider broad coupling between the buffers of the locomotive and the first car (vehicle) of the train. This effect can be enhanced also in the case in



which load is the minimum up to the axle of the locomotive. In this context, it is important to note that the load  $Q_l$  the axle locomotive downloaded directly proportional to speed at which the train, as shown in the diagram shown in Figure 2. This is due in particular to lower acceleration with decreasing walking speed according to the graph in Figure 4, and so consequently, and because of the variation of the dynamic nature of the locomotive loads.

The circulation study results a minimum curve radius that does not creep at the wheel - rail. In the case thought is a tendency by the first axle radial alignment. The equations presented indicates radial benefits management where small radius curves but with a corresponding adjustment elastic axle vehicle guidance system.

The results obtained from this study, they concluded that, to overcome the speed of about 160 km / h, the vehicle movement becomes unstable hunting phenomenon which would lead to unacceptable shear load of the tread, and even from endangering traffic safety. The maximum safe movement of a vehicle will be lower with 10 - 15% from the critical velocity to

hunting, if one takes into account the possible change in the elastic characteristics of the system for guiding axles. Relationships set allows to analyze the influence of various design parameters on the movement of the bogie hunting constructive arranges for the extension at higher speed drive system by hunting stability domain. Although the method applied is based on a number of simplifying assumptions, it can be used for fast performance evaluation of engineering bogies. The calculation vests belong to the author work, which is validated experimentally on a number of high-speed bogies analyzed in Railway Rolling Stock Department from the Polytechnic University of Bucharest.

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$$\Psi_1 = \alpha_1 + \frac{a}{R} + \frac{y_1 - y_2}{2.a}; \Psi_2 = \alpha_1 - \frac{a}{R} + \frac{y_1 - y_2}{2.a}; \quad (1)$$

$$F_{1x} = k_x.b.\Psi_1; F_{2x} = k_x.b.\Psi_2 \quad (2)$$

$$M_1 = 2.b.F_{1x} = 2.k_x.b^2.\Psi_1 = 2.k_x.b^2.\left(\alpha_1 + \frac{a}{R} + \frac{y_1 - y_2}{2.a}\right) \quad (3)$$

$$M_2 = 2.b.F_{2x} = 2.k_x.b^2.\Psi_2 = 2.k_x.b^2.\left(\alpha_2 - \frac{a}{R} + \frac{y_1 - y_2}{2.a}\right)$$

$$F_{1y} = k_x.(y_1 - y_{c1}); F_{2y} = k_y.(y_2 - y_{c2}) \quad (4)$$

$$\begin{aligned} T_{1x} = f_x v_{1x} = f_x \left( \frac{\gamma}{r} y_{c1} - \frac{e}{R} \right); \quad T_{1y} = f_y v_{1y} = f_y \alpha_1; \\ T_{2x} = f_x v_{2x} = f_x \left( \frac{\gamma}{r} y_{c2} - \frac{e}{R} \right); \quad T_{2y} = f_y v_{2y} = f_y \alpha_2. \end{aligned} \quad (5)$$

$$C_1 = k_g \cdot y_{c1}; C_2 = k_g \cdot y_{c2} \quad (6)$$

$$k_g = 2 \cdot Q \cdot \gamma / (\gamma_0 \cdot \rho_\chi) = 2 \cdot Q / (\rho_\chi - \rho_s) \quad (7)$$

$$\begin{aligned} C_1 - 2 \cdot F_{1y} + 2 \cdot T_{1y} = 0; \quad 2 \cdot e \cdot T_{1x} - M_1 = 0; \\ C_2 - 2 \cdot F_{2y} + 2 \cdot T_{2y} = 0; \quad 2 \cdot e \cdot T_{2x} - M_2 = 0; \\ 2 \cdot F_{1y} + 2 \cdot F_{2y} - F_n = 0; \quad M_1 + M_2 + 2 \cdot F_{1y} \cdot a - 2 \cdot F_{2y} \cdot a = 0; \end{aligned} \quad (8)$$

$$\begin{aligned} k_g \cdot y_{c1} - 2 \cdot k_y \cdot (y_1 - y_{c1}) + 2 \cdot f_y \cdot \alpha_1 = 0; \\ 2 \cdot e \cdot f_x \cdot \left( \frac{\gamma}{r} y_{c1} - \frac{e}{R} \right) - 2 \cdot k_x \cdot b^2 \cdot \left( \alpha_1 + \frac{a}{R} + \frac{y_1 - y_2}{2 \cdot a} \right) = 0; \\ k_g \cdot y_{c2} - 2 \cdot k_y \cdot (y_2 - y_{c2}) + 2 \cdot f_y \cdot \alpha_2 = 0; \\ 2 \cdot e \cdot f_x \cdot \left( \frac{\gamma}{r} y_{c2} - \frac{e}{R} \right) - 2 \cdot k_x \cdot b^2 \cdot \left( \alpha_2 - \frac{a}{R} + \frac{y_1 - y_2}{2 \cdot a} \right) = 0; \\ 2 \cdot k_y \cdot (y_1 - y_{c1}) + 2 \cdot k_y \cdot (y_2 - y_{c2}) - F_x = 0; \\ 2 \cdot k_x \cdot b^2 \cdot \left( \alpha_1 + \frac{a}{R} + \frac{y_1 - y_2}{2 \cdot a} \right) + 2 \cdot k_y \cdot b^2 \cdot \left( \alpha_2 - \frac{a}{R} + \frac{y_1 - y_2}{2 \cdot a} \right) + 2 \cdot k_y \cdot (y_1 - y_{c1}) a - \\ - 2 \cdot k_y \cdot (y_2 - y_{c2}) a = 0; \end{aligned} \quad (9)$$

$$\begin{aligned} y_{c1} = y_{c0} \left( 1 + \frac{f_y}{f_x} \cdot \frac{a^2}{e^2} \cdot \frac{A}{C} \right) + a \cdot \frac{F_n}{4 \cdot f_y} \cdot \frac{B}{C}; \\ A = 1 + \frac{f_x \cdot a}{k_x \cdot b^2} \cdot \frac{e \cdot \gamma}{r} \cdot \left( 1 + \frac{k_x \cdot b^2}{k_y \cdot a^2} \right); \end{aligned} \quad (10)$$

$$B = f_y \cdot a / (k_x \cdot b^2) - 1;$$

$$C = 1 + \frac{f_x \cdot f_y \cdot a^2}{(k_x \cdot b^2)^2} \cdot \frac{e \cdot \gamma}{r} \cdot \left( 1 + \frac{k_x \cdot b^2}{k_y \cdot a^2} \right);$$

$$y_{c2} = y_{c0} \left( 1 + \frac{f_y}{f_x} \cdot \frac{a^2}{e^2} \cdot \frac{2 - A}{C} \right) + a \cdot \frac{F_n}{a \cdot f_y} \cdot \frac{B + 2}{C} \quad (11)$$

$$\alpha_1 = -\alpha_2 = -(1/C).a/R \quad (12)$$

$$v_{1x} = \frac{v}{r} \left( 1 + \frac{f_y}{f_x} \cdot \frac{a^2}{e^2} \cdot \frac{A}{C} \right) \cdot y_{c0} - \frac{e}{R} = \frac{f_y}{f_x} \cdot \frac{a^2}{e.R} \cdot \frac{A}{C} \quad (13)$$

$$v_{1y} = (1/C).a/R$$

$$T = \sqrt{(f_x \cdot v_x)^2 + (f_y \cdot v_y)^2} = \frac{f.a}{R.C} \sqrt{1 + \frac{a^2}{e^2} \cdot A^2} \leq \mu.Q \quad (14)$$

$$R > \frac{f.a}{C.\mu.Q} \sqrt{1 + \frac{a^2}{e^2} \cdot A^2} \quad (15)$$

$$m_0 \cdot \ddot{\Psi}_1 + (2.\chi.Q/v) \cdot \dot{\Psi}_1 + k_y^* \cdot y_1 - k_y^* \cdot y_2 - (k_y^* \cdot a + 2.\chi.Q) \Psi_1 - k_y^* \cdot \Psi_2 = 0;$$

$$m_0 \cdot \ddot{\Psi}_2 + (2.\chi.Q/v) \cdot \dot{\Psi}_2 + k_y^* \cdot y_2 - k_y^* \cdot y_1 + k_y^* \cdot \Psi_1 + (k_y^* \cdot a - 2.\chi.Q) \Psi_2 = 0;$$

$$I_{0z} \cdot \ddot{\Psi}_1 + (2.\chi.Q/v) \cdot \dot{\Psi}_1 + (k_x \cdot b^2 + k_y^* \cdot a^2) \Psi_1 - (k_x \cdot b^2 - k_y^* \cdot a^2) \Psi_2 - (k_y^* \cdot a - 2.\chi.Q.e^2.\gamma/r) y_1 + k_y^* \cdot a \cdot y_2 = 0; \quad (16)$$

$$I_{0z} \cdot \ddot{\Psi}_2 + (2.\chi.Q.e^2/v) \cdot \dot{\Psi}_2 + (k_x \cdot b^2 + k_y^* \cdot a^2) \Psi_2 - k_y^* \cdot a \cdot y_1 + (k_y^* \cdot a + 2.\chi.Q.e.\gamma/r) y_2 - (k_x \cdot b^2 - k_y^* \cdot a^2) \Psi_1 = 0;$$

$$2 y_1^* = y_1 + y_2; \quad 2 y_2^* = y_1 - y_2;$$

$$2 \Psi_1^* = \Psi_1 + \Psi_2; \quad 2 \Psi_2^* = \Psi_1 - \Psi_2 \quad (17)$$

$$2m_0 \cdot \ddot{\Psi}_1 + \frac{4.\chi.Q}{v} \cdot \dot{\Psi}_2 - 4.\chi.Q \cdot \Psi_1^* = 0;$$

$$2m_0 \cdot \ddot{\Psi}_2 + \frac{4.\chi.Q}{v} \cdot \dot{\Psi}_2 + 4.k_y^* \cdot y_2^* - 4.k_y^* \cdot a \cdot \Psi_1^* - 4.\chi \cdot \Psi_2^* = 0; \quad (18)$$

$$2.I_{0z} \cdot \ddot{\Psi}_1 + \frac{4.\chi.Q.e^2}{v} \cdot \dot{\Psi}_1 + 4.k_y^* \cdot a \cdot \Psi_1^* + 4.\chi.Q \cdot \frac{e.\gamma}{r} \cdot y_1^* - 4.k_y^* \cdot a \cdot y_2^* = 0;$$

$$2.I_{0z} \cdot \ddot{\Psi}_2 + \frac{4.\chi.Q.e^2}{v} \cdot \dot{\Psi}_2 + 4.k_x \cdot b^2 \cdot \Psi_2^* + 4.\chi.Q \cdot \frac{e.\gamma}{r} \cdot y_2^* = 0;$$

$$\begin{aligned}
A_1 &= 4.k_y^* \left(1 - \frac{a^2}{e^2}\right) + 4.k_x \cdot \frac{b^2}{e^2}; \\
B_1 &= 4.k_y^* \cdot 4.k_x \cdot \frac{b^2}{e^2} \left(1 + \frac{a^2}{e^2}\right) + 2.(4.\chi.Q)^2 \cdot \frac{\gamma}{e.r} \\
C_1 &= (4.\chi.Q)^2 \cdot \frac{\gamma}{e.r} \left[4.k_y^* \left(1 + \frac{a^2}{e^2}\right) + 4.k_x \cdot \frac{b^2}{e^2}\right] \\
D_1 &= (4.\chi.Q)^2 \cdot \frac{\gamma}{e.r} \cdot 4.k_y^* \cdot 4.k_x \cdot \frac{b^2}{e^2} + (4.\chi.Q)^4 \left(\frac{\gamma}{e.r}\right)^2
\end{aligned} \tag{19}$$

$$\begin{aligned}
A_1 &= K_x + K_y \\
B_1 &= K_x \cdot K_y + 2.\Gamma \\
C_1 &= \Gamma \cdot (K_x + K_y) \\
D_1 &= \frac{K_x \cdot K_y}{1 + \frac{a^2}{e^2}} \cdot \Gamma + \Gamma^2
\end{aligned} \tag{20}$$

$$\begin{aligned}
&16.m_0^4 \cdot p^8 + 32.m_0^3 \cdot \frac{4.f}{v} \cdot p^7 + 8.m_0^2 \cdot \left[3 \left(\frac{4.f}{v}\right)^2 + m_0 \cdot A_1\right] \cdot p^6 + 4.m_0 \cdot \frac{4.f}{v} \cdot \\
&\left[2 \left(\frac{4.f}{v}\right)^2 + 3.m_0 \cdot A_1\right] \cdot p^5 + \left[\left(\frac{4.f}{v}\right)^4\right] + 6.m_0 \cdot \left[\left(\frac{4.f}{v}\right)^2 \cdot A_1 + 4.m_0^2 \cdot B_1\right] \cdot p^4 + \frac{4.f}{v} \cdot \\
&\left[\left(\frac{4.f}{v}\right)^2 \cdot A_1 + 4.m_0 \cdot B_1\right] \cdot p^3 + \left[\left(\frac{4.f}{v}\right)^2 \cdot B_1 + 2.m_0 \cdot C_1\right] \cdot p^2 + \frac{4.f}{v} \cdot C_1 \cdot p + D_1 = 0
\end{aligned} \tag{21}$$

$$\begin{aligned}
&16.m_0^4 \cdot \omega_c^8 - 8.m_0^2 \left[3 \left(\frac{4.f}{v_c}\right)^2 + m_0 \cdot A_1\right] \cdot \omega_c^6 + \\
&+ \left[\left(\frac{4.f}{v_c}\right)^4 + 6.m_0 \cdot \left(\frac{4.f}{v_c}\right)^2 \cdot A_1 + 4.m_0^2 \cdot B_1\right] \cdot \omega_c^4 - \left[\left(\frac{4.f}{v_c}\right)^2 \cdot B_1 - 2.m_0 \cdot C_1\right] \cdot \omega_c^2 + D_1 = 0;
\end{aligned} \tag{22}$$

$$g(\omega_c) = \left(\frac{4.f}{v_c}\right)^2 = \frac{32.m_0^3 \cdot \omega_c^6 - 12.m_0^2 \cdot A_1 \cdot \omega_c^4 + 4.m_0 \cdot B_1 \cdot \omega_c^2 - C_1}{(8.m_0 \omega_c^2 - A_1) \omega_c^2} \tag{23}$$

$$\begin{aligned}
f(\omega_c) &= 16.m_0^4 \cdot \omega_c^8 - 8.m_0^2 \cdot [3 \cdot g(\omega_c) + m_0 \cdot A_1] \cdot \omega_c^6 + \\
&+ [g^2(\omega_c) + 6.m_0 \cdot g(\omega_c) \cdot A_1 + 4.m_0^2 \cdot B_1] \omega_c^4 - [g(\omega_c) \cdot B_1 - 2.m_0 \cdot C_1] \omega_c^2 + D_1
\end{aligned} \tag{24}$$

$$\chi_x \approx \chi_y = \chi = \frac{300}{\sqrt[3]{Q}} \dots \frac{400}{\sqrt[3]{Q}} \quad (25)$$

Table 1

Axle	1	2	3	4	5	6
$Q_{is}$	$-0.707 \cdot F_0$	$+0.447 \cdot F_0$	$-0.311 \cdot F_0$	$+0.311 \cdot F_0$	$-0.447 \cdot F_0$	$+0.707 \cdot F_0$

Table 2

V[km/h]	R[daN]	$F_a$ [daN]	$F'_a$ [daN]	$-\Delta Q_{ls}$ [daN]	$-\Delta Q_{ld}$ [daN]	$Q_l$ [daN]	$\frac{dV}{dt}$ [ $m.s^{-2}$ ]
0	2696	5405	5639	3821	872	16307	0.158
7.2	2728	5098	5304	3604	816	16580	0.148
14.4	2826	4860	5045	3436	772	16792	0.140
21.6	2988	4671	4840	3302	733	16965	0.133
28.8	3215	4516	4672	3193	702	17105	0.127
36.0	3507	4388	4533	3102	673	17225	0.121
43.2	3864	4281	4416	3027	642	17331	0.116
50.4	4285	4183	4315	2962	616	17422	0.111

Table 3

V [km/h]	0	7,2	14,4	21,6	28,8	36,0	43,2	50,4
$\frac{dV}{dt}$ [ $m.s^{-2}$ ]	0.1581	0.1481	0.1400	0.1331	0.1269	0.1213	0.1160	0.1106

Table 4

$\mu_a$	0.340	0.360	0.380	0.400	0.420	0.440	0.460	0.480	0.486
$F_a$ [daN]	5513	5759	5998	6232	6459	6681	6897	7108	7170
$Q_l$ [daN]	16215	15996	15785	15580	15379	15184	14993	14807	14752

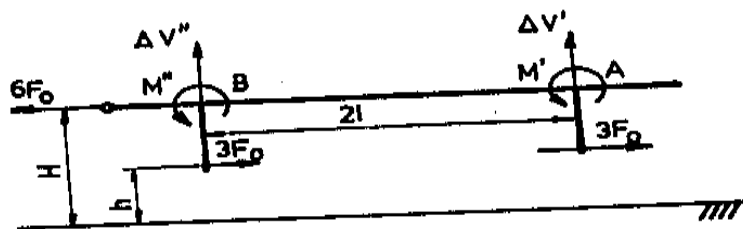


Fig. 1. *The forces and the moments acting on the box locomotive to the circulation on a portion of railway track curve with cant superelevation*

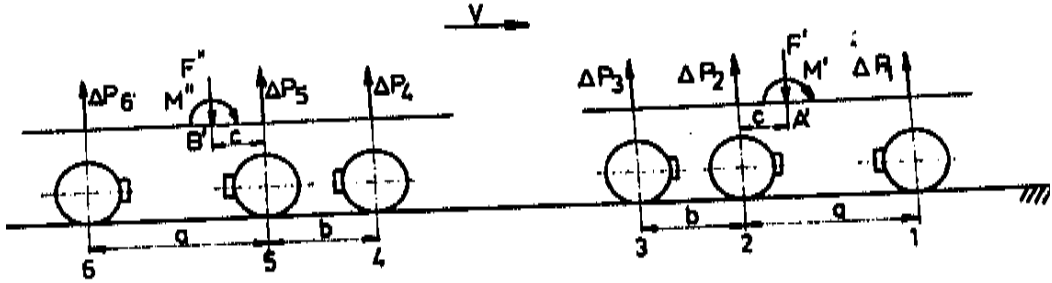


Fig. 2. The forces and the moments acting on the bogie from attacking of a line portion with an uphill declivity

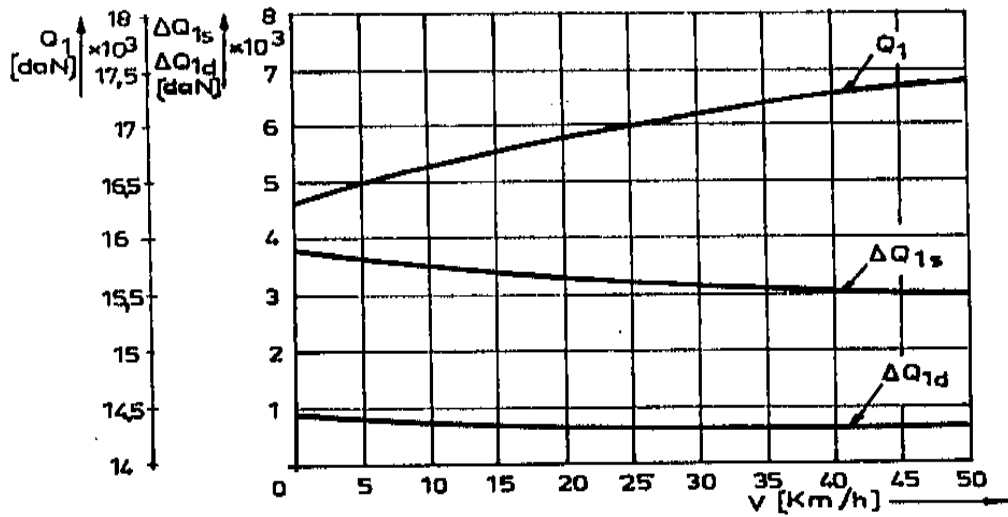


Fig. 3. The curves of variation of axle loads to overcome adhesion

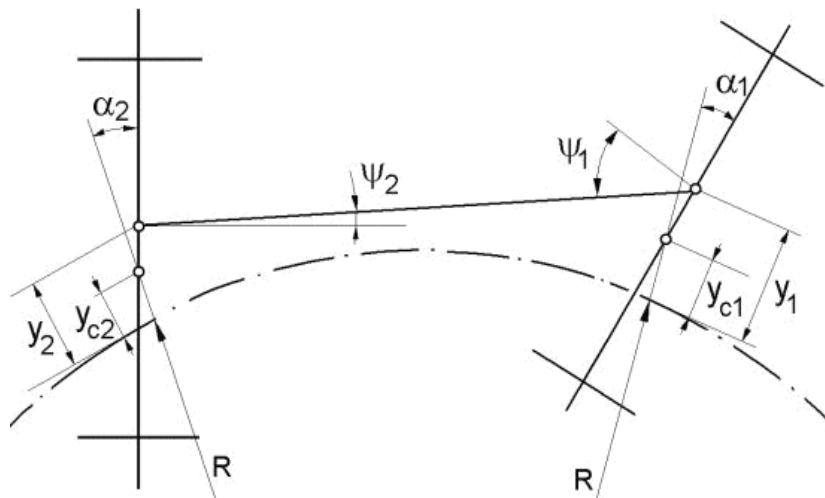


Fig. 4. Driving axle bogie spring curved movements



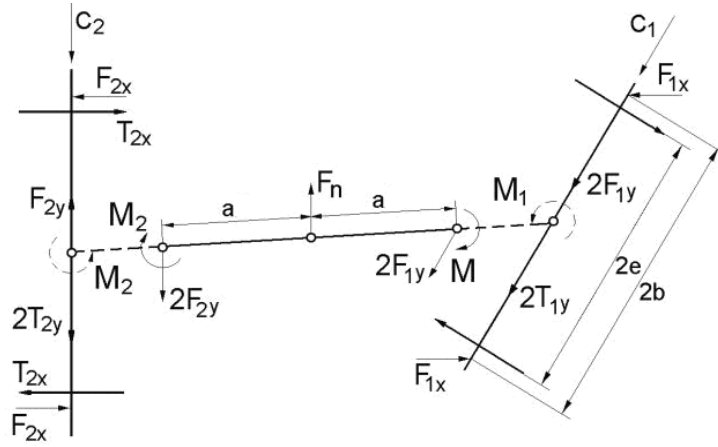


Fig. 5. Forces and moments acting on the axle and bogie frame

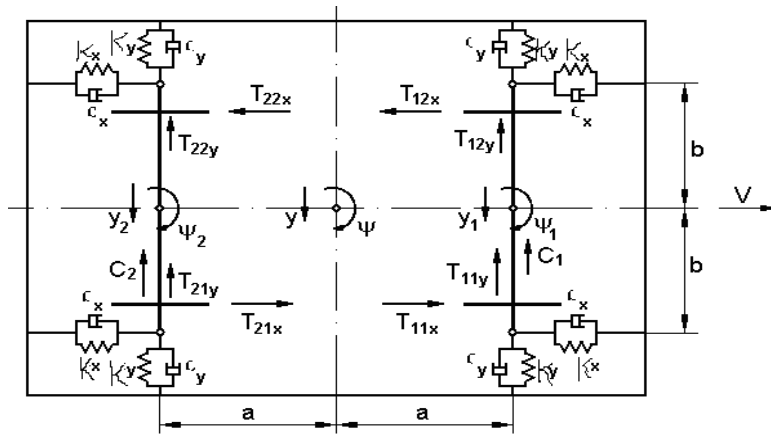


Fig. 6. Forces acting on an elastic driving bogie axle

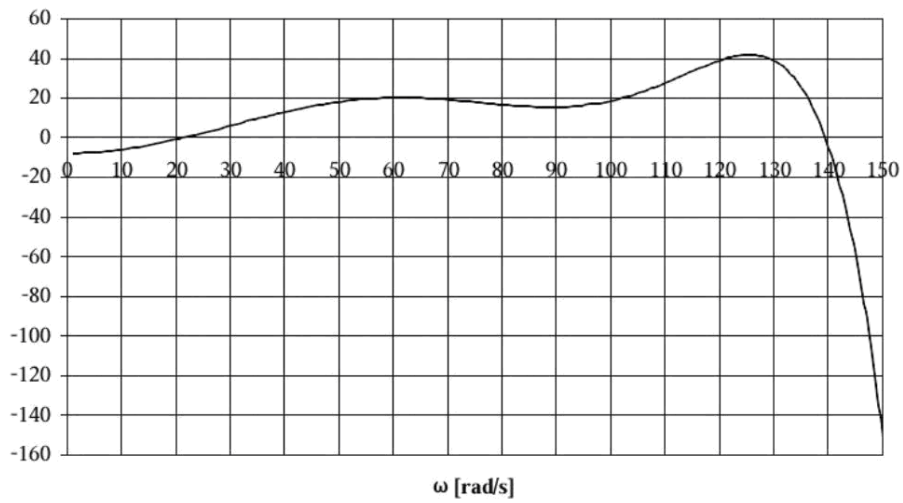


Fig. 7. Establishing the graphic values of the critical pulsations

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