

H_∞ CONTROLLER DESIGN TO CONTROL THE MEAN ARTERIAL PRESSURE AND CARDIAC OUTPUT

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Abstract: *Continuous research actions in the field of biomedical engineering and especially related to physiologic processes modeling, demand better performance and robustness for the designed automated systems. In the current paper, by using H_∞ technique, a robust control system is proposed and designed. Such a technique is used to gain a simultaneous control over two applied cardiovascular variables (i.e. blood pressure and cardiac output) when two drugs (i.e. Sodium Nitroprusside and Dopamine) are injected.*

Key words: *blood pressure, cardiac output, H_∞ controller.*

1. Introduction

There is a real need in knowing better how the biological systems are functioning. By mastering these unveiled details, more possibilities are allowed in having accurate results based on simulating available models.

In the field of medical engineering, complex simulations are done prior to any experimental prototypes that are built or tested. Starting with simple calculus and computation in order to establish the best possible treatments graduation, to complexity of having advanced control over the blood pressure while applying various technologies, the benefits for cardiovascular medicine are numerous. Moreover, all the implemented conclusions based on performed simulations increase the cardiovascular equipment reliability and decrease the chances of a failure or for administering inadequate treatment. As a response to such risks and as a mitigation

action, testing of that equipment is done on models which are simulated on computers.

Once the mathematical models for biological systems, and the appropriate interactive computer simulations have been developed, a new notion - the virtual patient - was defined [4]. Practically put, a virtual patient approximates (and as much as possible close to reality) the physiological variables and the associated evolution which is monitored for a given situation or for an application under study.

As a first step in building up the system proposed in this paper, a model for the human cardiovascular system (CVS) is considered while taking into accounts a big amount of uncertainties and parameters variations. The goal is to simultaneously control the mean arterial pressure (*MAP*) and cardiac output (*CO*) while two drugs Sodium Nitroprusside (*SNP*) and Dopamine (*DOP*) are injected. At the beginning, the process of monitoring the

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drugs infusion was started by using an open loop control system. The downside of this system, which is used in the vast majority of existing hospitals, is that the pump programming must be done manually (human intervention).

The next step was to establish a new closed loop control system. The idea behind this type of system is to have a controllable pump which can provide real-time adjustments and control of drugs infusion. Over time, different solutions have resulted based on studies of the newly proposed algorithms [5], [6], [13], [14], [19]. Thus, to gain the control on *MAP*, experts and some of the aforementioned authors [2], [3], [12], [20], [22], proposed a SISO (single-input and single-output) system which is a simple single variable control system with one input and one output. Some other experts and authors [7], [8], [15], [17], [18], proposed a MIMO (multiple-inputs and multiple-outputs) system in order to get the simultaneous control over *MAP* and *CO* while using *SNP* and *DOP*.

Controller designing is a complex operation due to:

- MIMO system that is supposed to be used.
- Required analysis of the side-effects of *SNP* and *DOP* on *CO* and *MAP*.
- Patient responses' time variation on various drugs dosing.

To have it solved, different strategies have been developed, much of these being treated in the literature. Some of the experts [10], [16], [21], [23], are using a reference adaptive model to control *MAP* and *CO* together. With considerable results, some other prefers the fuzzy control [3], [11], [12]. Another alternative (and a reasonable solution) is represented by the robust control which correctly handles parameters' big variation. In this paper, a robust control strategy based is proposed to gain a simultaneous control

over *MAP* and *CO* with a satisfactory performance.

2. The Physiological System Model

Different cardiovascular system (CVS) models have been defined and classified accordingly. Four (4) of them are detailed hereafter.

The pulse models (the first ones) include all system variables that are oscillating under the hart rate. Important effects are generated for a short time, usually over several heart beats. On the other hand, the variation amplitude's is rather small and the frequency is high, thus being ignored by some applications which handle long-term phenomena. As a consequence, the non-pulse models which represent the second model are more suitable for such applications.

The third ones which are represented by the extended model are equal to a more compact approach. They combine more sub-systems usually tied together, but not necessarily, with other physiological systems strictly related to CVS. As opposite to such models, the reduced models (the forth ones) are limited to a few facets and get more deeply into physiological phenomena details.

2.1. Common Injectable Drugs

To succeed in modeling a complete and useful CVS for automatic control applications, the whole dynamic and side-effects that are generated by the drugs needs to be considered. In some clinical situations, e.g. acute heart failure, the simultaneously control of *MAP* and *CO* is required.

In some other cases, *CO* is increased while *MAP* is decreased, then both of them are maintained to given thresholds. To control them, 2 (two) drugs are simultaneously injected: Dopamine (*DOP*)

- to increase the cardiac output, and Sodium Nitroprusside (SNP) - to expand the blood vessels with a positive influence on the blood pressure.

There is a permanent inter-action between CO and MAP : by increasing CO leads to a gain of MAP , and reducing of MAP determines an increase of CO . In other words, the two entry variables (and that are representing the infusion rate for DOP and SNP) of the pharmacological model are linked into a mutual effect: besides the main effect exercised on the directly controlled drug, there is a side effect on the one which is not directly controlled by the given variable.

2.2. Yu's Compact Model

A compact model which describes possible effects of the inotropic and vasoactive drugs on MAP and CO is the one proposed by Yu [23]. A large number of the model's shapes have been used in different applications to design and simulate control systems. The initial model that was proposed by Yu consists of 3 (three) parts:

- The circulatory system with implicit effect of body specific parameters (i.e. the arterial resistance (AR) and heart rate (HR)) that is monitored in a clinical situation over the controlled MAP and CO .
- The specific drugs infusion rates' effects on variation of body parameters.
- The CVS's natural self-control effects through baroreflex effect.

This model (based on baroreflex effect) is presented in Figure 1. The direct impact of both, DOP and SNP, on AR and HR can be observed. The original model proposed by Yu was derived into an extended and non-pulse model with 2 (two) inputs and 2 (two) outputs and with first order dynamic with dead time.

Input is considered the infusion rates of the applied drugs (DOP and SNP) that are measured as units of $[\mu\text{g}/\text{kg}/\text{min}]$. As

output, the MAP (expressed as units of $[\text{mmHg}]$) and CO (expressed as units of $[\text{mL}/\text{kg}/\text{min}]$) variations that are induced by drugs are monitored.

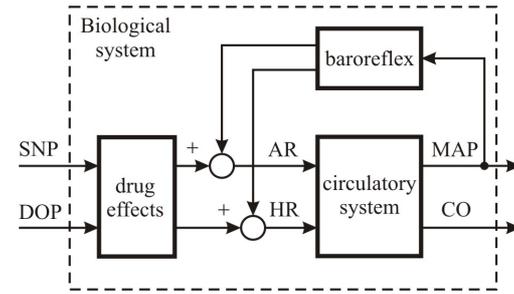


Fig. 1. The CVS block diagram

The whole model is mathematically described by [4]:

$$\begin{bmatrix} \Delta MAP \\ \Delta CO \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} I_{SNP} \\ I_{DOP} \end{bmatrix}, \quad (1)$$

where:

$$G_{ij}(s) = K_{ij} e^{-\tau_{ij}} / (sT_{ij} + 1).$$

The meaning of every parameter used in relation (2) is below provided:

- K_{ij} - is the patient's response to injected drugs;
- T_{ij} - represent the actions' time constants of given drugs over the examined physiological variables;
- τ_{ij} - are delays between the injection moment and the system reaction.

From a medical perspective, benchmarks of MAP and CO variations are computed as a difference between the patient's initial state (MAP_0, CO_0) and his normal state (MAP_{ref}, CO_{ref}):

$$\begin{aligned} \Delta MAP_{ref} &= MAP_{ref} - MAP_0, \\ \Delta CO_{ref} &= CO_{ref} - CO_0. \end{aligned} \quad (3)$$

with $MAP_{ref} = 100$ [mmHg], and $CO_{ref} = 6$ [mL/kg/min].

In Table 1 the parameters' ratings and field variation are displayed.

Table 1
The parameters' ratings and their variations

Parameter	Range	Typical	Units
K_{11}	[-1; -50]	-15	mL/ μ g
K_{12}	[0; 9]	3	mL/ μ g
K_{21}	[-15; 25]	12	mmHg. kg min/ μ g
K_{22}	[1; 12]	5	mmHg. kg min/ μ g
T_{11}	[30; 60]	40	[s]
T_{12}	[30; 60]	40	[s]
T_{21}	[70; 600]	150	[s]
T_{22}	[70; 600]	300	[s]
τ_{11}	[15; 60]	50	[s]
τ_{12}	[15; 60]	60	[s]
τ_{21}	[15; 60]	50	[s]
τ_{22}	[15; 60]	60	[s]

By analyzing the values shown in the Table 1 and the proposed model, some mutual influences between the 2 (two) control loop can be observed. Thus, when

injecting of one drug there is a direct impact on one parameter (for which drug was administered) and a side effect on the other parameter.

3. The CVS Dynamic Model

A new control system structure (Figure 2) results when CO and MAP are combined by adjusting the intravenous SNP and DOP dosage. Highlights of the block diagram are provided hereafter :

- benchmarks for the 2 (two) physiological variables (MAP_{ref} and CO_{ref});
- the initial system state described by MAP_0 and CO_0 ;
- desired values for the 2 (two) physiological variables (ΔMAP_{ref} and ΔCO_{ref});
- dosage for the 2 (two) material (I_{SNP} and I_{DOP});
- all the variations based on the injected material (ΔMAP and ΔCO).

By having this type of system structure, the 2 (two) material variations can be observed. Moreover, no direct control over them is allowed.

For every control loop presented in Figure 2, an H_∞ controller is built, while the materials' side effect is eliminated.

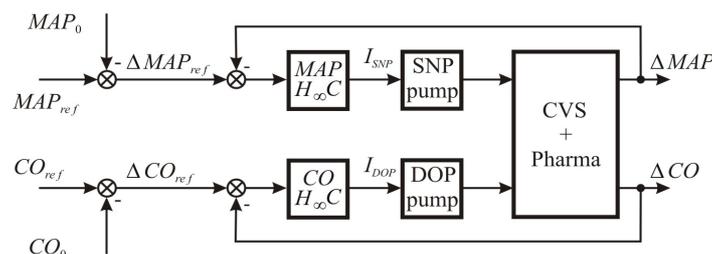


Fig. 2. The block diagram of the control system

3.1. General Description of an H_∞ Controller

A robust control system applicable to, or defining an H_∞ controller is considered to have 2 (two) parts. The known part is represented by the $P(s)$ (which stands for transfer function) and weighting control

structure. The second part is the unknown one, which is given by the controller's model $G_c(s)$. In the next figures all system's parts and their associated functions are shown and detailed.

A general description of the overall system is drawn in Figure 3. In Figure 4, where the weighting control structure is

represented, there are 3 (three) weighting functions (or weighting filters): $W_1(s)$, $W_2(s)$ and $W_3(s)$ [1], [9]:

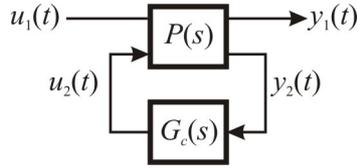


Fig. 3. General structure of H_∞ controller

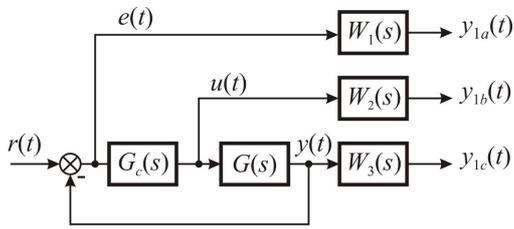


Fig. 4. The block diagram for weighting control structure

To know all the weighting transfer function and to be able to design the H_∞ controller by using the method of extended state space model, more steps, which are described further on, needs to be completed:

1. Re-organize Figure 2 to obtain the control structure represented in Figure 5, where the 3 (three) existing signals (error, input, and output) are prerequisites for the weighting functions. As a consequence, the output vector $y_1 = [y_{1a}, y_{1b}, y_{1c}]^T$ is computed based on all 3 (three) output function:

- $y_{1a}(t)$ is the weighted tracking error signal;
- $y_{1b}(t)$ is the weighted control signal;
- $y_{1c}(t)$ is the weighted plant error signal used for the control system's performance measurement.

2. It is assumed that the process model is given by the state space representation (A, B, C, D). In this case, the state space model for function $W_1(s)$ is provided by $(A_{W_1}, B_{W_1}, C_{W_1}, D_{W_1})$, for $W_2(s)$ by

$(A_{W_2}, B_{W_2}, C_{W_2}, D_{W_2})$, and for $W_3(s)$ is computed based on formula:

$$W_3(s) = C_{W_3}(sI - A_{W_3})^{-1} B_{W_3} + P_m s + \dots P_1 s + P_0.$$

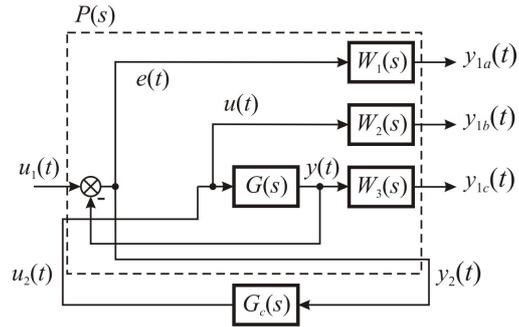


Fig. 5. Two port block diagram for weighting control structure

3. 2 (two) inputs need to be used for the extension of state space model:

$$P(s) = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}. \quad (4)$$

Applying all the aforementioned inputs, prerequisites, and formulas, a complete description in the state space is:

$$\dot{x} = Ax + \begin{bmatrix} B_1 & B_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad (5)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x + \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}.$$

In general, the plant model $P(s)$ can be written as:

$$P(s) = \begin{bmatrix} W_1 & -W_1 G \\ 0 & W_2 \\ 0 & W_3 G \\ I & -G \end{bmatrix}. \quad (6)$$

3.2. The H_∞ Controller Design for MAP and CO. Simulation Results

The studied control system is a feedback control system (Figure 2) with 2 (two) H_∞ controllers for a simultaneous control of changes in *MAP* and *CO* produced by drugs' infusion rates.

The weighting functions are chosen as follows:

- $W_1(s)$ is a low pass filter (to emphasize the tracking error in the low frequency band):

$$W_1(s) = \frac{25}{s^2 + 6s + 25};$$

- $W_2(s)$ is empty;
- $W_3(s)$ is a low pass filter:

$$W_3(s) = \frac{30}{s + 10}.$$

All the studied controllers were designed in Matlab and all the predefined tests passed for those that were correctly designed.

In order to check the system's performance, 9 (nine) individual scenarios which are combining 3 (three) different clinical situations are proposed:

1. Moderate hypertension with moderate heart failure: $MAP_0 = 120$ [mmHg], $CO_0 = 5$ [mL/kg/min];

2. Acute hypertension with moderate heart failure: $MAP_0 = 120$ [mmHg], $CO_0 = 3$ [mL/kg/min];

3. Moderate hypertension with acute heart failure: $MAP_0 = 150$ [mmHg], $CO_0 = 5$ [mL/kg/min];

Additionally 3 (three) types of reaction to infused drugs are considered further:

a) Patients with normal response (the parameters' nominal values presented in Table 1).

b) Patients with slower and less intense response (time constants are 75% bigger than and gain factors are 75% smaller than the parameters' nominal values).

c) Patients with faster and more intense response (time constants are 75% smaller than and gain factors are 75% bigger than the parameters' nominal values).

The simulation results are presented in Table 2. In the standard case ([1a]) the value of *MAP* decreases from 120 [mmHg] to benchmark values in 4 (four) minutes with an overshoot smaller than 0.5% (which can be neglected). After 5 (five) minutes, a slower answer for *CO* is observed. The overshoot in this case can be neglected, too. For the cases [1b] and [1c] a patient's slower answer to the administered medication is recorded (the settling time is bigger by 2, respectively by 4). This forces the injected medication to be increased having, in the same time, a negative impact on the patient overall health.

Simulation results Table 2

	Overshoot		Settling time	
	ΔMAO	ΔCO	ΔMAO	ΔCO
[1a]	0.24%	0.22%	~ 4 min	~ 5 min
[1b]	0.23%	0.42%	~ 5 min	~ 6 min
[1c]	0%	0%	~ 6 min	~ 9 min
[2a]	0.34%	0.22%	~ 5 min	~ 8 min
[2b]	0.2%	0.33%	~ 7 min	~ 9 min
[2c]	0%	0%	~ 7 min	~ 7 min
[3a]	0.4%	0.3%	~ 5 min	~ 6 min
[3b]	0.23%	0.42%	~ 6 min	~ 8 min
[3c]	0%	0.1%	~ 6 min	~ 6 min

4. Conclusions

The human cardiovascular system (CVS) can be successfully controlled with H_∞ control strategy that has been proven itself reliable in controlling the cardiovascular variables. The top characteristics of the presented methodology are the easiness to be understood and time effectiveness, while the model's satisfactory results were successfully demonstrated throughout the current paper.

Acknowledgements

We hereby acknowledge the structural funds project PRO-DD (POS-CCE, 0.2.2.1., ID 123, SMIS 2637, ctr. No. 11/2009) for providing the infrastructure used in this work.

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