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KINEMATIC MODELING OF THE ANKLE-FOOT ARTICULATION BY GEAR MECHANISM

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Abstract: *The mathematical modelling of the mobility at the human ankle joint level is essential for prosthetics and orthotic design. In the present paper, the kinematic modelling of the ankle-foot joint is approached, intending to determine the motion functions of the structures that model the ankle by gear mechanisms. The proposed mechanical models are composed by two bodies that materialize the foot and leg pair, and they can successfully replace the traditional universal and spherical joints (which are frequently used in the field).*

Key words: *gear, ankle, joint, mechanical models.*

1. Human Ankle and Gear Mechanisms

It is well known that the ankle joint allows the foot to perform three movements: flexion - extension around the transversal axis (*y*), pronation - supination around an axis that is tilted in the longitudinal direction (*x*), and pivoting around the vertical axis (*z*) (Figure 1) [1].

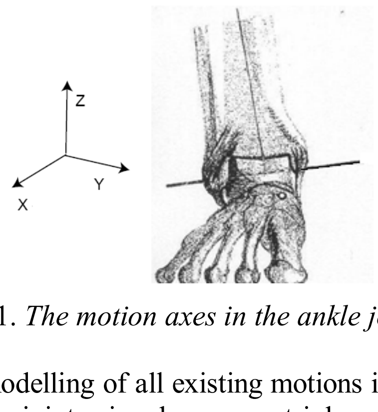

Fig. 1. *The motion axes in the ankle joint*

A modelling of all existing motions in the ankle joint involves a triple pivot connection, around Y (flexion), X (pronation) and Z (pivoting) (Figure 2) [1].

The equivalent spherical joint is not applicable as motor joint due to the difficulties involved by the driving through rotary motors.

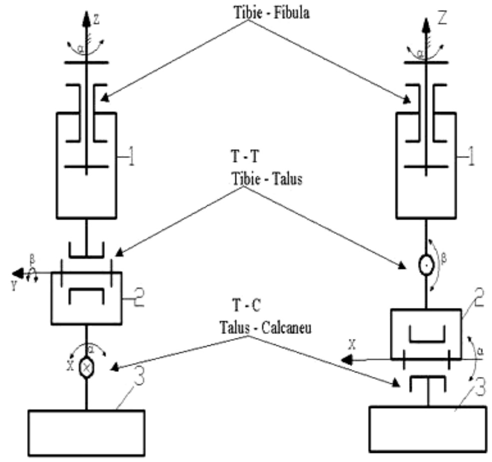

Fig. 2. *Constructive solution - front view and side view*

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The pivoting rotation is a combination of the possible movements in knee and ankle, and in the following this rotation is denoted by α and symbolized by the pair K-A (knee-ankle). The flexion-extension rotation is denoted by β and symbolized by T-T (tibia-talus), while the pronation - supination rotation is denoted by γ and symbolized by T-C (talus-calcaneus).

Under these terms, the purpose of the paper is to synthesize and develop several structures with gear mechanisms, which are able to realize (total or partial) the three necessary rotations.

2. Motion Functions for the Bimobil Model with Gear Mechanism

The motion functions of the gear mechanisms that model the ankle joint will be defined by the **transmission rates** expressions in the kinematic loops for the rotations α, β and γ [2].

Thus, for the **combination α-β (pivoting-flexion)**, in accordance with Figures 3 and 4, the following relationships are obtained:

$$\frac{\omega_2 - \omega_h}{\omega_4 - \omega_h} = -\frac{z_4}{z_2},$$
$$\frac{\omega_1 \frac{z_1}{z_2} - \omega_\beta}{0 - \omega_\beta} = -1 \quad (1)$$
$$\Rightarrow \omega_\beta = \omega_1 \frac{z_1}{2z_2}, i_{1h} = \frac{2z_2}{z_1}.$$

In Figure 3, $z_3 = z_1$, $z_4 = z_2$, ω_α and ω_1 are inputs, while $\omega_\beta = \omega_h$ is output (symmetric differential). Obviously, the rotation angles $\varphi_i = \omega_i \cdot t$ depend on the actuation time.

In Figure 4, ω_1 and ω_3 are inputs, $z_3 = z_1$, $\omega_\alpha = \omega_h$, $\omega_\beta = \omega_2$, thus resulting:

$$\frac{\omega_1 - \omega_h}{\omega_3 - \omega_h} = -1 \Rightarrow \omega_h = \omega_\alpha = \frac{\omega_1 + \omega_3}{2}, \quad (2)$$

$$\alpha = \omega_\alpha \cdot t, \quad \frac{\omega_1 - \omega_h}{\omega_2 - \omega_h} = -\frac{z_2}{z_1},$$
$$\omega_2 = \omega_\beta = \omega_1 \frac{z_1}{z_2} + \omega_h \left(1 - \frac{z_1}{z_2}\right), \quad (3)$$

where: $\beta = \omega_\beta \cdot t$, t - actuation time.

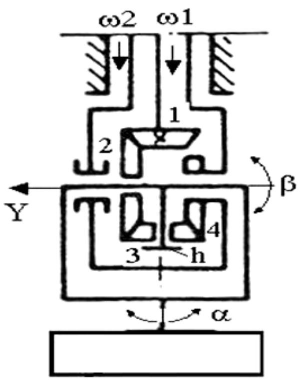


Fig. 3. *Kinematic scheme with symmetrical differential for the angles α - β (variant 1)*

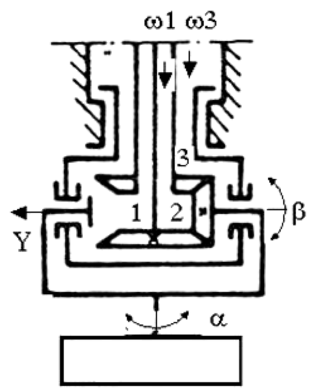


Fig. 4. *Kinematic scheme with symmetrical differential for the angles α - β (variant 2)*

The relationships for **the combination α-γ (pivoting-pronation)** are obtained in accordance with Figures 5, 6, and 7. Thus, in Figure 5, ω_1 and ω_4 are inputs, $z_3 = z_1$, $\omega_\alpha = \omega_3$, $\omega_\gamma = \omega_5$, resulting:

$$\omega_\gamma = \omega_5 = \omega_4 \frac{z_4}{z_5}, \quad (4)$$

the independent parameter being $\gamma = \omega_\gamma \cdot t$,

$$\frac{\omega_2 - \omega_h}{\omega_3 - \omega_h} = \frac{z_2}{z_3}, \quad \frac{\omega_1 \frac{z_1}{z_2} - \omega_4 \frac{z_4}{z_5}}{\omega_3 - \omega_4 \frac{z_4}{z_5}} = \frac{z_3}{z_2},$$

$$\omega_1 \frac{z_1}{z_2} - \omega_4 \frac{z_4}{z_5} = \omega_3 \frac{z_3}{z_2} - \omega_4 \frac{z_4}{z_5} \cdot \frac{z_3}{z_2}, \quad (5)$$

$$\omega_\alpha = \omega_3 = \left[\omega_1 \frac{z_1}{z_2} + \omega_4 \frac{z_4}{z_5} \left(\frac{z_3}{z_2} - 1 \right) \right] \frac{z_2}{z_3}. \quad (6)$$

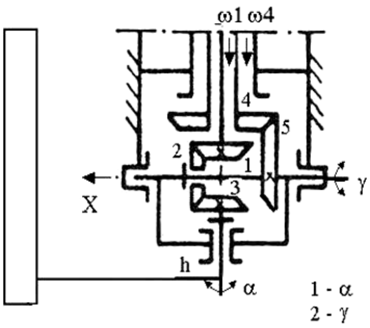


Fig. 5. *The kinematic scheme with planetary bevel gears for the angles α - γ (1st variant)*

In Figure 6, ω_1 and ω_5 are inputs, $\omega_\gamma = \omega_6$, while $\omega_\alpha = \omega_4$ are outputs, resulting:

$$\omega_\gamma = \omega_6 = \omega_h = \omega_5 \frac{z_5}{z_6}, \quad (7)$$

- for independent actuation,

$$\frac{\omega_3 - \omega_h}{\omega_4 - \omega_h} = \frac{z_4}{z_3}, \quad \frac{\omega_1 \frac{z_1}{z_2} - \omega_h}{\omega_4 - \omega_h} = \frac{z_4}{z_3},$$

$$\omega_4 = \left[\omega_1 \frac{z_1}{z_2} + \omega_h \left(\frac{z_4}{z_3} - 1 \right) \right] \frac{z_3}{z_4} \quad (8)$$

- for coupled actuations.

In Figure 7, ω_1 and ω_5 are inputs, $z_7 = z_3$, $\omega_\gamma = \omega_h$, $\omega_\alpha = \omega_4$ - outputs, resulting:

$$\frac{\omega_3 - \omega_h}{\omega_7 - \omega_h} = -1, \quad \frac{\omega_1 \frac{z_1}{z_2} - \omega_h}{\omega_5 \frac{z_5}{z_6} - \omega_h} = -1,$$

$$\omega_h = \omega_1 \frac{z_1}{2z_2} + \omega_5 \frac{z_5}{2z_6}; \quad (9)$$

$$\frac{\omega_3 - \omega_h}{\omega_4 - \omega_h} = \frac{z_4}{z_3},$$

$$\omega_4 = \omega_\alpha = \omega_1 \frac{z_1}{z_2} \frac{z_3}{z_4} + \omega_h \left(\frac{z_4}{z_3} - 1 \right) \quad (10)$$

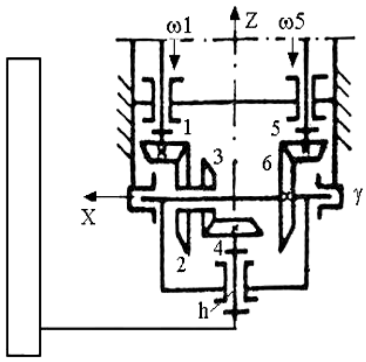


Fig. 6. *The kinematic scheme with planetary bevel gears for the angles α - γ (2nd variant)*

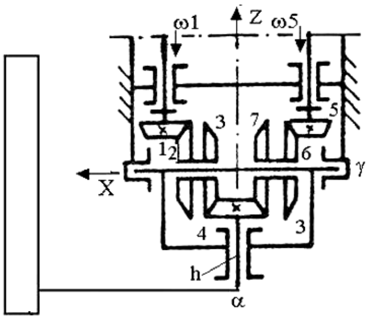


Fig. 7. *The kinematic scheme with planetary bevel gears for the angles α - γ (3rd variant)*

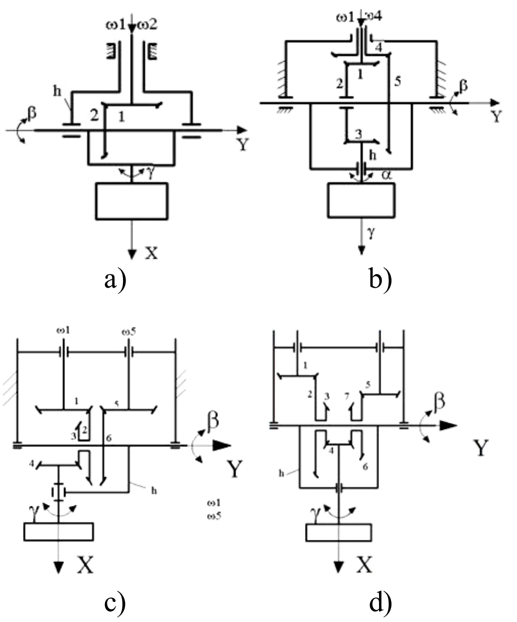
The relationships for the **combination β-γ (flexion-pronation)** are obtained in accordance with Figure 8 [3], where ω_1 and ω_h are inputs, $\omega_\gamma = \omega_h$ - outputs, and $\omega_\beta = \omega_2$, resulting:

$$\frac{\omega_1 - \omega_h}{\omega_2 - \omega_h} = \frac{z_2}{z_1},$$

$$\omega_\beta = \omega_2 = \omega_1 \frac{z_1}{z_2} + \omega_h \left(1 - \frac{z_1}{z_2}\right), \qquad (11)$$

for $\omega_h = 0$, $\omega_\beta = \omega_1 \frac{z_1}{z_2}$, $\omega_1 = 0$,

$$\omega_\beta = \omega_h \left(1 - \frac{z_1}{z_2}\right). \qquad (12)$$



a) b) c) d)

Fig. 8. *The kinematic scheme with bevel gear mechanism for the angles β - γ*

In Figure 8b, $z_3 = z_1$, ω_1 and ω_4 are inputs, $\omega_\gamma = \omega_3$ – outputs, and $\omega_\beta = \omega_h$, resulting:

$$\frac{\omega_1 - \omega_h}{\omega_3 - \omega_h} = -1, \quad \frac{\omega_1 - \omega_4 \frac{z_4}{z_5}}{\omega_3 - \omega_4 \frac{z_4}{z_5}} = -1,$$

$$\omega_3 = 2\omega_4 \frac{z_4}{z_5} - \omega_1, \qquad (13)$$

for $\omega_1 = 0$,

$$\omega_\gamma = 2\omega_4 \frac{z_4}{z_5}, \qquad (14)$$

for $\omega_h = 0$,

$$\omega_\gamma = -\omega_1. \qquad (15)$$

In Figure 8c, ω_1 and ω_5 are inputs, $\omega_4 = \omega_\gamma$ – outputs, and $\omega_6 = \omega_h = \omega_\beta$, resulting:

$$\frac{\omega_3 - \omega_h}{\omega_4 - \omega_h} = \frac{z_4}{z_3}, \quad \frac{\omega_1 \frac{z_1}{z_2} - \omega_5 \frac{z_5}{z_6}}{\omega_4 - \omega_5 \frac{z_5}{z_6}} = \frac{z_4}{z_3},$$

$$\omega_4 = \omega_\gamma = \omega_1 \frac{z_1}{z_2}\frac{z_3}{z_4} + \omega_5 \frac{z_5}{z_6}\left(1 - \frac{z_3}{z_4}\right). \qquad (16)$$

$$\omega_1 = 0, \ \omega_\gamma = \omega_5 \frac{z_5}{z_6}\left(1 - \frac{z_3}{z_4}\right) \qquad (17)$$

$$\omega_5 = 0, \ \omega_\gamma = \omega_1 \frac{z_1}{z_2}\frac{z_3}{z_4}. \qquad (18)$$

In Figure 8d, ω_1 and ω_5 are inputs, $\omega_\gamma = \omega_4$ - outputs, $z_7 = z_3$, and $\omega_\beta = \omega_h$, thus resulting:

$$\frac{\omega_3 - \omega_h}{\omega_7 - \omega_h} = -1, \quad \frac{\omega_1 \frac{z_1}{z_2} - \omega_h}{\omega_5 \frac{z_5}{z_6} - \omega_h} = -1,$$

$$\omega_h = \omega_\beta = \omega_1 \frac{z_1}{2z_2} + \omega_5 \frac{z_5}{2z_6} \qquad (19)$$

$$\omega_1 = 0, \ \omega_4 = \omega_5 \frac{z_5}{2z_6}, \qquad (20)$$

$$\omega_5 = 0, \ \omega_4 = \omega_1 \frac{z_1}{2z_2}. \qquad (21)$$

3. Motion Functions for the Trimobil Model with Gear Mechanism

For the general combination **α-β-γ, (pivoting-flexion-pronation)**, the specific relationships are obtained in accordance with Figures 9, 10 and 11. Thus, in Figure 9 [5], ω_h, ω_1 and ω_4 are inputs, ω_α, $\omega_\beta = \omega_h = \omega_5$ and $\omega_\gamma = \omega_3$ are outputs, and $z_3 = z_1$, resulting:

$$\omega_\beta = \omega_h = \omega_4 \frac{z_4}{z_5}, \quad \frac{\omega_1 - \omega_h}{\omega_3 - \omega_h} = -1,$$

$$\frac{\omega_1 - \omega_4 \dfrac{z_4}{z_5}}{\omega_3 - \omega_4 \dfrac{z_4}{z_5}} = -1,$$

$$\omega_3 = \omega_\gamma = 2\omega_4 \frac{z_4}{z_5} - \omega_1 . \tag{22}$$

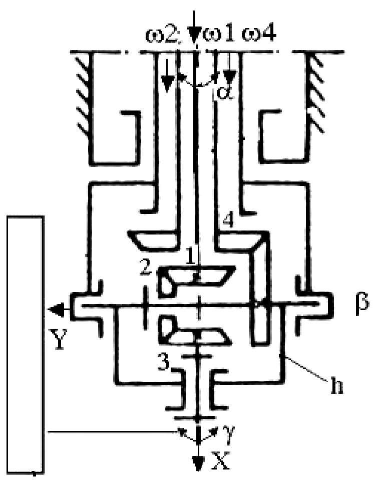


Fig. 9. *The kinematic scheme for the angles pair α - β - γ (1st variant)*

In Figure 10, ω_α, ω_7 and ω_1 are inputs, $\omega_\gamma = \omega_6$, and $\omega_\beta = \omega_h = \omega_{10}$ - outputs, resulting:

$$\omega_h = \omega_7 \frac{z_7}{z_8} \frac{z_9}{z_{10}}, \quad \frac{\omega_5 - \omega_h}{\omega_6 - \omega_h} = \frac{z_6}{z_5},$$

$$\frac{\omega_1 \dfrac{z_1}{z_2} - \omega_h}{\omega_6 - \omega_h} = \frac{z_6}{z_5},$$

$$\omega_5 = \omega_1 \frac{z_1}{z_2} \frac{z_3}{z_4} \frac{z_5}{z_6} + \omega_h \left(\frac{z_6}{z_5} - 1 \right). \tag{23}$$

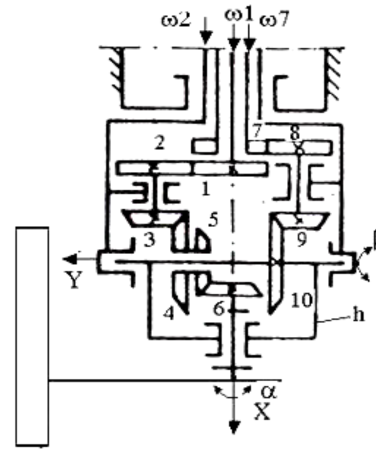


Fig. 10. *The kinematic scheme for the angles pair α - β - γ (2nd variant)*

In Figure 11, $z_9 = z_4$, ω_α, ω_1 and ω_6 are inputs, $\omega_\beta = \omega_h$ and $\omega_\gamma = \omega_5$ - outputs, resulting:

$$\frac{\omega_4 - \omega_h}{\omega_9 - \omega_h} = -1, \quad \frac{\omega_1 \dfrac{z_1}{z_2} \dfrac{z_3}{z_4} - \omega_h}{\omega_6 \dfrac{z_6}{z_7} \dfrac{z_8}{z_9} - \omega_h} = -1,$$

$$\omega_h = \omega_1 \frac{z_1}{2z_2} \frac{z_3}{z_4} + \omega_6 \frac{z_6}{2z_7} \frac{z_8}{z_9}, \tag{24}$$

$$\frac{\omega_4 - \omega_h}{\omega_5 - \omega_h} = \frac{z_5}{z_4},$$

$$\omega_5 = \omega_4 + \omega_h \left(\frac{z_5}{z_4} - 1 \right). \tag{25}$$

For the positioning angles of the leg, which are obtained by gearing the wheels, there are simple relationships, as follows [4]:

$$\alpha = \omega_\alpha \cdot t + \alpha_0,\ \beta = \omega_\beta \cdot t + \beta_0,$$
$$\gamma = \omega_\gamma \cdot t + \gamma_0. \quad (26)$$

For a constant transmission ratio $i = \omega_i / \omega_e$, the motion functions (26) are linear (depending on time). The slope of this line is given by the transmission ratio value.

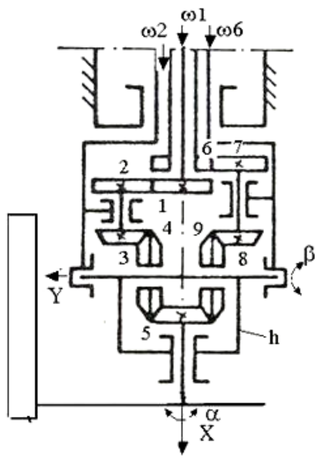


Fig. 11. *The kinematic scheme for the angles pair α - β - γ (3rd variant)*

4. Conclusions

The modelling of the lower limbs joints is a particular challenge for researchers due to the complexity of this system. The objective of this study was to develop an innovative model of the ankle-foot articulation, which is defined by functional performance and bio-fidelity.

The geometrical parameters that define the system (the number of teeth), the input kinematical parameters (the angular displacements or velocities), and the output parameters (the transmission ratios) have been presented, the kinematic functions being expressed by specific methods from the mechanisms theory.

Considering the main movements, the combinations of flexion - pronation (β - γ) are recommended. These structures are relatively simple, so easier to adopt. The results show that the proposed model offers an accurate characterization of the ankle joint, and it can be used for the simulation of the joint movements, as well as for the control design and development.

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