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# ABOUT THE OWN FREQUENCY OF AN OSCILLATOR

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***Abstract:*** *This paper deals with aspects connected to the own frequency. The oscillator, to which the own frequency is analysed, is formed of a cylindrical and elastic sleeve, having at one its sides a body. When deducing the characteristic reaction of the oscillator's frequency, there were taken into account both the mass of the elastic sleeve and the mass of the body. Further on, I present the relation of the own frequency to an acoustic resonator (Helmoltz). This relation is deduced, taking into consideration both the theoretical and practical aspects. The experimental determinations have proved the available relation of the oscillator's own frequency, in which were taken into account both the mass of the elastic sleeve and the mass of the body.*

***Key words:*** *frequency, oscillator, resonator.*

## 1. Introduction

This paper deals with aspects connected to the own frequency of an oscillator. In the first part, I demonstrate that when deducing the own frequency of an oscillator, one must take into consideration, not only the mass of the body, which is a part of the oscillator, but also the mass of the spring (cylindrical sleeve). In the second part of the paper, I present, analysing both the theoretical and practical aspects, the way in which the own frequency of an acoustic resonator, considered as being an oscillator, is determined.

## 2. Theoretical and Practical Aspects

An oscillator [3] can be represented in a schematic way, as shown in Figure 1, where:

$l_0$ - the initial length of the spring;
$x_0$ - the extension of the whole spring;
$da$ - an element of length from the spring;
$a$ - the length of a part/fraction of an spring;
$x$ - the extension that corresponds to the "a" part from the spring;
$m$ - the mass of the spring;
$M$ - the mass of the body attached to the spring.

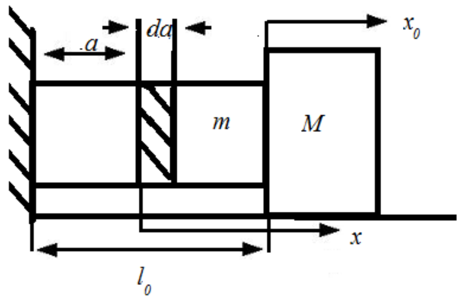


Fig. 1. *Oscillator*

In accordance with the Hooke's law:

$$x_0 = \frac{Fl_0}{ES}, \tag{1}$$

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$$x = \frac{Fa}{ES}, \quad (2)$$

where:

$E$ - the longitudinal module of elasticity (Young's module);

$S$ - the section of the spring (of the cylindrical, elastic sleeve);

$F$ - the force that deforms.

Taking [5] into account the relations (1) and (2), between the extensions $x_0$ and $x$, there is a dependence given by the next relation:

$$x = \frac{x_0}{l_0} a. \quad (3)$$

The movement of the oscillator is an oscillatory movement with the amplitude, $A$, constant, because the resistance forces can be neglected, which means that only the elastic force ($f = -ky$) acts on the oscillator.

For the oscillator presented in Figure 1, the fundamental principle of dynamics is written as it follows:

$$M \frac{d^2 y}{dt^2} + ky = 0, \quad (4)$$

where:

$M$ - the mass of the body attached to the spring:

$\frac{d^2 y}{dt^2} \equiv a$ - the acceleration of the movement;

$y$ - the elongation of the oscillatory movement;

$k$ - the elastic constancy.

The solution of the differential equation that characterises the oscillator from Figure 1 is given by the relation:

$$y = A \sin \omega t, \quad (5)$$

where:

$y$ - is the elongation of the oscillatory movement;

ω - the pulsation;

$t$ - the time.

Replacing the relation (4), we get the pulsation of oscillation of the oscillator, given by the equation:

$$\omega = \sqrt{\frac{k}{M}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k}{M}}, \quad (6)$$

where $f$ represents the frequency of oscillation of the oscillator.

The speed [6] of the oscillatory movement can be acquired by deriving, according to time, the elongation of the movement given by the relation (5), and we finally get:

$$v = \frac{\partial y}{\partial t} \equiv \dot{y} = \omega A \cos \omega t. \quad (7)$$

The acceleration of this movement is given by the relation:

$$a = \frac{d^2 y}{dt^2} + = -\omega^2 A \sin \omega t. \quad (8)$$

Due to the fact that the resistance forces can be neglected, the total energy of the oscillator (the relation 9) is conserved, thus it remains constant.

$$V = T + U = T_{\max.} = U_{\max.} = \text{ct}, \quad (9)$$

where:

$V$ - the total energy of the oscillator;

$T$ - the kinetic energy of the oscillator;

$U$ - the potential energy of the oscillator.

The total energy $V$, is given by the relation:

$$V = \frac{kA^2}{2}, \quad (10)$$

where:

$k$ - the elastic constancy;

$A$ - the amplitude of the oscillatory movement.

When calculating the kinetic energy, one takes into consideration both the kinetic energy of the body of the mass $M$ and the kinetic energy that corresponds to the spring of the mass $m$. The kinetic energy, $T_1$ that corresponds to the spring of the mass $m$ is calculated starting from the relation:

$$dT = \frac{dm}{2}\dot{x}^2 = \frac{1}{2}\frac{m}{l_0}da\frac{a^2}{l_0^2}\dot{x}_0^2, \tag{11}$$

where:

$dm$ - the elementary mass corresponding to the length $da$;

$dT$ - the kinetic energy corresponding to the element of the mass $dm$;

$\dot{x}$ - the speed corresponding to the extension $x$;

$\dot{x}_0$ - the speed corresponding to the extension $x_0$.

Adding the relation (11), we get:

$$T_1 = \frac{1}{2}\frac{m\dot{x}_0^2}{l_0^3}\int_0^{l_0} a^2 da = \frac{1}{2}\frac{m}{3}\dot{x}_0^2. \tag{12}$$

The kinetic energy, $T_2$ that corresponds to the body of the mass $M$, is given by the relation:

$$T_2 = \frac{M\dot{x}_0^2}{2}, \tag{13}$$

Taking into account the relations (12) and (13), the maximum kinetic energy of the oscillator is:

$$T_{max} = \frac{1}{2}\frac{m\dot{x}_0^2}{3} + \frac{M\dot{x}_0^2}{2}. \tag{14}$$

The maximum speed $\dot{x}_0$ is given by the relation:

$$\dot{x}_0 = \omega A, \tag{15}$$

where ω is the pulsation of oscillation of the oscillator.

By introducing the relations (13) and (14) in the relation (9), we get:

$$\frac{kA^2}{2} = \frac{1}{2}\frac{m\omega^2 A^2}{3} + \frac{M\omega^2 A^2}{2}. \tag{16}$$

The own pulsation results from the relation (15):

$$\omega = \sqrt{\frac{k}{M + \frac{m}{3}}}. \tag{17}$$

In almost all the specialized books, the own pulsation is determined without taking into consideration the mass of the spring, as in the relation (6).

In order to demonstrate that relation (1) is much closer to the values that we can get in an experimental way, further on, I am going to determine [4], according to some theoretical and practical aspects, the own pulsation of an acoustic resonator.

Resonators absorb [1] a maximum quantity of acoustic energy when the frequency of the sound wave is equal to the own frequency of the resonator, and this situation corresponds to the phenomenon of resonance. The resonator from Figure 2 can be considered as an oscillator with a single degree of liberty, where the mass of the air from the cavity of volume $V$ acts as an spring (resort), and the mass m is the mass of the air from the neck. The mass of the air from the neck of the resonator is given by the relation:

$$m = \rho_0 S l, \tag{18}$$

where:

$\rho_0$ - the density of the air;

$l$ - the length of the neck;
$S$ - the section of the neck.

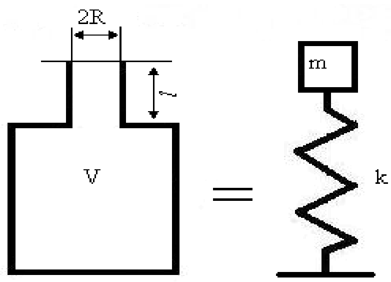


Fig. 2. *The Helmoltz resonator*

It is considered [2] that the movement takes place in the neck of the resonator and the force of returning in the position of equilibrium comes completely from the difference of pressure of the air from the volume $V$.

The return force can be calculated with the relation:

$$F = dp\pi R^2 = dpS\,, \tag{19}$$

where:
$R$ - the radius of the neck;
$dp$ - the pressure variation.

During the movement [10], the air from the cavity $V$ goes through an adiabatic process, which leads to the following way of writing:

$$pV^{\gamma} = \text{const}\,, \tag{20}$$

where $\gamma$ is the exponential adiabatic, which - for the air - has the value $\gamma = 1, 4$.

Turning the relation (19) into a logarithm and then making a difference, we get:

$$\ln p + \gamma \ln V = const. \Rightarrow$$
$$\Rightarrow \frac{dp}{p_0} + \gamma \frac{dV}{V_0} = 0\,, \tag{21}$$

where:
$p_0$ - the equilibrium pressure;
$V_0$ - the volume of the air from the cavity to the equilibrium;
$dV = Sx$ - the volume variation;
$x$ - the change of position towards the equilibrium position of the mass $m$.

From the relation (20), one obtains the pressure variation that is given by the relation:

$$dp = -\frac{\gamma p_0 dV}{V_0} = -\frac{\gamma p_0 Sx}{V_0}\,. \tag{22}$$

If we replace the relation (21) in the relation (17), we get the return force, that is an elastic force given by the relation:

$$F = -\frac{\gamma p_0 S^2 x}{V_0} = -\mathrm{k}\,x, \tag{23}$$

where $k$ (the elastic constancy) is the rigidity of the oscillator.

The equation that characterises the movement of the oscillatory system from the Figure 2 is as it follows:

$$\rho_0 Sl \frac{d^2 x}{dt^2} + \mathrm{k}\,x = 0\,. \tag{24}$$

The solution of this equation can be written in the form:

$$x = A \sin \omega t\,. \tag{25}$$

If we replace [4] the solution given by the relation (24) in the Equation (23), we get:

$$\rho_0 Sl\omega^2 = \mathrm{k} = -\frac{\gamma p_0 S^2}{V_0}\,. \tag{26}$$

Due to the fact [8] that the gas from the cavity goes through the adiabatic process, the propagation speed of the sound (the sound wave) is given by the relation:

$$c = \sqrt{\frac{\gamma p_0}{\rho_0}}\,, \tag{27}$$

in which:

$p_0$ - the air pressure in normal conditions ($p_0 \cong 10^5$ N/m$^2$);

$\rho_0$ - the air density in normal conditions ($\rho_0 \cong 1.3$ kg/m$^3$);

$c$ - the propagation speed of the sound in the air in normal conditions ($c \cong 340$ m/s).

From the relation (21), one obtains the oscillation frequency, whose formula is:

$$\omega = c\sqrt{\frac{S}{V_0 l}} \Rightarrow f = \frac{c}{2\pi}\sqrt{\frac{S}{V_0 l}} . \qquad (28)$$

If we replace in the relation (27), the relations (17) and (25), we get the relation (6), that, as it is known when determining the own frequency, the mass of the spring will be neglected.

In order to get accurate results, Hemholtz proposed a practical relation of calculation, in the form:

$$f = \frac{c}{2\pi}\sqrt{\frac{S}{V_0(l + 1.6R)}} . \qquad (29)$$

If one makes a comparison between the relation (27) and the relation (28) it will be noticed that the length $l$ is replaced with the „effective" (equivalent) length of the neck, which is equal to the real length plus 1.6 multiplied with the radius of the neck.

Helmholtz made this correction, because of the fact that in the oscillatory movement there is involved, besides the air from the neck of the resonator, a mass of air from the cavity of the resonator also.

The resonator [7] from Figure 2 is characterized by the following values: $V_0 = 10^{-4}$ m$^3$; $l = 5 \cdot 10^{-2}$ m; $R = 2 \cdot 10^{-2}$ m.

Replacing in the relation (23) and the relation (24) the values that are characteristic to the resonator, we will get two values of the resonance frequency.

The value got from the relation (27) is $f_t = 860$ Hz, and the one got from the relation (28) is $f_p = 670$ Hz. The real value of the resonance frequency is $f_p$.

Practically, the absorption of the resonator is made possible only by means of a narrow waveband of frequencies, situated around the resonance frequency (Figure 3).

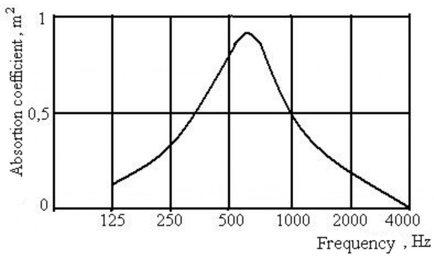


Fig. 3. *The graphics of variation of the absorption coefficient function of frequency*

In Figure 3 there is represented the graphics of variation of the absorption coefficient function of frequency, graphics that is rendered by means of the values that were obtained in an experimental way.

From the graphics [9], one can notice that the maximum value of the absorption coefficient corresponds to the frequency that has a value of approximately $f_p = 670$ Hz. It can be observed that this value is the same as the value obtained in the relation (28), as far as the size order is concerned.

Taking into [4] account the relations (17), (25), (26), then the relation (28) can be written also:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{\rho_0 Sl + \rho_0 S1,6R)}} . \qquad (30)$$

As it can be noticed the first term from the denominator represents the mass of the air $m$ from the neck of the resonator.

Replacing the values of the sizes from the second term, we get $\rho_0 S 1.6R \cong 5.10^{-5}$ kg .

Knowing the value of the cavity volume and the air density, we get the mass of the air $M \cong 1.3 \cdot 10^{-4}$ kg from the cavity of the resonator. Making a comparison between the two values, it can be observed that the second term from the relation (29) is approximately equal to a third from the mass of the air from the resonator cavity. Thus, the relation (29) is identical to the relation (6).

## 3. Conclusions

As it can be noticed, this paper's aim was to prove an aspect - that the theoretical models characterise the systems only in a general way. Applying these theoretical models to some concrete, practical models leads to the negligence or the addition of some terms in order to make the theoretical values of some sizes to be as close to the practical values that were obtained for the respective sizes.

In our case, in order to make the theoretical values of the own frequency of the acoustic resonator, considered as being an oscillator, to coincide with the experimental values of the own frequency, both the mass of the body attached to the spring (the mass of the air from the neck of the resonator) and the mass of the spring (the mass of the air from the resonator cavity), had to be taken into consideration.

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