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OPTIMIZATION OF RECTANGULAR SLOT AREA AT ELECTRICAL MACHINES

M. COSTIN[1, 3] **I. VONCILA**[2]

Abstract: *The electrical machines design take place a major role in energy conversion systems. In this paper a procedure for optimization of rectangular slot area of stator has been developed. By a proper analytic approach, it has been founded that slot area may be expressed as a function of specific Joule losses. According to these work, economic criteria for total cost or consumption has been also developed. Numerical simulations performed in Matlab software for a case study confirm our theoretical approach.*

Key words: *rectangular slot area, permanent magnet synchronous machine, optimization, Poynting vector.*

1. Introduction

Energy conversion systems with electrical machines are present in a large area of applications: industrial electric drive with adjustable speed for motion control, power plant application, robotics, conversion of renewable resources, special applications etc. In all fields of applications, the design of electrical machines takes place particularly requirements according to specificity and particularity of applications [2], [3], [5], [11] and [16].

Over time many approaches has been developed that lead to developing of several design methodology of electrical machines [2], [5] and [14]. A crystallization of the first approach has been appearing on the start of 20th century. This one has been developed according to electric circuit theory. This methodology has been successful applied in practice only with the help of monograms obtained by practical experiences of already machines realized [5]. In this situation the experiences of designer play a major role. But, there appear some drawbacks, because there is no information - as magnetic field distribution, or others - on details. That is why the modern solution is performed by solving partial differential equations of field, usually, with the help of finite elements method. Sometimes, when the geometry of machine is not a complicate one, it is using the analytic form of partial differential equations. This result is a strong one, but is valid for a short class of situations where the simplification assumptions are taken into account.

In the effective design procedure of electrical machines must be fallowed several steps [14]: electromagnetic computing,

[1] Research Centre "Integrated Systems of Energy Conversion and Advanced Control of Complex Process", *Dunărea de Jos* University of Galaţi.
[2] Dept. of Automatic Control and Electrical Engineering, *Dunărea de Jos* University of Galaţi.
[3] Cross-border Faculty of Humanities, Economic and Engineering Sciences of Cahul, *Dunărea de Jos* University of Galaţi

thermal computing, and mechanical computing and performances tests. In the electromagnetic step are computed the main geometric dimensions (stator inner diameter and ideal machine length) and electromagnetic stress. The thermal one is about heat transfer in various parts of machine and computing of the fan of the machine. Mechanical stress step design is about mechanical stresses that are developed according to motion energy transients and steady state. Finally, at the end of the project is checked if the machine performances match the desired one.

In general, the design procedure is well known for a long time, but many problems rise on the attention of researchers [12], [13].

The goal of the paper is to propose an alternative approach performed for develop a procedure of rectangular slot area based on electromagnetic stress.

The subject of the paper is about optimization of rectangular slot area of permanent magnet synchronous machine. A first approach of this subject - based on a thermal computing - has been presented in [8]. In the current paper the work is continued on an electromagnetic forces approach.

2. Slot Area Computing

From the starting project of any electrical machines design, requirements are imposed. The design methodology considered on the rest of the paper is based on electric circuit equivalent scheme design methodology.

Now, from the electromagnetic point of view, in electrical machine is fallowed to find the adequate values of electromagnetic stress as: current density J_1 [A/mm^2], linear current density A_1 [A/mm], and magnetic flux density on the air gap B_δ [T]. If is known the electromagnetic stress, it is founded the geometric dimensions of the machine.

Based on the practical experiences of electrical machines design, we have [12], [13]:

$$A_1 \cdot J_1 \approx 3000\,(I.P.44) \div 3200\,(I.P.23)\left[\frac{\mathrm{A}^2}{\mathrm{mm}^2}\right], \tag{1}$$

where, *I.P* is the international protection degree.

Based on the last condition, we compute the thermal flow of losses in the active part of stator winding:

$$Q_{1T} \approx 3000(I.P.44) \div 8000(I.P.23)\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right], \tag{2}$$

where the dependence relation of thermal flow of losses is described by relation:

$$Q_{1T} = \rho_1 A_1 J_1\,, \tag{3}$$

where ρ represent the cooper resistivity.

The work is completed with a general dependence of both all important geometric dimensions and other kind of quantities, as a dependence of a general factor defined according to relationship [13]:

$$k_f = 1 + \frac{L_f}{L}, \tag{4}$$

where L_f is the length of the frontal part of stator winding and L is the ideal length of the machine.

In the order to illustrate the elements of the geometric factor, in Figure 1 has been depicted a simplified geometry of machine.

In the above figure has been represented, on the machine geometry, the length of front part of stator winding (in some situation may approximated by a semicircle).

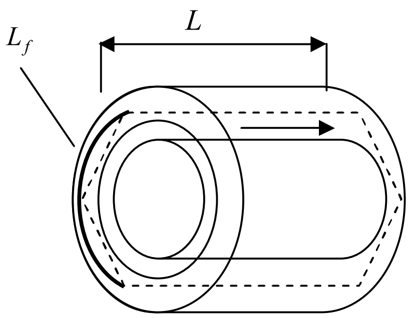

Fig. 1. *Geometry elements of machine*

The height of slot area is computed according to electric stress [5]:

$$h_{c1} \cong \frac{A_1}{J_1 k_{U1}(1-k_{z1})}\,[\mathrm{m}], \tag{5}$$

where k_{U1} is the fill factor and k_{z1} is the ration between the air gap magnetic flux density on the one of teeth.

The arhitecture factor of the slot is described by:

$$\gamma_1 = \frac{h_{c1}}{b_{c1}} \geq 3, \tag{6}$$

where b_{c1} is the width of the slot.

Based on previous relationships, the slot area can be expressed by relationship:

$$S_{c1} = h_{c1} \cdot b_{c1} = \frac{h_{c1}^2}{\gamma_1}. \tag{7}$$

The architecture factor is computed by relationship [8]:

$$\gamma_1 = \frac{Q_{1T} - 2(1-k_{z1})Q_{1c}}{2(1-k_{z1})Q_{1c}}, \tag{8}$$

where the thermal flow of stator insulation is described by relationship:

$$Q_{1c} = \frac{\lambda_{iz1}}{\Delta_{iz1}} \cdot \theta_{iz1}, \tag{9}$$

$$\theta_{is} \cong \frac{\Delta_{iz1}}{\lambda_{iz1}} \cdot Q_{1c} \leq \theta_{isad}(30^0 \div 40^0). \tag{10}$$

For our approach we will use the relationship of force densities in air-gap, radial and tangential component [12]:

$$f_\delta = \frac{1}{\sqrt{2}} \cdot JB_\delta, \tag{11}$$

$$f_r = \mu_0 \cdot AJ, \tag{12}$$

$$f_t = \frac{1}{\sqrt{2}} \cdot JB_\delta \cdot \frac{1}{\mu_d \cdot k_{z1}}. \tag{13}$$

From relationships (3) and (12) the thermal flow on the active part of stator winding may be expressed as a dependence of the radial component of force density:

$$Q_{1T} = \frac{\rho_1}{\mu_0} \cdot f_r. \tag{14}$$

Based on relationship (13) the slotted factor may be expresses as:

$$k_{z1} = \frac{1}{\mu_d} \cdot \frac{1}{\sqrt{2}} \cdot JB_\delta \cdot \frac{1}{f_t}, \tag{15}$$

or, in addition, by using relationship (11), we define the slotted factor as function of the ratio of force densities air gap one and tangential one:

$$k_{z1} = \frac{1}{\mu_d} \cdot \frac{f_\delta}{f_t}. \tag{16}$$

Based on relationships (14) and (16), the architecture factor of slot (8) becomes:

$$\gamma_1 = \frac{\frac{\rho_1}{\mu_0} f_r - 2\left(1 - \frac{1}{\mu_d} \cdot \frac{f_\delta}{f_t}\right) Q_{1c}}{2\left(1 - \frac{1}{\mu_d} \cdot \frac{f_\delta}{f_t}\right) Q_{1c}}. \tag{17}$$

Now, from relationships of rectangular slor height (5) and of architecture factor of slot, the relationship of rectangular slot area becomes:

$$S_{c1} = \frac{A_1^2}{J_1^2} \cdot \frac{\mu_d^2 f_t^2}{(\mu_d f_t - f_\delta)} \cdot \ldots$$
$$\ldots \cdot \frac{2}{\rho_1} \cdot \frac{\mu_0}{\mu_d} \cdot \frac{\mu_d f_t - f_\delta}{f_r} \cdot \frac{1}{Q_{1c}}. \quad (18)$$

Now, take into account the relationship of Joule power losses:

$$P_J = \iiint_{V_\Sigma} \overline{E}\overline{J} dv = \iiint_{V_\Sigma} \rho J^2 dv = \iiint_{V_\Sigma} p_J dv, \quad (19)$$

where the Joule specific power may be expressed by:

$$p_J = \frac{dP_J}{dv}. \quad (20)$$

Finally, be using the relationship (19) and (20) of specific Joule losses, the rectangular slot area (18) becomes:

$$S_{c1} = \frac{A_1^2}{p_J} \cdot \frac{\mu_d^2 f_t^2}{(\mu_d f_t - f_\delta)} \cdot \frac{2\mu_0}{\mu_d} \cdot \frac{1}{f_r} \cdot \frac{1}{Q_{1c}}. \quad (21)$$

When the slot area has a minimal value, the slot number per pole and phase and the total number of slots have maximal values.

Based on the previous work of authors [7] and [8], in Figure 2 has been depicted this kind of situation, where the link between the extreme points of curves of slot area, slot number per pole and phase and total slots has been highlighted.

Based on the analytical approach developed for stator rectangular slot area, two different criteria may be developed. The first criterion leads to compute the slot area in a parallel optimization with specific active materials consumption:

$$\begin{cases} S_{c1} = f(k_{f1}), \\ c_m[kg/VA] = f(k_{f1}) = \min. \end{cases} \quad (22)$$

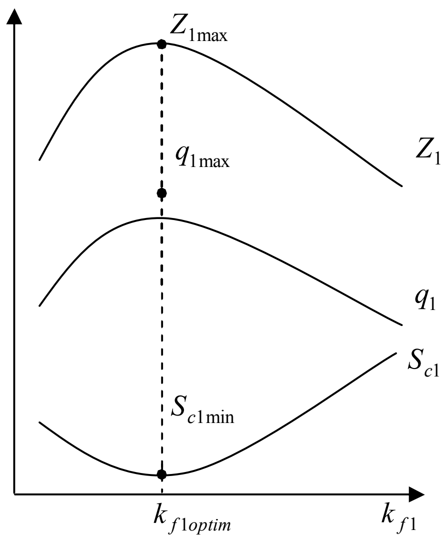

Fig. 2. *Graphics of slot area, slot number per pole and phase and total slots*

In the second case, the criterion take place into consideration the rectangular slot area optimization in parallel with specific cost:

$$\begin{cases} S_{c1} = f(k_{f1}), \\ k_m[m.u/VA] = f(k_{f1}) = \min. \end{cases} \quad (23)$$

These criteria - developed in this work - can be used and integrated on the general scheme block of Poynting vector algorithm [13]. Thus, the developed relationship is placed on the sizing electromagnetically system (Figure 3).

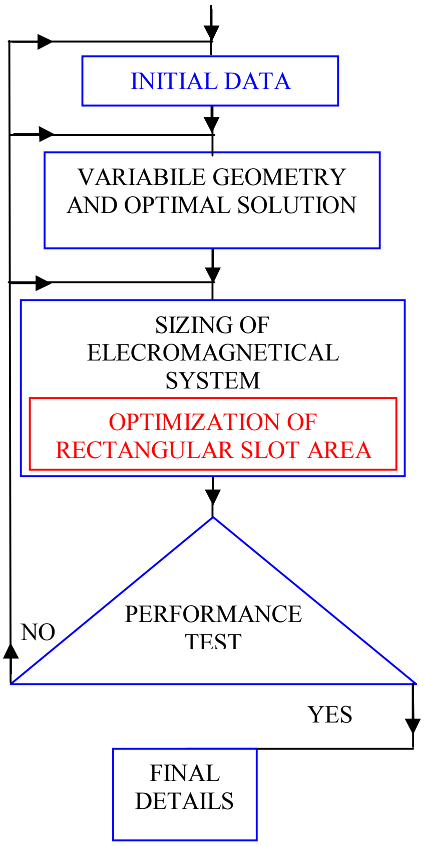

Fig. 3. *Improved algorithme for generator design*

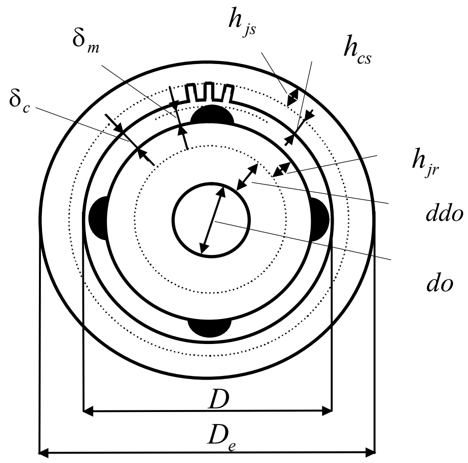

Fig. 4. *A cross section on permanent magnet synchronous machine*

3. Case Study: Optimization of Permanent Magnet Synchronous Generator

Based on the above considerations and observations, in this section will be performed a case study for a permanent magnet synchronous machine - in generator regime - with rotor surface magnets. The machine topology is depicted in Figure 4.

The work of the paper is based on Poynting vector algorithm [13].

The imposed data by design project are:

- rated power: P_n = 500 kW;
- rated voltage: U_n = 400 V;
- rated efficiency: η_n = 0.93;
- rated power factor: $\cos\varphi_n$ = 0.92;
- pairs of poles: p = 8;
- protection degree: IP 44;
- insulation class: F.

The design procedure is divided in two steps. The first step is composed by an analyses and the second one is a syntheses one.

Based on project data - as a rated one - and on some estimations, in the first step (analyze) are determinate the specific characteristics according to electric circuit equivalent scheme (Figure 5).

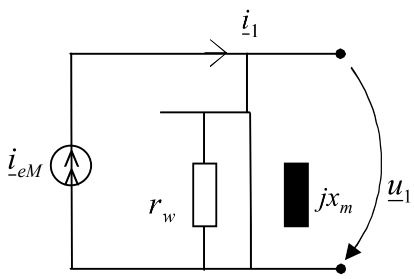

Fig. 5. *Electric circuit equivalent scheme of permanent magnet machine*

The characteristics as efficiency, power factor and stator current (expressed in per-unit) are represented in Figure 6. Two types of representation are used, the first denoted by "T" as electric circuit equivalent scheme used for its computing and other characteristics denoted by "A" from its analytic relationship of computing.

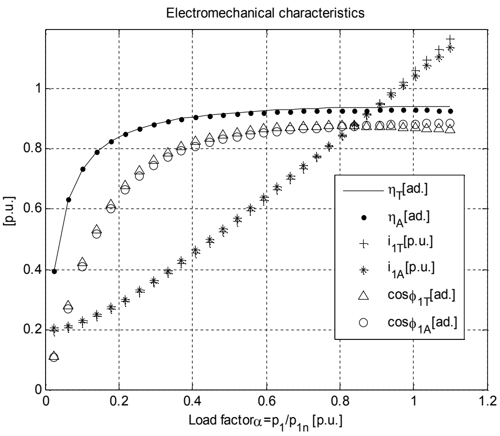

Fig. 6. *Electromechanical characteristics*

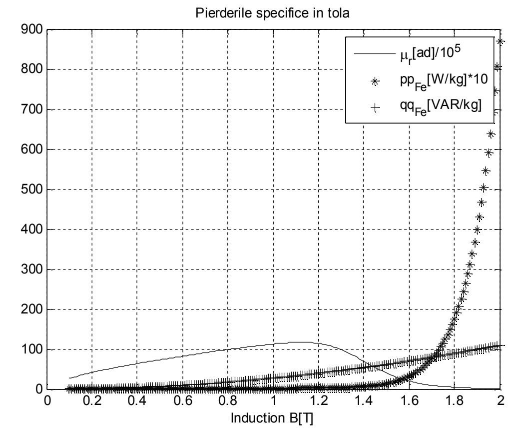

Fig. 8. *Characteristics of iron steel*

The angular characteristics where represented in Figure 7. At this step is computed the intern angle - from angular characteristics - by a graph-analytical method. The value of internal angle is obtained on the intersection of all characteristics. Thus, the obtained value of internal angle is about $\theta = 28^0$.

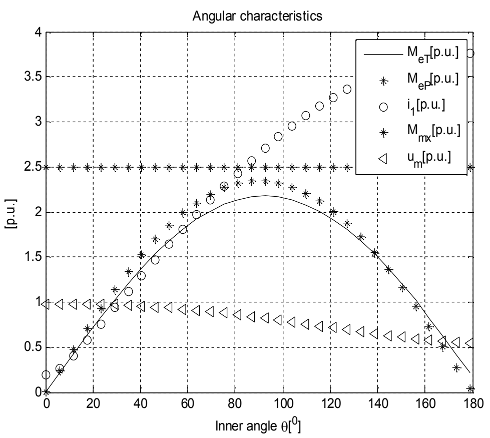

Fig. 7. *Estimated angular characteristics*

The iron steel characteristics as specific active and reactive losses (expressed in [W/kg], respectively [VA/kg]), and relative magnetic permeability - as a dependence of magnetic flux density - were represented in the Figure 8.

Geometric dimensions (inner and outer diameter, machine ideal length, height of slot and yoke) of stator have been represented in Figure 9 with the help of Matlab software, based on Poynting vector algorithm [13], as a function of geometric factor.

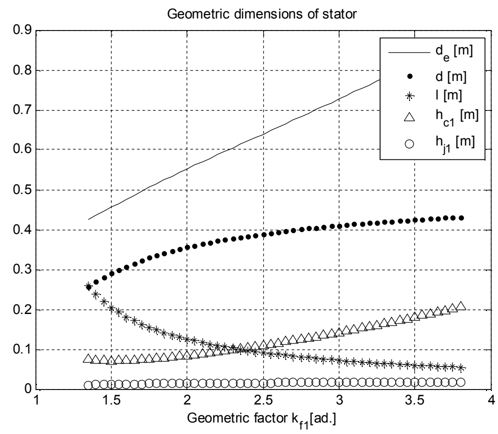

Fig. 9. *Geometric dimensions of stator*

In Figure 10 the geometric dimensions of rotor, as a function of geometric factor, were represented (diameters and air gaps).

In Figure 11 were represented the specific statoric consumption of active material as copper-Cu, iron-Fe and total one, and specific statoric cost. As can be seen from this figure, the total consumption of active materials has a minimal point as an extreme one.

Specific total cost details is presented in the Figure 12.

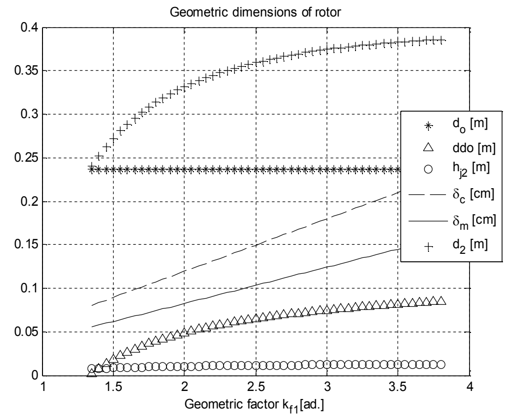

Fig. 10. *Geometric dimensions of rotor*

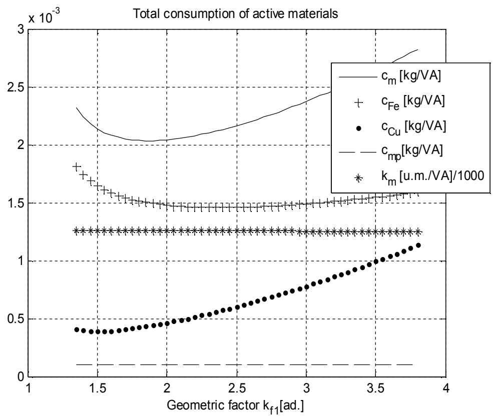

Fig. 11. *Total consumption and cost*

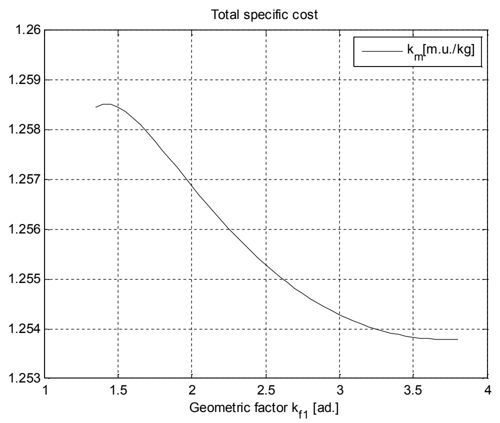

Fig. 12. *Total specific cost* (*detail*)

Finally, in Figure 13, where represented the rectangular slot area, slot number per pole and phase, and the number of total slots.

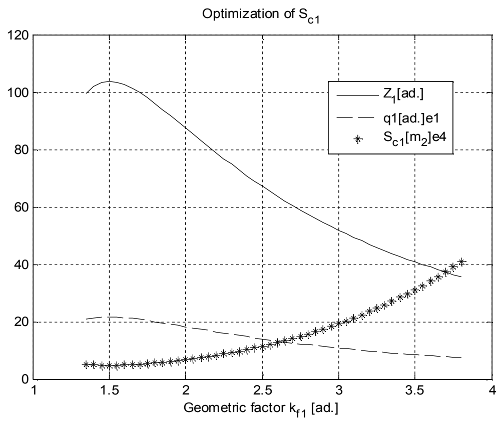

Fig. 13. *Optimization of slot area*

The theoretical considerations established on Figure 2 were matched by Figure 13.

By using optimization criteria developed, the next results are obtained:

Optimization results Table 1

Parameter	Consumption optimization	Cost optimization
k_{f1}	1.85	3.5
γ_1	10	10
S_c [m^2]	0.0006	0.0031
c_m [kg/VA]	0.0020	0.0026
k_m [m.u/VA]	1.2574	1.2538

5. Conclusions

Slot rectangular of permanent magnet synchronous machine may be computed by different approaches, but an integrated one is developed according to electromagnetic stress.

Because electrical machines and, in the particularly case, permanent magnet synchronous one are converters which develops torques and, in principle, forces that may be used for an utile effect, the work of the paper is orientated to a unitary approaches as to express the important quantities as a dependence of forces.

The Poynting vector design algorithm - used for optimal design of classical and special electric machine class may open new opportunities in the design of electrical machines for both field as industry and residential applications.

The work developed according this paper may be substantially developed for the other cases of slot area, but in this situation may be tacked into consideration different assumptions.

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