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INERTIAL DISTRIBUTION OF SEISMIC ENERGY INTO MULTI-STORY FRAMES

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Abstract: *Current study offers an analytical tool to assess the distribution of seismic input energy along the height of the structure. The study is conducted on multi-degree-of-freedom systems with lumped masses located at story levels. The seismic action are a set of three recorded and scaled accelerograms (El Centro 1940, Vrancea 1977 and Focşani 1986 respectively). Performed analyses are of time-history type and are conducted in elastic domain on a set of five level steel frame considered in three states of their lateral stiffness. The computed results are presented numerically and graphically in a comparative manner, commented and relevant concluding remarks are inferred.*

Key words: *seismic energy distribution, lumped story masses.*

1. Introduction

The objective of proposed contribution is to work out an analytical instrument to assess the inertial distribution of seismic input energy and of its structural components (elastic strain energy, kinetic energy, dissipated energy via viscous damping) in the case of multi-story frames seismically acted upon. The inertial distribution may play a certain role in evaluating the structural concept and, certainly, an important role in anticipating seismic behaviour of design structure. Once computed, the inertial distribution of seismic energy allows for computing and using a new analytical tool in seismic analysis – inertial participation to the total seismic response.

The inertial distribution of seismic energy is an aspect somehow less referred to in the literature. Instead, the story distribution [1], [2], [3] of seismic energy into multi-story frames is rather preferred and analysed as an energy distribution by structural components [4], [5]. Nevertheless, the story distribution does not have to be equated with inertia distribution since not always the story means mass, i.e. inertia property. Also, not always an MDOF dynamic system represents a multi-story structure. In other words, not always mass m_i is identical to story i. Present study refers to inertia, i.e. mass distribution of seismic input energy and of its structural components. From inertia distribution to inertia participation it is a small step. Therefore, the inertia participation factors are defined and computed.

The adopted methodology of inertial distribution of seismic input energy is based on the seismic energy balance equation adapted to elastic linear seismic analysis of multi-story structures equipped with added viscous damp [6], [7], [8], [9].

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$$E_k + E_s + E_d + E_{ad} = E_i \qquad (1)$$

The terms in (1) have the usual meaning: E_i is the seismic input energy, E_k is the kinetic energy, E_s is the elastic strain energy, E_d is the energy dissipated by modal damping while, E_{ad} is the energy dissipated by added viscous damping [9], [10]. The energy involved terms in equation (1) are expressed in terms of the n vector **u** of the n degrees of freedom (story lateral horizontal displacements) and of its derivatives $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ with respect time and of ground recorded acceleration $\ddot{u}_g$ seismically induced. The general coordinates **u** are transformed into modal coordinates expressed by n vector **x** by the linear coordinate transformation

$$\mathbf{u} = \mathbf{\Phi}\cdot\mathbf{x} \qquad (2)$$

- where $\mathbf{\Phi}$ is the nxn modal matrix [11].

Vector **x** collects the n x_j (j = 1,n) modal coordinates. Substitution of (2) into (1) leads to a system of n decoupled of each other differential equations in x_j [11]. Each of the n equations constitutes the analytical model of vibration by the j^{th} natural mode of vibration of analysed structure. The scalar nature of energy allows for expressing the total energy amount as a sum of the n energy quantities associated each to the natural modes of vibration and of lumped masses m_i located at story levels [11], [12].

The amount of energy (input, kinetic, elastic strain, dissipated) $E^{(i)}$ associated to mass m_i expresses the inertial distribution of seismic energy and allows for the evaluation of inertial (mass) energy participation. The set of results have been obtained on several multi-story steel structures acted upon by recorded reference earthquakes (El Centro 1940, Vrancea 1977 and Focsani 1986). Each analysed structure has been considered in three different states of its lateral stiffness and in several cases of added viscous damping. The large amount of computed numerical results associated to such a spectrum of structural states allows emphasizing the versatility of proposed analytical tool (inertial distribution of seismic energy) and its utility in seismic analysis and conceiving of structures located in seismic areas. The computed results are associated to an eight level planar frame and are numerically and graphically presented and commented. The results are not aimed to suggestions regarding the design of seismically acted upon structures, but to emphasize the validity and versatility of proposed concepts of inertial distribution and inertial participation of seismic input energy and of its structural components.

2. Theoretical Fundaments

The share $E^{(i)}$ that is associated to mass m_i of the total energy E, is computed from the modal quota E_j of the seismic (input, kinetic, elastic strain, dissipated via viscous damping) energy. The modal quota is computed, in its turn, via modal coordinates x_j [11]. Mathematically, the analytical expressions of seismic input energy and its structural components take the following forms [11]:

Seismic input energy:

$$E_i = -\int_0^t \sum_{j=1}^{n} \mathbf{m}^T\cdot \mathbf{\Phi}_j\cdot\dot{x}_j\cdot \ddot{u}_g\cdot dt \qquad (3)$$

Here, $\mathbf{m}^T$ denotes the transpose of inertia n vector **m** and $\mathbf{\Phi}_j$ is the normalized eigen n vector associated to natural mode j of vibration. Kinetic energy:

$$E_k = \sum_{j=1}^{n} \frac{1}{2}\cdot M_j\cdot\dot{x}_j^2 \qquad (4)$$

In (4), M_j is the classical modal mass j.

Elastic strain energy:

$$E_s = \sum_{j=1}^{n} \frac{1}{2} \cdot K_j \cdot \dot{x}_j^2 \qquad (5)$$

where, K_j is the generalized stifness of natural mode j of vibration.

Energy dissipated by inherent viscous damping:

$$E_d = \int_0^t \sum_{j=1}^{n} C_j \cdot \dot{x}_j^2 dt \qquad (6)$$

Here, C_j is the generalized modal linear viscous damping j [12].

By convenient arrangement of terms in (3), the seismic input energy takes the form:

$$E_i = -\sum_{j=1}^{n} \int_0^t m_i \cdot \Phi_{ij} \cdot \dot{x}_j \cdot \ddot{u}_g \cdot dt \qquad (7)$$

where, the energy associated to mass m_i is emphasized in the form:

$$E_i^{(i)} = -\int_0^t \sum_{j=1}^{n} m_i \cdot \Phi_{ij} \cdot \dot{x}_j \cdot \ddot{u}_g \cdot dt \qquad (8)$$

Expression (8) allows for casting the total seismic input energy E_i as a summ of its inertial (mass) components $E_i^{(i)}$:

$$E_i = \Sigma E_i^{(i)} \qquad (9)$$

Following the same analytical procedure, the structural inertial components $E^{(i)}$ of the structural components of seismic energy take the forms. Kinetic energy:

$$E_k^{(i)} = \frac{1}{2} \cdot \int_0^t \cdot \dot{x}_j^2 \cdot dt \qquad (10)$$

with the total kinetic given by:

$$E_k = \Sigma E_k^{(i)} \qquad (11)$$

Elastic strain energy:

$$E_s^{(i)} = \frac{1}{2} \cdot \int_0^t K_j \cdot \dot{x}_j^2 \cdot dt \qquad (12)$$

giving the total elastic strain energy:

$$E_s = \Sigma E_s^{(i)} \qquad (13)$$

And, finally, the mass component of energy dissipated by viscous damping:

$$E_d^{(i)} = \int_0^t C_j \cdot \dot{x}_j^2 \cdot dt \qquad (14)$$

while, the total dissipated energy is:

$$E_d = \Sigma E_d^{(i)} \qquad (15)$$

and it is associated to a level of 2% of ineherent viscous damping.

Above mentioned integrals with respect time are computed via classical numerical techniques.

3. Structures and Seismic Actions

The numerical results presented further on are associated to a five level planar frame considered in three states of lateral rigidity by convenient allocation of member cross sections [11]: stiff, moderate stiff (Fig. 1) and flexible.

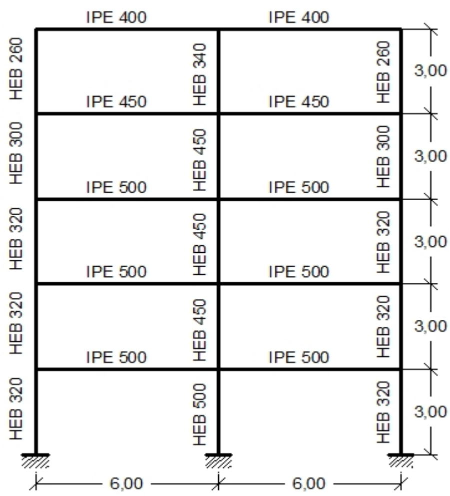

Fig. 1. *Moderate stiff structure*

Three seismic actions acted upon the structures (El Centro 1940, Focsani 1986 – Fig. 2 and Vrancea 1977) have been selected for both, their distinct seismic features and their popularity among the practising researchers and designers.

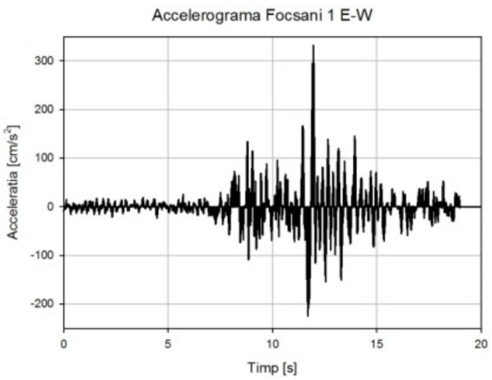

Fig. 2. *Focsani 1986 accelerogram*

4. Numerical Results

A reduced set of numerical results are presented aimed at underlying the capacity and versatility of proposed analytical tool to assess the inertial distribution of seismic input energy.

4.1 Inertial Components of Energy

Presented numerical results refer to the total seismic input energy E_i given by the definition expression (3) and by its decomposition into its mass components (9) respectively for the case of 5 levels frame and of 8 levels frame. Several states of added viscous damping have been considered: an inherent level of 2% (fraction of critical damping) and 5%, 10% and 15% added damping levels.

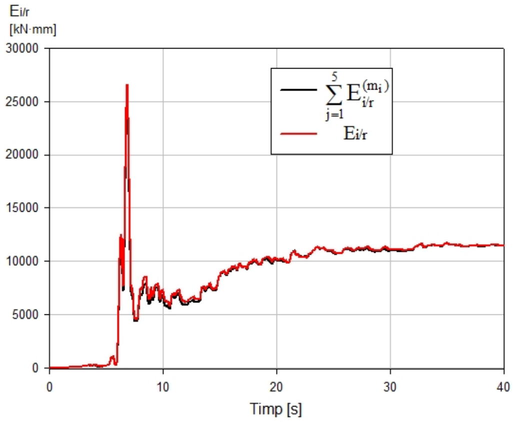

Fig. 3. *Total seismic input energy. Five levels frame moderate stiff. Vrancea, 2%*

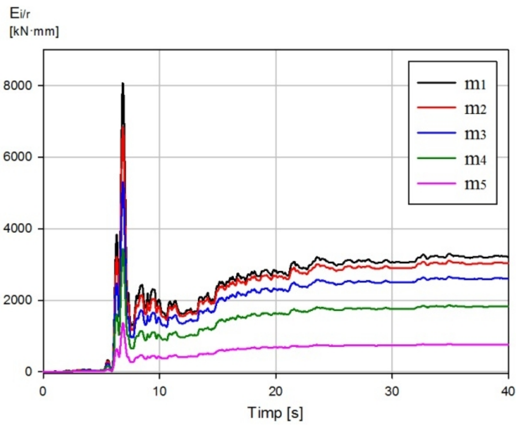

Fig. 4. *Mass components of seismic input energy. Five levels stiff frame Vrancea, 2%*

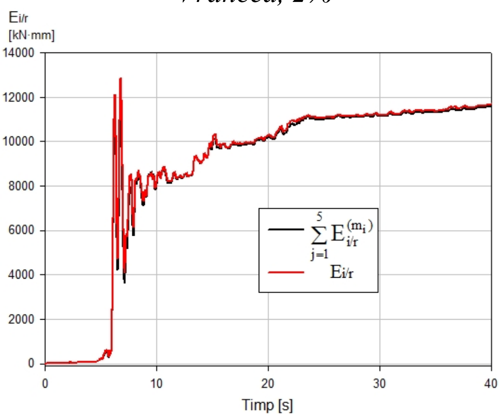

Fig. 5. *Total seismic input energy five levels stiff frame. Vrancea, 5%*

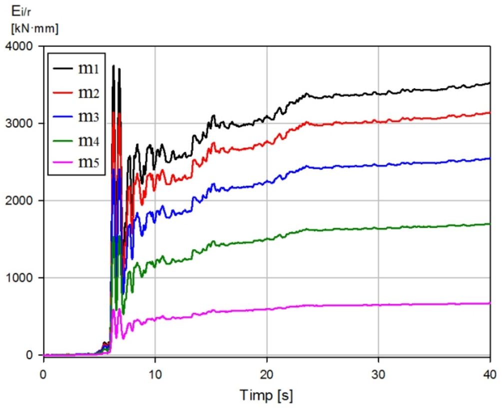

Fig. 6. *Mass components of seismic input Energy five levels stiff frame. Vrancea, 5%*

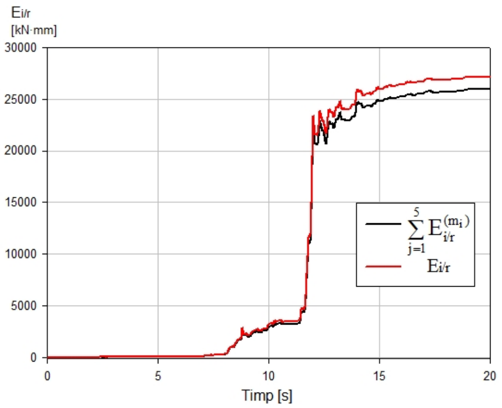

Fig. 7. *Total seismic input energy five levels stiff frame. Focsani, 10%*

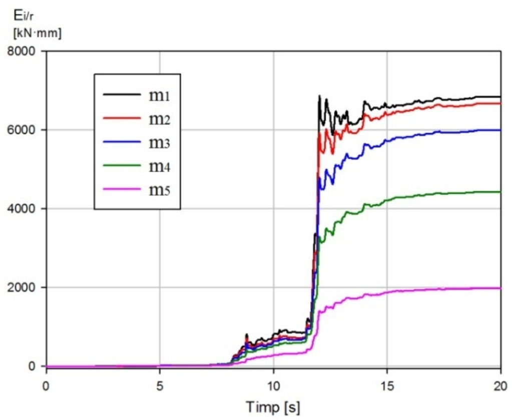

Fig. 8. *Mass energy components five levels stiff frame. Focsani, 10%*

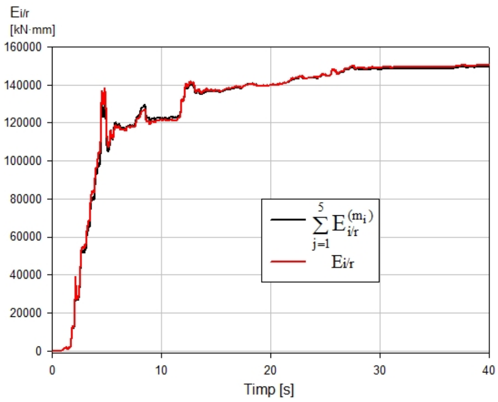

Fig. 9. *Total seismic input energy five levels flexible frame. El Centro, 5%*

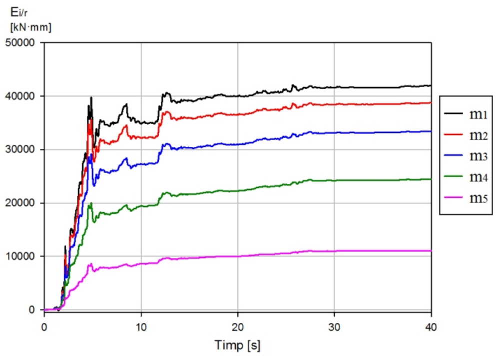

Fig. 10. *Mass energy components five levels flexible frame. El Centro, 5%*

As it can be noticed, the total amounts of seismic input energy are given by both: their relations of definition (3) to (6) and their expressions as the sum of mass components (9), (11), (13) and (15). To emphasize the accuracy of the performed computations, the numerical results are presented accordingly. In all cases, a good agreement of the two compute values of the total seismic input energy may be concluded (Fig. 3, Fig. 5, Fig, 7, Fig. 9).

4.2 Mass Participation

The concept of mass participation may be viewed as an extension of the more popular and traditional concept of modal participation [11], [12]. The coefficient that numerically express the mass participation (referred to as γ^i) is the ratio of mass energy component $E_i^{(i)}$ given by (8) to the total amount of seismic input energy E_i given by (9):

$$\gamma^i = E_i^{(i)} / E_i \qquad (16)$$

The following are a set of numerical results associated to the analyzed cases that express the energy mass participation to seismic input energy.

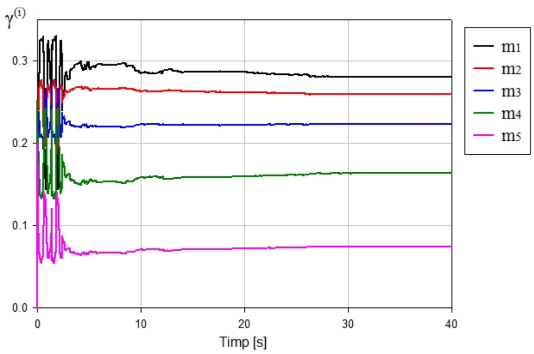

Fig. 11. *Mass participation coefficients. Five levels stiff frame. Vrancea, 5%*

The rule of $\Sigma\gamma^i = 1$ (the sum extending over all masses) and its observance may be noticed (in all cases) as in the case of modal participation factors.

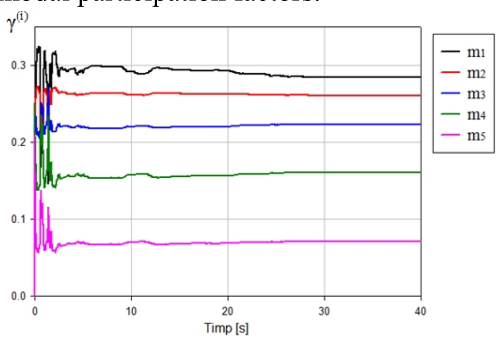

Fig. 12. *Mass participation coefficients. Five levels stiff frame El Centro, 15%*

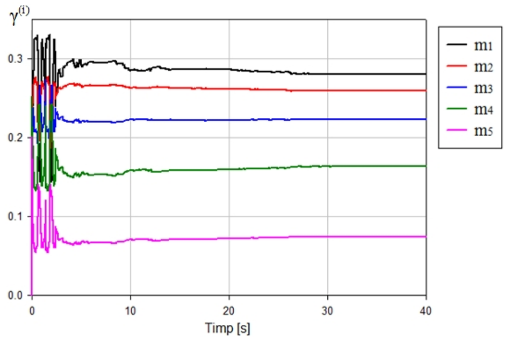

Fig. 13. *Mass participation coefficients. Five levels flexible frame. El Centro, 5%*

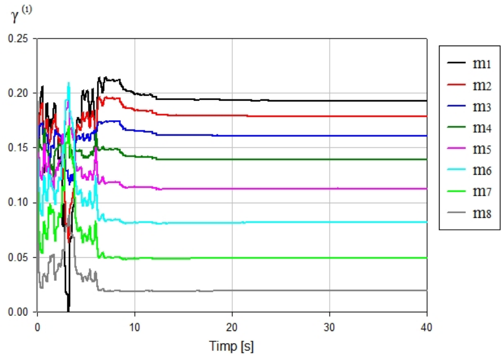

Fig. 14. *Mass participation coefficients. Eight levels flexible frame. Vrancea, 5%*

As it can be noticed, the proposed mass participation coefficients γ^i vary in time (Fig. 11 to Fig. 13).

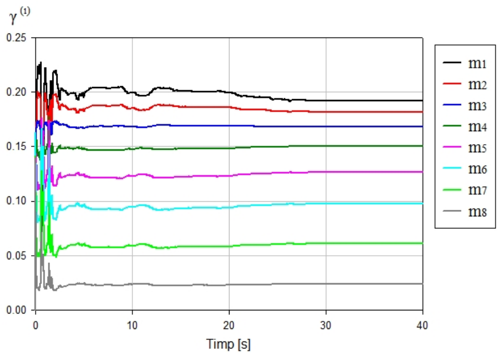

Fig. 15. *Mass participation coefficients. Eight levels stiff frame. El Centro, 10%*

A set of time instances of such values are presented bellow for the case of five levels frame (Fig. 16 to Fig. 18).

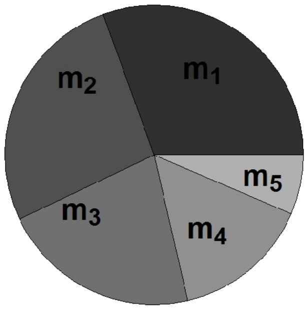

Fig. 16. *Mass participation state in sec. 12. Focsani, stiff,10%*

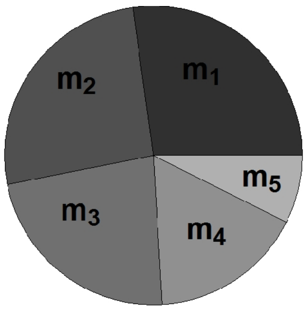

Fig. 17. *Mass participation state in sec. 19. Focsani, stiff, 10%*

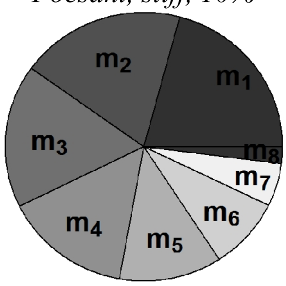

Fig. 18. *Mass participation state – sec. 20. Vrancea, stiff, 10%*

It may be of interest to assess their state not just as maximum values, but at any required time.

A useful conclusion may be inferred from the variation of mass participation coefficients in terms of added damping level. The variation of mass participation coefficients with added damping level are given in Fig. 19 – for the case of five levels frame and in Fig. 20 – for the case of eight level frame in all their states of added linear viscous damping.

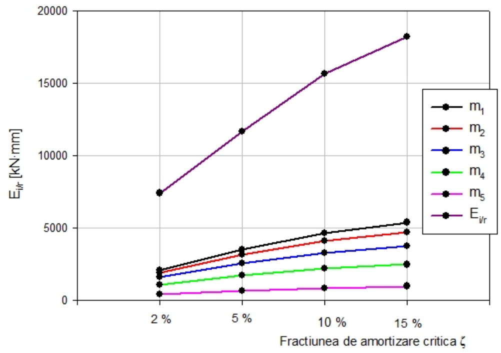

Fig. 19. *Vrancea seismic action. Rigid 5 levels frame*

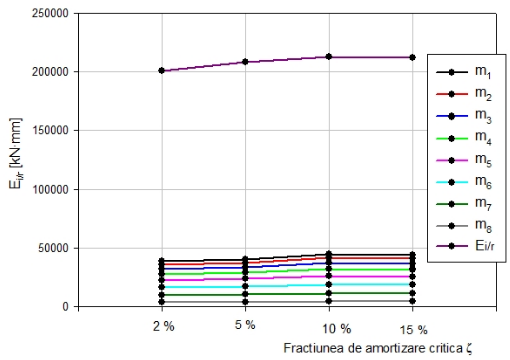

Fig. 20 *Vrancea seismic action. Flexible 8 levels frame*

5. Conclusions

The inertia or mass decomposition of seismic input energy emphasize a rather equilibrated status of the this mass distribution – as it can be seen in Figure 10 and Figure 20. The associated seismic input energy allotted to mass m_1 (top level), in the case of five levels frame for instance, is the largest, indeed. But this amount is comparable with the amounts that are allotted to the other masses. Going numerically, in the case of five levels frame, the top level mass m_1 absorbs cca. 25% af entire amount of seismic input energy, while the energy associated to mass m_2 is around 20%. Extension of the concept of participation over the inertia state of the structure provides useful information in the conceiving process of structural design of structures located in seismic areas. In the case of multi-story steel structures, an analytical tool associated to the distribution of seismic input energy along the height of the structure is important and may offer a powerful instrument regarding the optimization of location of seismic dampers. Such location is more effective if the dampers are placed at the levels that absorb more seismic energy. Combined view regarding the distribution of seismic input energy along the height of the structure and its dependence on the added damping level constitute a valuable tool in conceiving multi-story frame type structures seismically acted upon.

References

1. Chou, C.- C., Uang, C. – M.: *Evaluating distribution of seismic energy in multistory frames*. In: 13th World Conference on Earthquake Engineering Vancouver, B.C., Canada, August 1-6, Paper No. 24, 2004.
2. Chapman, M.C.: *On the use of elastic input energy for seismic hazard analysis*. In Earthquake Spectra, 15(4), 1999, pp. 607 – 634.
3. Motamedi, M., Nateghi-A. F.: *A proposed lateral load pattern using seismic energy distribution along the height of buildings*. In: The 14th World Conference on Earthquake Engineering Beijing, China, October 12 – 17, 2008.
4. Nariyuki, Y., Hirao, K.: *Effect of Structural and Hysteretic Characteristics on Distribution of Input and Hysteretic Energy of MDOF Systems subjected to Seismic Excitation*. In: Proceedings of The 9th World Conference on Earthquake Engineering (9WCCE), Tokyo-Kyoto, No.Vol.5, Aug., 1988, pp.123-128.
5. Wang, F., Yi, T.: *A Methodology for Estimating Seismic Hysteretic Energy of Buildings*. In: Civil Engineering and Urban Planning, 2012, pp. 17-21.
6. Uang, C. M., Bertero, V. V.: *Evaluation of seismic energy in structures*. In: Earthquake Engineering and Structural Dynamics, Vol. 19, 1990, pp. 77–90.
7. Bayat, M., Abdollahzadeh, G.: *Analysis of steel braced frames equipped with ADAS devices under the far field records*. In: Latin American Journal of Solids and Structures 8, 2011, pp. 163 - 181.
8. Ghodrati Amiri, G., Abdollahzadeh Darzi, G., Khanzadi, M.: *Earthquake Duration and Damping Effects on Input Energy*. In: International Journal of Civil Engineering, Vol. 5, No. 1, 2007, pp. 14-29.
9. Kalkan, E., Kunnath ,S. K.: *Relevance of Absolute and Relative Energy Content in Seismic Evaluation of Structures*. In: Advances in Structural Engineering, Vol. 11, No.1,2008, p.18.
10. Nakashima, H., Saburi, K., Tsuji, B.: *Energy input and dissipation behaviour of structures with hysteretic dampers*. In: Earthquake Engineering and Structural Dynamics, 19(1), 1996.
11. Ladar, Ioana: *Seismic response of multistory steel structures. An energy approach.* PhD Thesis. Technical University of Cluj – Napoca (Romanian). 2013.
12. Chopra, A.K.: *Dynamics of Structures. Theory and Applications to Earthquake Engineering*. Prentice Hall International, 1995.