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# STEADY-STATE MAGNETIC FIELD COMPUTING BY USING THE INTEGRAL MODEL OF MOLECULAR CURRENTS

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***Abstract:*** *The issue of magnetic field in a media where the magnetic permeability is defined as a function of point is not a simple one. In this paper is presented a computing method of steady-state magnetic field, in this type of media, that using a mathematical model of integral equations on the boundary. This model corresponds to a physical equivalent and is using, in this sense, fictive repartitions of the density of sources, on the boundary, as fictive currents (molecular currents). The physical premises for this kind of equivalent are represented by the magnetizing of magnetic bodies introduced in an exterior magnetic filed.*

***Key words:*** *steady-state magnetic field, integral model, molecular currents, magnetic permeability.*

## 1. Introduction

For problems of steady-state electromagnetic field are used different formulations as: differential, integro-differential and integral one. The integral models corresponds to some physics equivalents, and are using, in this sense, fictive repartition of field sources density as magnetic polarization charges or molecular currents.

The physical premises for this equivalence are represented by the magnetization of magnetic body placed in an external magnetic field.

In this paper is presented a method for computing of steady-state magnetic field which is using molecular currents as secondary sources of magnetic field.

In the case of linear media, the volume density of the molecular currents is zero and the magnetization state of the body is characterized by an equivalent surface distribution of molecular currents. In this situation the problem of steady-state magnetic field is solved by boundary element method (BEM) [1]. The main advantage in this case is that only surface integrals need to be discretized.

For nonlinear magnetic bodies, the magnetization state is characterized by equivalent distribution molecular currents repartized with surface density and volume density. Therefore, in this case, the problem is described by the two field integral equations, in where take place the volume integral, and thus requires a volume meshing of the body. The problem is solved by using the boundary element method and the method of volume integral equations of volume (VIEM) [2-4].

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## 2. The Integral Mathematic Model of Steady-State Magnetic Field

Let be a ferromagnetic body with magnetic permeability $\mu(\overline{r})$, occupying the volume $v_m$ (bounded) of closed surface $\Sigma_m$. We consider a current source with density $\overline{J}$ which produces a magnetic field with magnetic field density $\overline{B}_s$, which leads to magnetising of ferromagnetic body. The magnetic field in a point $P(\overline{r}_P)$ of volume $v_m$ may be computed as a superposition of $\overline{B}_s$ and the magnetic flux density produced by magnetized body $\overline{B}_m$:

$$\overline{B}(\overline{r}_P) = \overline{B}_s(\overline{r}_P) + \overline{B}_m(\overline{r}_P), \tag{1}$$

where:

$$\overline{B}_s(\overline{r}_P) = \frac{\mu_0}{4\pi}\iiint_{v_J}\frac{\overline{J}(\overline{r}_Q)\times(\overline{r}_P-\overline{r}_Q)}{\left|\overline{r}_P-\overline{r}_Q\right|^3}dv_J, \tag{2}$$

$$\overline{B}_m(\overline{r}_P) = \frac{\mu_0}{4\pi}\cdot\left\{\begin{array}{l}\displaystyle\iiint_{v_\Sigma}\frac{\overline{J}_m(\overline{r}_M)\times(\overline{r}_P-\overline{r}_M)}{\left|\overline{r}_P-\overline{r}_M\right|^3}dv + \\ \displaystyle + \oiint_{\Sigma}\frac{\overline{J}_{m\ell}(\overline{r}_N)\times(\overline{r}_P-\overline{r}_N)}{\left|\overline{r}_P-\overline{r}_N\right|^3}d\Sigma\end{array}\right\}, \tag{3}$$

In relationship (3), $\overline{J}_m(\overline{r}) = \nabla\times\overline{M}(\overline{r})$ is molecular currents volume density; $\overline{J}_{m_\ell}(\overline{r}) = \overline{M}(\overline{r})\times\overline{n}(\overline{r})$ represent the molecular currents body surface density and $\overline{n}$ is the normal unitary vector of the surface $\Sigma$.

Taking into account the relationships:

$$\overline{B}(\overline{r}) = \mu_0\mu_r(\overline{r})\overline{H}(\overline{r}), \tag{4}$$

$$\overline{M}(\overline{r}) = [\mu_r(\overline{r})-1]\overline{H}(\overline{r}), \tag{5}$$

the molecular currents volume density becomes:

$$\begin{aligned}\overline{J}_m(\overline{r}) &= \nabla\times\left\{\frac{\overline{B}(\overline{r})}{\mu_0}\left[1-\frac{1}{\mu_r(\overline{r})}\right]\right\} \\ &= \left[1-\frac{1}{\mu_r(\overline{r})}\right]\nabla\times\frac{\overline{B}(\overline{r})}{\mu_0} \\ &\quad+\nabla\left[1-\frac{1}{\mu_r(\overline{r})}\right]\times\frac{\overline{B}(\overline{r})}{\mu_0}.\end{aligned} \tag{6}$$

Due to the fact that in the inner of the body there are no free currents, the strength of magnetic field satisfied the equation $\nabla\times\overline{H}(\overline{r}) = 0$, from relationship:

$$\overline{B}(\overline{r}) = \mu_0[\overline{H}(\overline{r}) + \overline{M}(\overline{r})], \tag{7}$$

it results:

$$\nabla\times\frac{\overline{B}(\overline{r})}{\mu_0} = \nabla\times\overline{M}(\overline{r}) = \overline{J}_m(\overline{r}). \tag{8}$$

By substituting the relationship (8) into (6), it is obtained:

$$\overline{J}_m(\overline{r}) = \mu_r(\overline{r})\nabla\left[1-\frac{1}{\mu_r(\overline{r})}\right]\times\frac{\overline{B}(\overline{r})}{\mu_0}. \tag{9}$$

Due to the next development:

$$\begin{aligned}\nabla\left[1-\frac{1}{\mu_r(\overline{r})}\right] &= -\nabla\frac{1}{\mu_r(\overline{r})} \\ &= -\frac{d}{d\mu_r(\overline{r})}\left[\frac{1}{\mu_r(\overline{r})}\right]\nabla\mu_r(\overline{r}) \\ &= \frac{\nabla\mu_r(\overline{r})}{\mu_r^2(\overline{r})},\end{aligned} \tag{10}$$

the relationship (9) in the point $P(\overline{r}_P)$ becomes:

$$\bar{J}_m(\bar{r}_P) = \frac{1}{\mu_r(\bar{r}_P)} \nabla \mu_r(\bar{r}_P) \times \frac{\bar{B}(\bar{r}_P)}{\mu_0} = \frac{1}{\mu_0 \mu_r(\bar{r}_P)} \nabla \mu_r(\bar{r}_P) \times \bar{B}(\bar{r}_P). \quad (11)$$

Now, by substituting into relationship (11) the relationship of resulted magnetic flux density $\bar{B}(\bar{r}_P)$ described by relationship (1), and taking into account the relationship (3), it is obtained:

$$\begin{aligned} \bar{J}_m(\bar{r}_P) &= \frac{1}{\mu_0 \mu_r(\bar{r}_P)} \nabla \mu_r(\bar{r}_P) \times \bar{B}_s(\bar{r}_P) + \frac{1}{4\pi \mu_r(\bar{r}_P)} \nabla \mu_r(\bar{r}_P) \\ &\times \left\{ \iiint_{v_\Sigma} \frac{\bar{J}_m(\bar{r}_M) \times (\bar{r}_P - \bar{r}_M)}{\left|\bar{r}_P - \bar{r}_M\right|^3} dv + \oiint_{\Sigma} \frac{\bar{J}_{m\ell}(\bar{r}_N) \times (\bar{r}_P - \bar{r}_N)}{\left|\bar{r}_P - \bar{r}_N\right|^3} d\Sigma \right\}. \end{aligned} \quad (12)$$

On the surface of ferromagnetic body, the magnetic permeability has a step variation, which leads to vary the gradient of magnetic permeability to infinite and, thus, the density of molecular current to infinite. But, in the same time the density of molecular current on the ferromagnetic body surface has a finite value. In this situation the molecular currents has the sense of surface currents.

We consider two points $\bar{G}_1(\bar{r}^+)$ and $\bar{G}_2(\bar{r}^-)$ which are situated symmetrical along the boundary between vacuum and ferromagnetic body, being outside and inside of ferromagnetic body, and the point $\bar{G}(\bar{r}_G)$ situated on the surface of ferromagnetic body, thus:

$$\bar{r}_G^{\pm} = \bar{r}_G \pm \frac{\Delta h}{2} \bar{n}_G, \quad (13)$$

where $\bar{n}_G$ is the positive normal unitary vector on the surface $\Sigma$ of ferromagnetic body.

In the point $\bar{G}(\bar{r}_G)$ of ferromagnetic body surface, the magnetic permeability has a value equal with the average one:

$$\mu_r(\bar{r}_G) = \frac{\mu_r + 1}{2}. \quad (14)$$

For the case of $\Delta h \to 0$, the average value of gradient tends to the value of surface gradient:

$$\frac{\nabla \mu_r(\bar{r}_G)}{\mu_r(\bar{r}_G)} = -2 \frac{\mu_r(\bar{r}_G) - 1}{\mu_r(\bar{r}_G) + 1} \bar{n}_G \lim_{\Delta h \to 0} \frac{1}{\Delta h}. \quad (15)$$

By substituting the relationship (15) into account the relationship (11), it is obtained:

$$\bar{J}_m(\bar{r}_G) = -\frac{2}{\mu_0} \frac{\mu_r(\bar{r}_G) - 1}{\mu_r(\bar{r}_G) + 1} \cdot [\bar{n}_G \times \bar{B}(\bar{r}_G)] \lim_{\Delta h \to 0} \frac{1}{\Delta h}. \quad (16)$$

It can be observed that for $\Delta h \to 0$, results $\bar{J}_m(\bar{r}_G) \to \infty$. But, if we multiplying the both member of relationship (16) with $\Delta h$, and if we passing to limit $\Delta h \to 0$, it is obtained a finite value:

$$\bar{J}_{m\ell}(\bar{r}_G) = \lim_{\Delta h \to 0} \bar{J}_m(\bar{r}_G) \Delta h = -\frac{2}{\mu_0} \frac{\mu_r(\bar{r}_G) - 1}{\mu_r(\bar{r}_G) + 1} \bar{n}_G \times \bar{B}(\bar{r}_G), \quad (17)$$

where $\bar{J}_{m_\ell}(\bar{r}_G)$ represents the density of surface molecular currents which take into account the influence of the interface.

Now, by substituting the relationship (17) into relationship (1) and into account the relationship (3), it is obtained, behind the first Equation (12), the second Equation described by:

$$\begin{aligned}\overline{J}_{m\ell}(\overline{r}_G) = -\frac{1}{2\pi}\frac{\mu_r(\overline{r}_G)-1}{\mu_r(\overline{r}_G)+1}\cdot\overline{n}_G \\ \times\left\{\iiint_{v_J}\frac{\overline{J}(\overline{r}_Q)\times(\overline{r}_G-\overline{r}_Q)}{\left|\overline{r}_G-\overline{r}_Q\right|^3}dv_J\right. \\ +\iiint_{v_\Sigma}\frac{\overline{J}_m(\overline{r}_M)\times(\overline{r}_G-\overline{r}_M)}{\left|\overline{r}_G-\overline{r}_M\right|^3}dv \\ \left.+\oint\!\!\!\oint_{\Sigma}\frac{\overline{J}_{m\ell}(\overline{r}_N)\times(\overline{r}_G-\overline{r}_N)}{\left|\overline{r}_G-\overline{r}_N\right|^3}d\Sigma\right\},\end{aligned} \tag{18}$$

where the points $\overline{G}(\overline{r}_G)$ and $\overline{N}(\overline{r}_G)$ apart to surface $\Sigma$ of ferromagnetic body.

The field problem consist on solving of both coupled Equations (12) and (18), the unknown being the current densities $\overline{J}_m(\overline{r})$ and $\overline{J}_{m\ell}(\overline{r})$ of molecular currents. Knowing the molecular currents densities, it is determinate, with the help of the (3), based on relation (1) by superposition method, the resultant magnetic flux density.

On the discontinuity surface of magnetic body, must be fulfilled the continuity conditions of normal components of magnetic flux density:

$$\overline{n}_G(\overline{r}_G)\cdot\overline{B}(\overline{r}_G^{+}) = \overline{n}_G(\overline{r}_G)\cdot\overline{B}(\overline{r}_G^{-}), \tag{19}$$

and of the tangential components of magnetic strength of field is expressed by:

$$\overline{n}_G(\overline{r}_G)\times\frac{\overline{B}(\overline{r}_G^{+})}{\mu_0} = \overline{n}_G(\overline{r}_G)\times\frac{\overline{B}(\overline{r}_G^{-})}{\mu_0\mu_r}, \tag{20}$$

where:

$$\begin{aligned}\overline{B}(\overline{r}_G^{\pm}) = \overline{B}(\overline{r}_G) \\ \pm\frac{\mu_0}{2}\overline{J}_{m_\ell}(\overline{r}_G)\times\overline{n}_G(\overline{r}_G).\end{aligned} \tag{21}$$

By using the Equation (21), it can be observed that the condition (19) is automated satisfied. If it is substituted the relationship (21) into condition (20), it is obtained the relationship (16) of molecular currents densities. Thus, the continuity conditions are satisfied.

## 3. Numerical Simulation

In the order to prove the theoretical background developed in the above section, in the current section it is considerate, as an application, a DC electromagnet with culisant armature with open magnetic circuit. The geometric dimensions of the cylindrical armature has the radius $r$ = 21 mm, the length $z$ = 128 mm and the cylindrical coil has the same length as the magnetic core.

Due to the fact that the problem is 2D, the Equations (12) and (18) are simplified and the volume integral becomes a surface one, and the surface integral becomes on the line.

In the current operation of such devices it is considerated the non-saturated state, which means that the magnetic permeability to be constant $\mu(\overline{r}) = ct.$, and it results $\nabla\mu(\overline{r}) = 0$, which implies a zero value of molecular currents volume density. In this situation the problem is reduced to a one Equation:

$$\begin{aligned}\overline{J}_{m\ell}(\overline{r}_G) + \frac{\alpha(\overline{r}_G)}{2\pi}\overline{n}_G \\ \times\oint_{\Gamma}\frac{\overline{J}_{m\ell}(\overline{r}_N)\times(\overline{r}_G-\overline{r}_N)}{\left|\overline{r}_G-\overline{r}_N\right|^3}d\Gamma \\ = -\frac{\alpha(\overline{r}_G)}{2\pi}\overline{n}_G \\ \times\iint_{S_j}\frac{\overline{J}(\overline{r}_Q)\times(\overline{r}_G-\overline{r}_Q)}{\left|\overline{r}_G-\overline{r}_Q\right|^3}dS_j.\end{aligned} \tag{22}$$

In the Equation (22), by $N$ and $Q$ has been denoted the arbitrary points apart to contour $\Gamma$ of the magnetic circuit cross section, respectively to section $S_j$ of excitation winding, and we have:

$$\alpha(\bar{r}_G) = \frac{\mu_r(\bar{r}_G) - 1}{\mu_r(\bar{r}_G) + 1}. \tag{23}$$

In angular points where the direction of external normal vector is not univocal definite, it is associated to the angular point, two discretization nodes, geometrical decalated, corresponding to both values of normal surface in the angular points. In this case, we have:

$$\alpha(\bar{r}_G) = 4\frac{\mu_r(\bar{r}_G) - 1}{5\mu_r(\bar{r}_G) + 3}, \tag{24}$$

The field domain has been discretizated on the contour of ferromagnetic core in 60 of elements. In simulation have been considerate two different values of relative magnetic permeability and armature position.

In Figures 1a-4a are represented the variations of molecular density currents along the contour for the above mentionedconditions as armature position and relative magnetic permeability values.

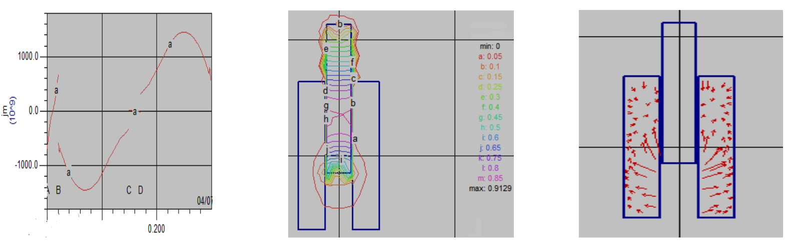
From Figures 1b-4b, where was represented the lines of $B$ = ct. of 0.05 T, it can observed. Note that the magnetic flux is practically constant in successive cross sections of ferromagnetic core, which may provide the elements of equivalent scheme in terms of permeances, inductances and force developed.

Calculating the normal component of magnetic induction $B_n$ and the tangential one of the magnetic field $H_t$ on the surface of ferromagnetic core, we can determine the density of surface force that is acting on the core by:

$$\begin{aligned}\bar{f}_s &= \left(-\frac{1}{2}\int_{\mu_1}^{\mu_2} H^2 d\mu\right)\bar{n}_{12} \\ &= \frac{1}{2}(\mu_1 - \mu_2)\left(\frac{B_n^2}{\mu_1\mu_2} + H_t^2\right)\bar{n}_{12},\end{aligned} \tag{25}$$

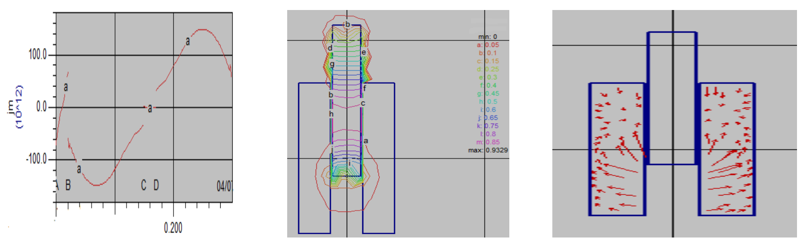
and $\mu_1 = \mu_0\mu_r$, $\mu_2 = 0$, and $\bar{n}_{12}$ is the external normal vector on armature surface.

In Figures 1c-4c has represented the vector of force.



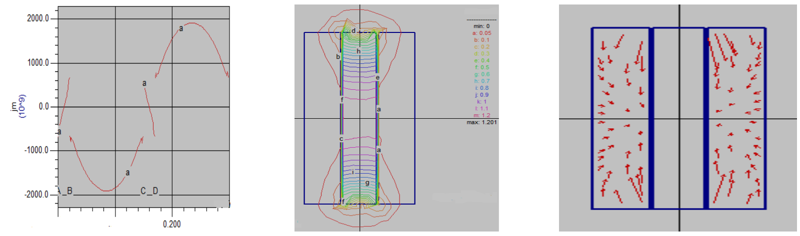
a) b) c)

Fig. 1. *Simulations for $\mu_r$ = 1000 and $z_c$ = 113 mm: a) molecular current density; b) Contour of B = ct.; c) Magnetic force*



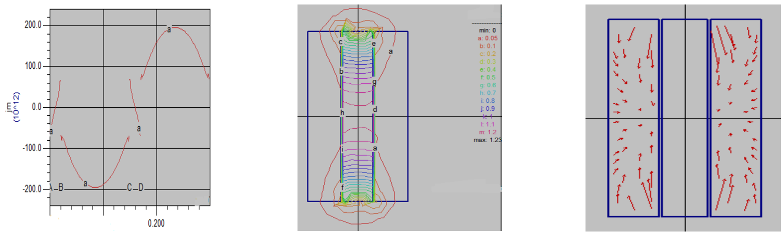
a) b) c)

Fig. 2. *Simulations for $\mu_r$ = 10000 and $z_c$ = 113 mm: a) molecular current density; b) Contour of B = ct.; c) Magnetic force*



a) b) c)

Fig. 3. *Simulations for $\mu_r$ = 1000 and $z_c$ = 128 mm: a) molecular current density*; *b) Contour of B = ct.*; *c) Magnetic force*



a) b) d)

Fig. 4. *Simulations for $\mu_r$ = 10000 and $z_c$ = 128 mm: a) molecular current density; b) Contour of B = ct.; c) Magnetic force*

## 4. Conclusion

The presented method for computing of steady state magnetic field allows the determination of resulted steady state magnetic field produced by field primary source (conduction current density from coil) and by primary filed source (molecular current density from ferromagnetic core). The method may be extended for more complex magnetic systems, with multiple primary filed sources.

The advantage of method is given by the fact that the computing of magnetic field is done only in the interest points, not in all the space as in the case of classical methods of finite elements as for finite elements method.

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