THERMAL EXPANSION OF COMPOSITE LAMINATES

M.N. VELEA[1]   S. LACHE[1]

Abstract: *When connecting composite materials and isotropic materials, their different thermal expansion coefficients may introduce additional stress in the structure. This article shows the way the fibre composite laminates may be designed and optimized in order to fit the thermal expansion of the pair isotropic material and thus resulting similar thermal deformations in both of the materials. An optimization model is developed based on the micromechanics theory of composite laminates where the plies angles represent the design variables while the objective are represented by the thermal expansion coefficients of the laminate.*

Key words: *thermal expansion, lamina, laminate, composites.*

1. Introduction

Composite materials and structures may provide multiple benefits, first of all with respect to weight. Their mechanical properties may be tailored according to the imposed loading conditions by changing the fibre or matrix, the fibre orientation or the stacking sequence. All these degrees of freedom allow optimizing the structure in order to obtain the best stiffness and strength to weight ratio [4]. In addition to the applied mechanical loads, thermal loads may also occur when a system is working within a large interval of temperatures. Especially when metal parts are in contact with the composite parts, care must be taken within the design process due to different thermal behaviour of materials. Therefore, one of the problems when joining dissimilar materials is referring to thermal expansion. Connecting materials with different thermal expansion coefficients usually generates additional stress in the structure. If not

properly design, the connection may fail under the combined thermal and mechanical load.

This article presents a design method of the fibre reinforced composite laminates in order to obtain thermal expansion coefficients equal to the one of the metal that the composite part must be in contact with. After presenting the theoretical background a calculation example is given in order exemplify the presented design method.

2. Thermal Expansion of Lamina

Having given the fibre and matrix individual properties, the overall properties of the lamina (one single ply) may be determined by using the Rule of Mixture approach [5]. The thermal expansion coefficients of the lamina in local coordinates (1-2), α_1 and α_2 will therefore be determined with Equation (1) for 1-direction and with Equation (2) for 2-direction, Figure 1.

---

[1] Dept. of Mechanical Engineering, *Transilvania* University of Braşov.

$$\alpha_1 = \frac{v_f \alpha_{f1} E_{f1} + v_m \alpha_m E_m}{v_f E_{f1} + v_m E_m}, \qquad (1)$$

$$\alpha_2 = v_f \alpha_{f2} \left(1 + \frac{\nu_{f12} \alpha_{f1}}{\rho_f} \right) \qquad (2)$$
$$+ v_m \alpha_m (1 + \nu_m) - (v_f \nu_{f12} + v_m \nu_m) \alpha_1,$$

where:

α_{f1} - thermal expansion coefficient of the fibre, in the fibre direction;

α_{f2} - thermal expansion coefficient of the fibre perpendicular to the fibre direction;

α_m - thermal expansion coefficient of the matrix;

v_f - fibre volume fraction;

v_m - matrix volume fraction;

E_{f1} - Young's modulus of the fibre, along the fibre direction;

E_m - Young's modulus of the matrix;

ν_{f12} - Poisson's ratio of the fibre;

v_m - Poisson's ratio of the matrix;

ρ_f - density of the fibre.
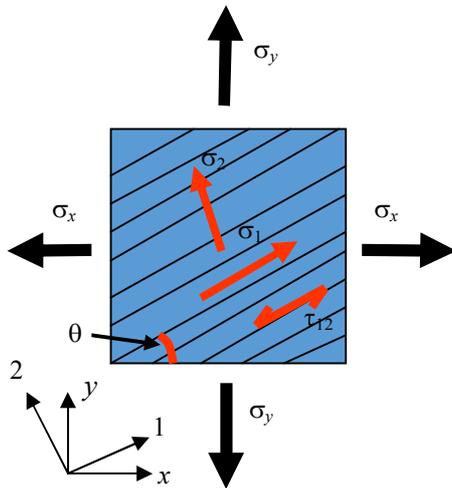
Fig. 1. *Local and global coordinates of the lamina*

3. Thermal Expansion of Laminate

Once the thermal expansion coefficient are determined in local coordinates (1-2)

for individual plies and based on a predefined stacking sequence, the thermal expansion coefficients of the laminate in global coordinates α_x, α_y and α_{xy} may be determined using Equation (3). In Equation (3), **A** represents the extensional stiffness matrix, calculated using Equation (4):

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} = 0.5 [\mathbf{A}]^{-1} \begin{bmatrix} K_1 V_{0A} + K_2 V_{1A} \\ K_1 V_{0A} - K_2 V_{1A} \\ K_3 V_{2A} \end{bmatrix}, \qquad (3)$$

$$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}. \qquad (4)$$

Equations (5) to (11) represent the components of the extensional stiffness matrix **A**:

$$A_{11} = U_1 V_{0A} + U_2 V_{1A} + U_3 V_{3A}, \qquad (5)$$

$$A_{22} = U_1 V_{0A} - U_2 V_{1A} + U_3 V_{3A}, \qquad (6)$$

$$A_{12} = U_4 V_{0A} - U_3 V_{3A}, \qquad (7)$$

$$A_{66} = U_5 V_{0A} - U_3 V_{3A}, \qquad (8)$$

$$A_{16} = 0.5 U_2 V_{2A} + U_3 V_{4A}, \qquad (9)$$

$$A_{26} = 0.5 U_2 V_{2A} - U_3 V_{4A}, \qquad (10)$$

with:

$$V_{0A} = h, \qquad (11)$$

$$V_{1A} = \sum_{k=1}^{n} t_k \cos 2\theta_k, \qquad (12)$$

$$V_{2A} = \sum_{k=1}^{n} t_k \sin 2\theta_k, \qquad (13)$$

$$V_{3A} = \sum_{k=1}^{n} t_k \cos 4\theta_k, \qquad (14)$$

$$U_5 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}), \quad (20)$$

$$V_{4A} = \sum_{k=1}^{n} t_k \sin 4\theta_k, \qquad (15)$$

$U_1 ... U_5$ represents the Tsai-Pagano material invariants [3] and they are calculated through Equations (16)-(20), where:

where:

h - represents the total thickness of the laminate;

n - represents the number of lamina;

θ_k - represents the orientation angle of the kth lamina.

$$Q_{11} = \frac{E_1}{(1 - \nu_{12}\nu_{21})},$$

$$U_1 = \frac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}), \quad (16)$$

$$Q_{22} = \frac{E_2}{(1 - \nu_{12}\nu_{21})},$$

$$U_2 = \frac{1}{2}(Q_{11} - Q_{22}), \qquad (17)$$

$$Q_{12} = \frac{\nu_{12}E_2}{(1 - \nu_{12}\nu_{21})},$$

$$U_3 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}), \quad (18)$$

$$Q_{66} = G_{12}.$$

K_1, K_2, K_3 from Equation (3) represents the thermal material constants, defined through the Tsai-Pagano constants by Equations (21)-(23):

$$U_4 = \frac{1}{8}(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}), \quad (19)$$

$$K_1 = (U_1 + U_4).(\alpha_1 + \alpha_2) + U_2(\alpha_1 - \alpha_2), \qquad (21)$$

$$K_2 = U_2(\alpha_1 + \alpha_2) + (U_1 + 2U_3 - U_4)(\alpha_1 - \alpha_2), \qquad (22)$$

$$K_3 = U_2.(\alpha_1 + \alpha_2) + 2(U_3 + U_5)(\alpha_1 - \alpha_2). \qquad (23)$$

4. Case Study

A tube is made of Carbon Fibre Reinforced Plastics with the external radius $R_1 = 50$ mm. A metal component is fixed inside the shaft, having the radius $R_2 = 50$ mm, Figure 2.

Let us consider that this assembly will operate in a range of temperatures from 20 to 200 Celsius degrees. In terms of the thermal expansion coefficients, thermal deformations will occur in both materials, according to Equation (24):

$$\Delta R = \alpha_y R \Delta T. \qquad (24)$$
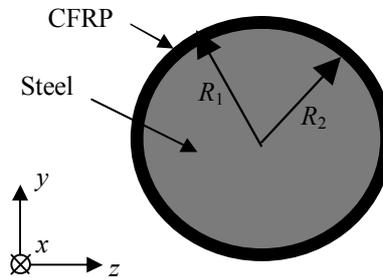
Fig. 2. *Steel-composite connection*

While for steel, polymer matrix and carbon fibre the thermal expansion coefficients are known, Table 1, in case of

the CFRP laminates the thermal expansion coefficient will strongly be influenced by the individual properties of the matrix and fibre and also by the plies' orientation angles [2], Equation (2).

According to the example given above, and considering Equation (4) at 200 Celsius degrees, the radius of the steel part R_2 will increase from 50 mm to 50.1449 mm while the radius of the CFRP part R_1 will increase from 50 mm to only 50.0071 mm. This means that the steel part will press on the CFRP part and thus generating additional in-plane stress within the structure.

Table 1
Thermal expansion coefficients [1]

Thermal expansion [1/°C]	Steel	Carbon fibre	Epoxy resin
α_1	16E-6	-0.6E-6	55E-6
α_2	16E-6	8.5E-6	55E-6

As it can be noticed from Table 1, the steel has a larger thermal expansion coefficient than the fibre but lower than the matrix. Moreover, the fibre is actually contracts at high temperature, having a negative thermal expansion coefficient. In order to avoid additional stress caused by the larger thermal deformations of the steel part in comparison to the one of the CFRP part, it is therefore important to design the laminate such that it has similar thermal expansion coefficient as the steel has. This problem may be formulated as an optimization problem where the objective is to find the configuration of the laminate that will give the desired thermal expansion coefficient. Considering that fibre and matrix properties are given, Table 1, the problem is actually reducing to the optimization of the orientation angles θ_k where $k = 1…n$, and n represents the total number of plies.

The mathematical model described in section 2 was defined within Excel.

HyperStudy software [6] was then used to solve the optimization problem by linking the Excel file to the solver. The objective function consist of reaching a target value of 16E-6 [1/°C] for α_y - thermal expansion along Y direction of the composite laminate, Figure 1, and thus to obtain the same thermal deformation as steel along the radial direction of the tube, Figure 2. The predefined stacking sequence is [45/-45/45/-45/45/-45]$_S$ which represents a symmetric laminate with 12 layers. θ_k represent the design variables; the laminate being symmetric, we have only 6 different θ angles as independent variables. The in-plane stiffness parameters of the laminate E_x and E_y are also considered within the optimization as constraints: $E_x \geq 10000$ MPa and $E_y \geq 10000$ MPa.

5. Results and Conclusion

Figure 3 illustrates the values obtained for α_x and α_y. As expected, while α_y reach target value of 16E-6 [1/°C], α_x is decreasing and becomes negative (the material contracts at high temperatures).
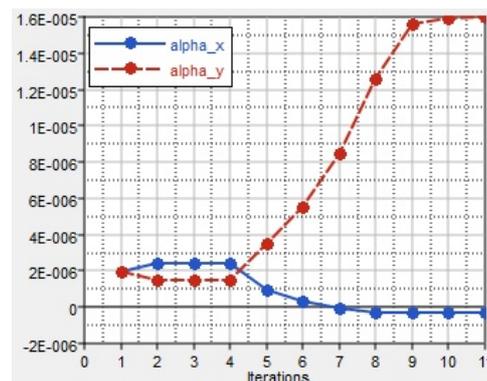
Fig. 3. T*hermal expansion coefficients of the laminate*

Figure 4 shows the values obtained for the plies' orientation angles that allow obtaining the desired value of α_y.

Although the desired thermal expansion behaviour is reached, the mechanical

properties of the composite laminate are altered along Y-direction due to the resulted staking sequence [20/-20/20/-20/20/-20]$_S$.
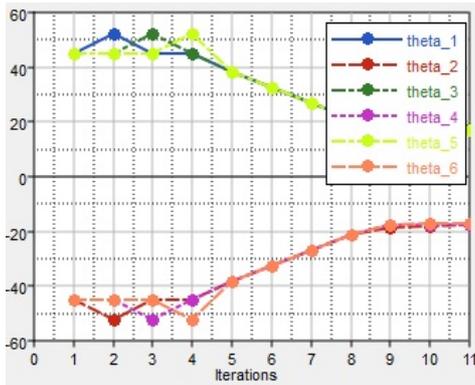
Fig. 4. *Values of the plies' orientation angles*

Figure 5 presents the values obtained for the in-plane stiffness parameters E_x and E_y of the laminate. By looking on both Figure 4 and Figure 5, it turs out that the laminate stiffness increase while the thermal expansion coefficient decrease, along each of the in-plane orthogonal direction X and Y.
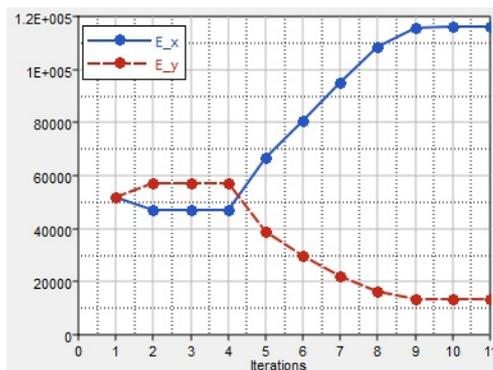
Fig. 5. *The resulted stiffness of the laminate*

This observation is also visible from Figure 6 and Figure 7: small values for the thermal expansion coefficients imply large values for the laminate stiffness and vice versa. This behaviour is disadvantageous for

the here-in considered case study. A trade of between high stiffness and low thermal expansion coefficient is needed. However, there are situations where high stiffness and low thermal expansion are of interest.
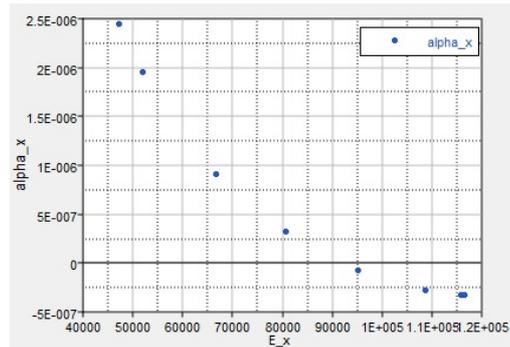
Fig. 6. *Thermal expansion coefficient* α_x *of the laminate in terms of the laminate stiffness* E_x *along X-direction*

Future studies may also consider the stiffness and strength of the laminate along both in-plane directions as objectives to be maximized. A trade-off solution may be selected this way by considering both mechanical and thermal loads.
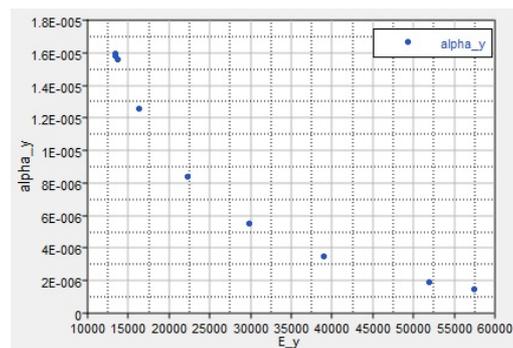
Fig. 7. *Thermal expansion coefficient* α_y *of the laminate in terms of the laminate stiffness* E_y *along Y-direction*

Acknowledgements

References

1. Åström, B.T.: *Manufacturing of Polymer Composites*. Nelson Thornes Ltd., 2002.
2. Marín, L., Trias, D., Badalló, P., Rus, G., Mayugo, J.A.: *Optimization of Composite Stiffened Panels Under Mechanical and Hygrothermal Loads Using Neural Networks and Genetic Algorithms*. In: Composite Structures **94** (2012), p. 3321-3326.
3. Tsai, S.W., Melo, J.D.D.: *An Invariant-Based Theory of Composites.* In: Composites Science and Technology **100** (2014), p. 237-243.
4. Velea, M.N., Wennhage, P., Zenkert, D.: *Multi-Objective Optimisation of Vehicle Bodies Made of FRP Sandwich Structures*. In: Composite Structures **111** (2014), p. 75-84.
5. Vinson, J.R.: *Plate and Panel Structures of Isotropic, Composite and Piezoelectric Materials, Including Sandwich Construction.* Dordrecht. Springer, 2005.
6. Altair. HyperWorks 11: *Online Help and Documentation*, 2012.