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KINEMATIC ANALYSIS OF AN ANALOGUOUS HUYGENS PENDULUM BEHAVIOUR USING INVERTED TOOTH CHAIN

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Abstract: *This paper presents the results of the kinematic analysis of an analogous Huygens pendulum using Inverted Tooth Chain (I.T.C); the simulation was done by the aid of MSC ADAMS VIEW. During the simulation process one could see the behaviour of the Inverted Tooth Chain by observing the motion of the chain plates in relation to the global coordinate axes. Also, it presents the possibility of obtaining various measurements of different angular displacements, velocities and accelerations of each plate individually, which in return give relevant information on the system behaviour.*

Key words: *multibody approach, inverted tooth chain, angular displacement, velocity, acceleration.*

1. Introduction

As there are different types of chain driving systems, it is important to question what type of chain systems behave better in an engine and why. It is essential to understand how the chains affect different systems. For this purpose different types of chains have to be compared in order to understand how they behave in different scenarios.

The Inverted Tooth chain consists of special designed inner and middle plates, assembled together with riveted pins. Outer plates ensure the chain guidance around the sprockets. The plates with the pins joints can be press fit or slip fit. An example of I.T. chain construction is given in the Figures 1 and 2. The inner and middle plates are designed in a way to decrease the contact forces and reduce the noise of high speed engines. The outer plates act like a planar constraint to assure

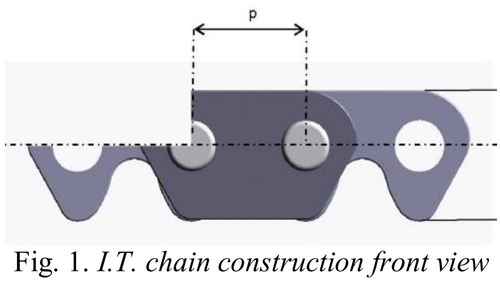

Fig. 1. *I.T. chain construction front view*

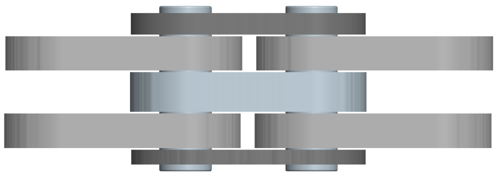

Fig. 2. *I.T. chain construction top view*

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that the links won't deviate from the plane and all the plates are well pressed. The I.T. chains are characterized by the pitch "p" which represents the distance between the axes of two consecutive pins, and the lacing (number of inner plates and pairing).

The inverted tooth chains are characterized by high power density and produce less noise hence, they are also called silent chains.

This paper analyses the kinematic behaviour of an analogue Huygens pendulum with Inverted Tooth Chains (I.T.C) and a sprocket.

2. The Theoretical Approach

There are "N" links in a chain that are connected together with revolute joints.

A revolute joint gives one degree of freedom and blocks the rest of the motions. Therefore, it can be assumed that there is no vibration emitted on the Y-Z plane. The rest of the chain parts that are not in contact with the sprocket could be considered as a semi rigid body due to an enforced force that would strain the chain. Figure 3 shows the analogue Huygens pendulum model, developed with MSC ADAMS software.

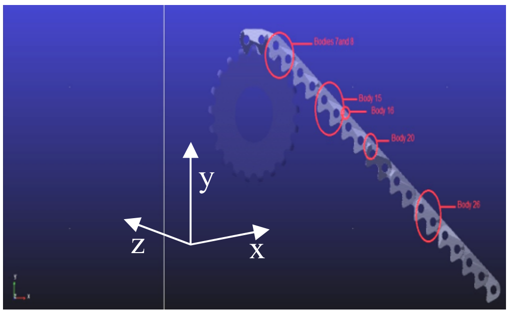

Fig. 3. *Huygens' pendulum model by MCS ADAMS software*

Due to the possibility of creating markers on every plate with a centre of mass "*c*" and location of interests, as shown in Figure 4, it is possible to anticipate the next movement of a specified plate having the general formula of a marker added on the plate [3], [4]:

$$r^i = R^i + A^i \overline{U}^i, \tag{1}$$

where: r_i is the vector of the displacement in rotation of a given marker on a rigid body; A_i is the transformation matrix from the local coordinate system to the global coordinate system; R_i is the global position vector; U_i is the position vector of a point "P" in the body coordinate system.

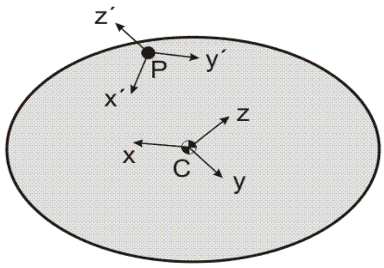

Fig. 4. *The body after adding a marker point P* [3]

The marker point is generally created at an important point of a chain plate; the important markers are the points created at the center of the holes of each plate and its center of mass of the plate.

Since there is an anologue Huygens pendulum effect due to a fixed centroid around which the chain oscilates against the sprocket, a mono-involute motion is being created. In general, the tangent to an involute motion is normal to the evolution of the intersections of the curves made by the motion of the chain.

If we use the cycloid parameters in Huygens pendulum effect in plane we have [2]:

$$x = \frac{\rho}{4}(2\theta + \sin 2\theta), \quad y = \frac{\rho}{4}(1 - \cos 2\theta), \tag{2}$$

where: *x, y* are the coordinates of the plate in the XY plane; ρ is the distance between the point where the oscillations begin and the centre of mass of the plate; θ is the angle of rotation between the point where the oscillations begin and the centre of mass of the plate.

3. Setting the Model in MSC ADAMS

In order to develop the Huygens pendulum model, the Hiller-Anantharaman Stiff Integrator (HASTIFF) was used. Its main benefit is that it does not stop easily the simulation calculations as GSTIFF and it typically requires fewer function evaluations to proceed with the simulation. One of the main priorities of using HASTIFF integrator is an emphasis on accuracy rather than speed. It is mainly used where importance is given to the analysis of velocity, acceleration and forces. The HASTIFF results should have fewer numerical discontinuities. A sub-integrator S11 is used as a fixed step size so it won't decrease or change the step size of the integrator as the model analysed is relatively complicated [5].

However, there are many limitations when analysing large models, as for example when using large constants, or when having a lot of body parts that are actively dynamic rather than static, as well as limitations due to the dependency on the processor power of the PC used [1].

4. Simulation Results

4.1. Angular Displacements Graphs

A general notice to consider is that the sprocket has global coordinates, while all the rest of the bodies have their own local coordinates that change in time due to the forces affecting the system at different intervals of time. The angular displacement graphs help to define the position of a certain body "*j*" in time.

Figure 6 represents the angular displacement of the body number 20 which has a complete oscillation and the body number 16 which has an impact with the sprocket. Figure 5 represents the angular displacement of the impacted body number 16 with respect to the sprocket which is fixed to the ground (representing the global coordinates).

Figures 7 and 9 represent different bodies at different positions of the chain which perform a complete oscillation around the sprocket. These bodies oscillate in respect to the global coordinates positioned at the centre of mass of the sprocket. Figure 8 represents the oscillation of two bodies that would have to complete an oscillation, but due to the effect of the double pendulum they prove not to keep the alignment, even though the forces that move the pendulum are high and should tension the pendulum as a whole.

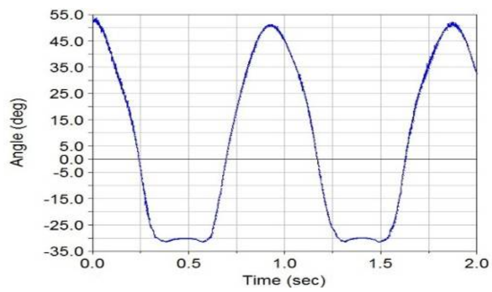

Fig. 5. *The angular displacement of body_16, that is the last body subjected to impact and the sprocket*

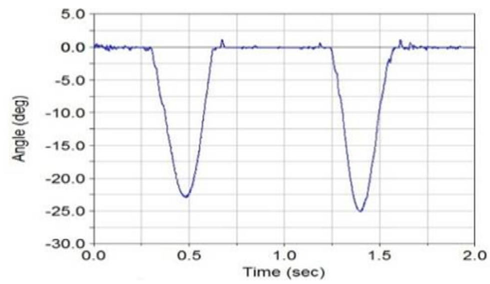

Fig. 6. *The angular displacement between body_20 and body_16*

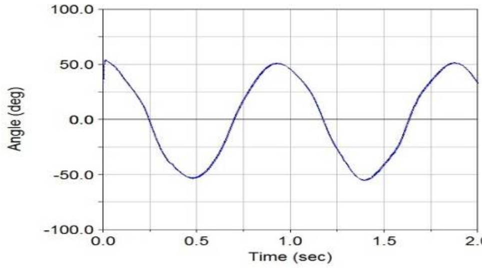

Fig. 7. *The angular displacement between body_20 and the sprocket*

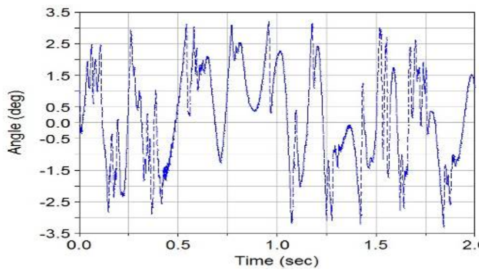

Fig. 8. *The centre of mass body_26, related to the centre of mass of body_20*

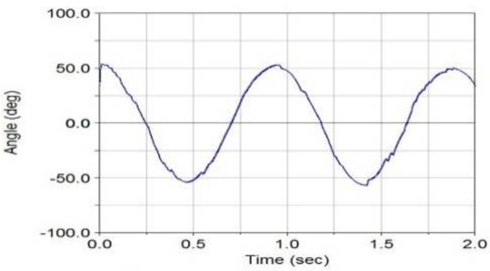

Fig. 9. *The centre of mass of body_28 related to the sprocket*

4.2. Angular Velocities and Acceleration Graphs

• **Angular velocities**

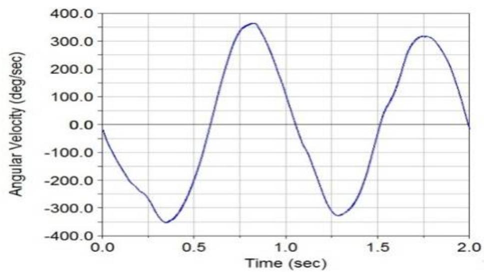

Fig. 10. *The angular velocity of body_20 about "Z" axis*

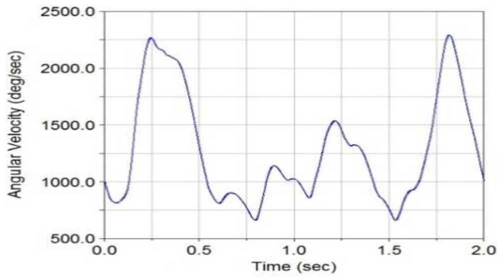

Fig. 11. *The magnitude of the angular velocity of the body_20*

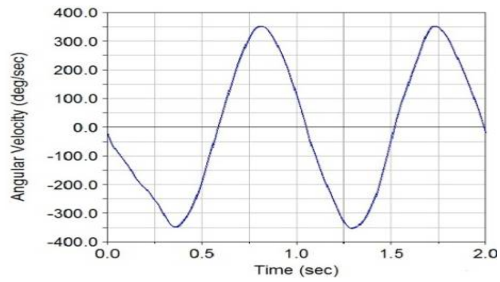

Fig. 12. *The angular velocity about "Z" axis of body_26*

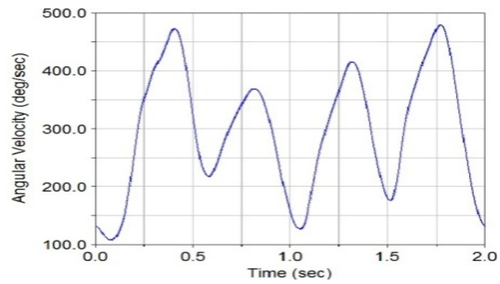

Fig. 13. *The magnitude of the angular velocity of body_26*

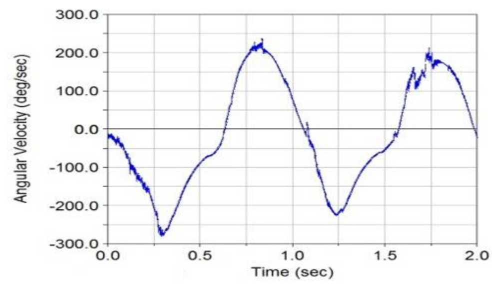

Fig. 14. *The angular velocity about "Z" axis of body_16*

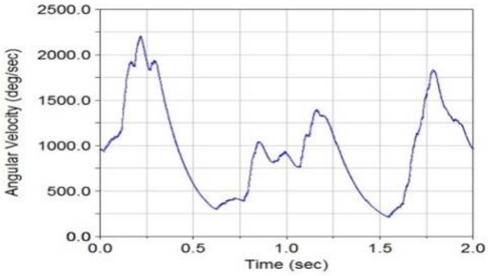

Fig. 15. *The magnitude of the angular velocity of body_16*

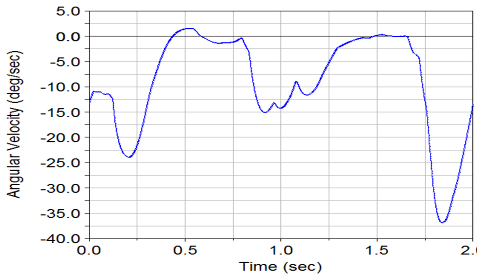

Fig. 16. *The angular velocity about "Z" axis of body_10*

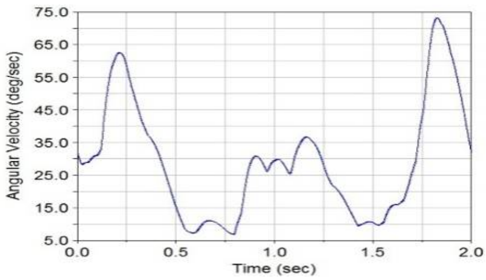

Fig. 17. *The magnitude of the angular velocity of body_10*

- **Angular accelerations**

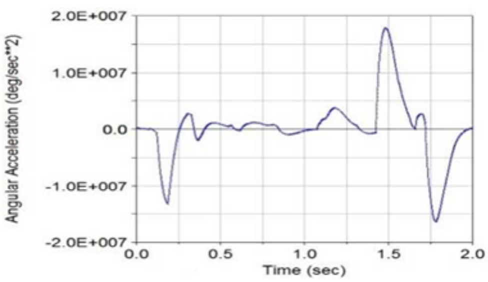

Fig. 18. *The angular acceleration of body_20 about "Z" axis*

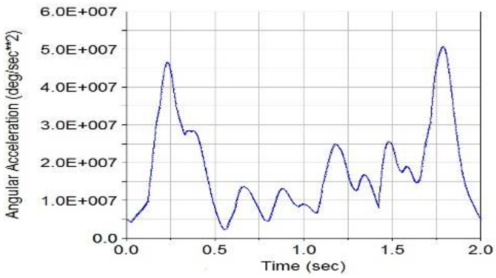

Fig. 19. *The magnitude of the angular acceleration of body_20*

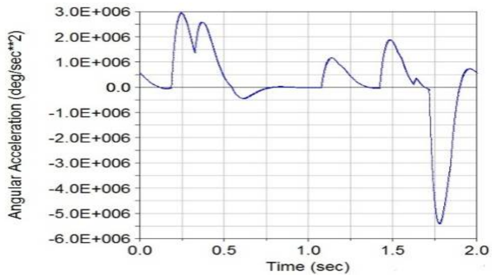

Fig. 20. *The angular acceleration of body_26 about "Z" axis*

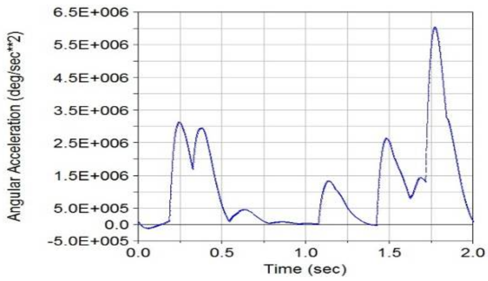

Fig. 21. *The magnitude of the angular acceleration of body_26*

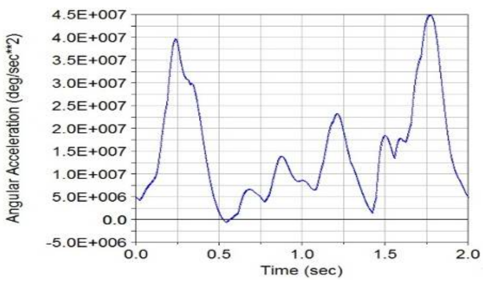

Fig. 22. *The magnitude of the angular acceleration of body_16*

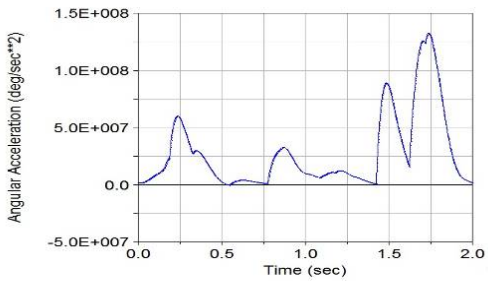

Fig. 23. *The magnitude of the angular acceleration of body_10*

Analysing the angular acceleration and velocity graphs, one may formulate the following observations:

1. From Figures 10 and 18 it is concluded that the bodies which perform a complete oscillation positioned near the bodies that are subjected to impacts inherit much of the vibration and cause a noisy signal, while Figures 12 and 20 the bodies which perform a complete oscillation positioned further away from bodies exposed to impacts have cleaner signals and are more stable.
2. The angular velocities and accelerations of bodies subjected to impacts react against the movement of the rest of the chain or the pendulum system. Yet, bodies further away from impact situations tend to respect the total velocity and acceleration of the pendulum system itself.
3. Each link of the chain tends to have its own properties admitting that Huygens' pendulum does not give a uniform harmonic oscillation due to the generated forces such as the centrifugal and centripetal forces and the centroid which is generated by the impacting bodies which tend to tilt and un-align the pendulum system.

5. Conclusions

The kinematic simulation of a Huygens pendulum with an inverted tooth chain and sprocket system is successfully done. The effect of double pendulum is clearly observed. It's surely obvious that the system depends on the rigidity of the chain. When the rigidity is higher the effect of double pendulum will tend to disappear. The most stressed links are those exactly after the last link subjected to contact or impact with the sprocket teeth.

Acknowledgements

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