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EFFECT OF FRICTION ON TRANSMISIBILITY IN COULOMBIAN DAMPER

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Abstract: *The friction modifies the all properties of Coulombian damper (dry and mixed friction). The theoretical dynamic model of the gasket movement in the tube of the washing machine damper is proposed. The friction is analyzed that a function to the sliding velocity (Striebeck curve-continuum and derivable curve). The force transmissibility and the displacement transmissibility of damper are modified by the static friction coefficient and the gradient of the kinetic friction with the sliding velocity. The Coulombian damper can be the excitatory when the friction decreases with the sliding velocity.*

Key words: *Friction damper; Striebeck curve; Transmissibility by friction; Dynamic model.*

1. Introduction

The undesirable effects of vibration are reduced with the isolation device. Basically, it involves the insertion of a resilient member (or isolator) between the vibrating mass and the source of vibration so that a reduction in the dynamic response of the system is achieved under specified conditions of vibration excitation. An isolation system is said to be active or passive depending on whether or not external power is required for the isolator to perform its function. A passive isolator consists of a resilient member (stiffness) and an energy dissipater (damping). Examples of passive isolators include metal springs, cork, felt, pneumatic springs, and elastomer (rubber) springs [1-3]. Vibration isolation can be used in two types of situations. In the first type, the foundation or base of a vibrating machine is protected against large unbalanced forces. In the second type, the system is protected against the motion of its foundation or base. The dampers with the dry and limit friction have the functional key element two cushioning gasket (ring). This cushioning gasket is made of rubber, soaked with lubricant till saturation (oil or grease). These dampers also are named the Coulombian dampers (shock-absorbers). The Coulombian dampers are frequently used in the machine building, washing machines, and so on.

The Coulombian damper are classified in direct absorbers (fixed in the outer shell), or reverse absorbers (attached to the rod)

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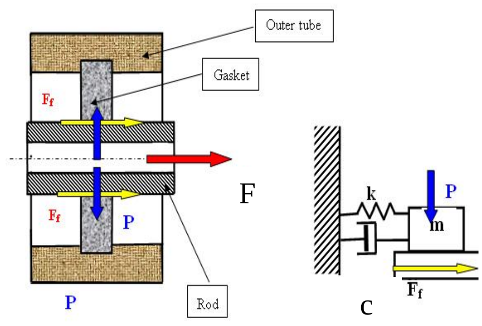
as a function to the method of gasket radial fixing. The physical phenomenon of damping vibration is based on the friction between the cushioning gasket and the output regulator of the damper. The output regulator is the steel rod of the direct damper or the damper housing for the “reverse” damper. The friction can be dry, limit, mixed or fluid as a function to the velocity and the contact pressure for the work conditions of damper.

In these conditions, it is proposed that the friction may be characterized by a Striebeck curve type (friction coefficient μ - sliding velocity *v*) [4]. The friction may damp a vibratory motion or can maintain it (self-vibration), depending on the friction coefficient variation with the velocity (it can increases, decreases or it can be relative constant).The friction force is a disturbing (excitation) force which is partial or total transferred through the damper (absorber) or it can maintain the vibratory motion [5-7].

The aims of this paper is to analyze the damping phenomenon in a system with the variable friction (Striebeck type) and the harmonically excitation of rod (Figure 1), and to determine the transmissibility of the direct Coulombian damper. Some discussions are made regarding the performances of an automatic washing machine damper.

2. Analytical Model for Vibratory Movement in the Colombian Damper

The geometry of the Colombian damper is presented in Figure 1a. The damping phenomena appear in the contact of the rod with two gaskets, when the mechanical energy is transferred to outer tube by friction. The move of the gaskets and the outer tube is analyzed with the theoretical model (Figure 1 b). The mass of the gasket and outer tube (*m*), the axial rigidity (*k*) and the damping coefficient (*c*) of gasket material (rubber or sponge), the gasket geometry (inner diameter D_g, width b_g) and the friction curve of the rubber or the sponge (Striebeck type) are known.



a) b)

Fig. 1. *Scheme forces (a) and the dynamic model (b) in the Coulombian damper.*

As a model it is proposed the system from Figure 1b, consisted of a material point having “*m*” mass, supported with the Striebeck friction (F_f) on a rigid body. This body is loading with harmonically variable force:

$$F = F_o \cos(\omega t), \quad (1)$$

where F_o – is the axial force amplitude of the rod; ω - the frequency of the rod force; *t* – the time.

The friction force between the material point and the rigid support (F_f) depends on the relative velocity between rod and gasket and the contact pressure. The sliding velocity appears only when the force *F* is greater than the friction force F_f. Thus, upon the material point will act the forces: – $c\,(dx/dt)$, kx and ($F_f - F$) (Fig. 1b). The friction force (F_f) depends on the friction coefficient (μ) and the normal force (F_n). The normal force is generated by the contact pressure (p_m) between the rod and the gasket.

Thus,

$$F_f = \mu F_n = \mu_S \mu_a 2\pi\, D_g b_g \cdot p_m = k_a \mu_a \quad (2)$$

where the parameter $k_a = \mu_S 2\pi D_g b_g p_m$ depends on the static friction characteristics of the gasket material (μ_S), the gasket geometry (D_g, b_g) and the radial contact pressure (p_m). The radial contact pressure is defined the fitting assemblage and the elastic characteristics of the gasket material. The sliding friction coefficient relative to the static friction, $\mu_a = \varepsilon / \varepsilon_s$, is defined in paper [4]:

$$\mu_a = 1 \; if \; 0 \le v_{am} \le v_{om};$$

$$\mu_a = a_a v_{am}{}^2 + b_a v_{am} + c_a \; if \; v_{om} \le v_{am} \le v_{ocr}$$

$$\mu_a = c_{ha} v_{am} + \mu_o \; if \; v_{am} \ge v_{ocr}, \qquad (3)$$

where a_a, b_a, c_a, c_{ha} and ε_o are the friction characteristic of the Striebeck curve (continuum and derivable) and $v_{am} = v / v_m$; $v_{om} = v_o / v_m$; $v_{ocr} = v_{cr} / v_m$ dimensionless velocities: v - current sliding velocity, v_m - velocity at the minimum friction coefficient, v_o ї minimum velocity when beginning the slide between rod and gasket, v_{cr} - minimum velocity when beginning the fluid friction.

This system (fig. 1b) can be idealized as a single degree of freedom and the differential equation of motion with the excitation will be:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F - F_f = F_o \cos(\omega t) - k_a \mu_a \cdot \qquad (4)$$

The damping coefficient of the rubber or the sponge (c) is assumed to be inversely proportional to the frequency as $c = h / ɣ$, where h is called the hysteresis damping constant and б=h/k is dimensionless hysteresis damping [8].

The equation (4) can be written in the classic mode [8], [9], [10]. In the dynamic model of the friction damper, the equation (4) has the form:

$$\frac{d^2x}{dt^2} + 2\alpha\, p\frac{dx}{dt} + p^2 x = f\cos(\omega t) - q\mu_a, \qquad (5)$$

where: $2\alpha = c/(mp)$ = б/(rm), (Ŭ -the damping ratio); $p^2 = k/m$, (p- the natural (proper) circular frequency of gasket (rubber ring); r = ɣ /p ї the frequency ratio; $f = F_o/m$ ї the excitation parameter; $q = k_a/m$- the friction parameter.

The solutions of equation (5) have different forms, depending on the mechanical properties of the gasket (α, p), on the excitation of the rod (f, ɣ), on the friction parameter (q) and the friction coefficient (ε_a).

Since this equation is nonhomogeneous, its general solution $x(t)$ is given by the sum of the homogeneous solution ($x_h(t)$), and the particular solution ($x_p(t)$). The homogeneous solution (transient solution), which is the solution of the homogeneous equation (left part of (5)) is the free vibration. This vibration dies out with time under each of the three possible conditions of damping (underdamping, б < 2r, critical damping, б = 2r, and overdamping, б > 2r) and under all possible initial conditions. Since the transient solution dies out after some time, only the steady-state solution (particular) will be left. In this case, the friction coefficient depends only to sliding velocity of rod ($v = dx_p/dt$) and the points of the Striebeck curve. The characteristics points (velocity and friction coefficient) of Striebeck curve are: the initial point $A(0, \varepsilon_s)$;- the static friction point $B(v_o, \mu_s)$, the minimum friction point $C(v_m, \mu_m)$ and the critical point $D(v_{ocr}, \mu_o)$. This curve is necessary to be obtained by experiments for the material combination. In the case of Coulombian damper, the material combination is rubber (sponge) ї steel. The preliminary experimental results (rubber - steel), obtained in the Tribology laboratory of the POLITEHNICA University of Bucharest, are A(0, 1.2), B(0.001, 1.2), C(0.8, 0.06) and D(2.6, 0.08). The unit measure of velocities is m/s. Thus, preliminary dimensionless parameters of rubber- steel friction curve are

$v_{am} = v / v_m$; $v_{om} = v_o / v_m = 0.0012$;
$v_{ocr} = v_{cr} / v_m = 2.6$; $\mu_o = 0.08$;
$\mu_m = 0.05$; $a_a = 0.95$; $b_a = 1.90$;
$c_{ha} = 0.13$; $v_{acr} = 1.07$; $n = -0.088$.

The theoretical Striebeck curve which has the points A, B, C and D is shown in the Figure 2.

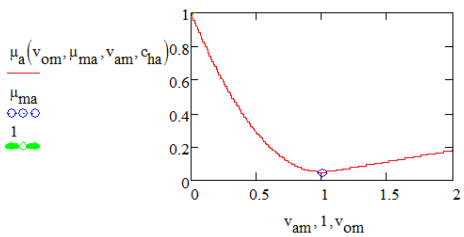


Fig. 2. *Dimensionless Striebeck theoretical curve.*

The slip between gasket and rod appears only when it is satisfied the condition

$$f\cos(\omega t) - q\mu_a \geq 0\,. \qquad (6)$$

This restriction can explain the stick- slip phenomenon. The amplitude of stick- slip movement increases when the friction coefficient decreases and the velocity increases [4]. The particular solution of Eq. (5) is also expected to be harmonic ($x_{pf}(t)$); we assume it in the form

$$x_{pf}(t) = X_1\cos(\omega t - \phi) - X_2\, sign(dx_{pf} / dt) \qquad (7)$$

where the amplitudes (X_1 and X_2), the angle (◖) are defined by Eq. (5) and *sign(x)* is the *signus* function of the *x* argument. Thus,

$$X_1 = \frac{F_o}{k\sqrt{\left(1-r^2\right)^2 + (2\alpha\, r)^2}} = \frac{\delta_{st}}{\sqrt{\left(1-r^2\right)^2 + (2\alpha\, r)^2}} \qquad (8)$$

and $X_2 = \dfrac{F_f}{k} = \mu_s \mu_a \delta_r$,

where ŭ$_{st}$ is the deflection under static force F_o, and ŭ$_r$ is the radial deflection under static normal force F_n,

$$\delta_r = \frac{\pi p_m D_g b_g}{k}\,;\ \phi = \tan^{-1}\left(\frac{2\alpha\, r}{1-r^2}\right). \qquad (9)$$

The velocity of vibration

$$v_p(t) = \frac{dx_{pf}}{dt} = -X_1\omega\sin(\omega t - \phi)\,. \qquad (10)$$

This velocity will be used for the dependence of the friction coefficient Eq. (3). The main parameters of elongation Eq. (7) are damping ratio (Ŭ) of material, the friction static coefficient (ε$_s$) and the excitation (F_o, ɤ). Thus, Figures 3, 4 and 5 show, for example, the effect of these parameters about the elongation. The elongation curves were obtained in MATCHAD soft for the following parameters: p =10 rad/s, r = 2, ŭ$_r$ = 1 mm, v_{om} = 0.01, v_m = 0.8 m/s, μ_{ma} = 0.1 and c_{ha} = 0.01.

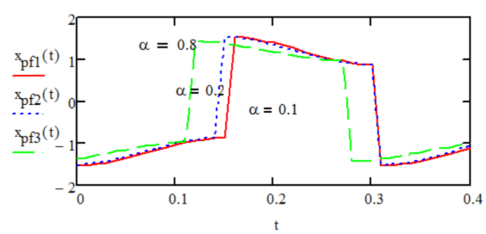


Fig. 3 *Effect of damping ratio (α) ~~about~~ on elongation $x_{pf}(t)$ (mm- s).*

2. The Transmissibility of Damper

Vibration isolation is a procedure by which the undesirable effects of vibration are reduced [1], [4], [8-10]. This isolation involves the insertion of a resilient member (or isolator) between the vibrating mass and the source of vibration so that a reduction in the dynamic response of the system is achieved under specified conditions of vibration excitation. Vibration isolation can be used in two types of situations.

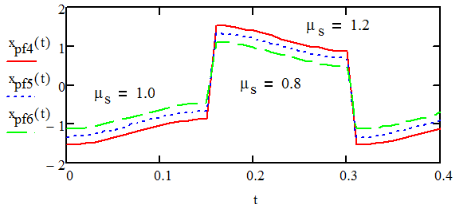


Fig. 4. *Effect of static friction coefficient (μ_s) ~~about~~ on elongation $x_{pf}(t)$ (mm- s).*

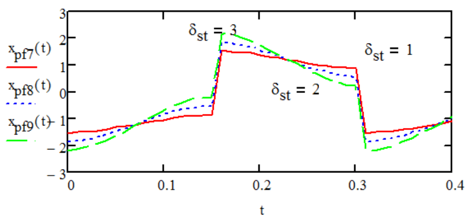


Fig. 5. *Effect of static deflection (δ_{st}) ~~about~~ on elongation $x_{pf}(t)$ (mm- s).*

a) *The first type of isolation* is used when a mass is subjected to a force or excitation. For example, in the case of reciprocating and rotating machines, the inherent unbalanced forces are transmitted to the base of the machine. In such cases, the force transmitted to the base, varies harmonically, and the resulting stresses in the base bolts also vary harmonically, which might lead to fatigue failure. The Coulombian damper can be used in both types of isolation.

The force transmitted to the base through the spring and the dashpot of material damper (rubber or sponge), is given by

$$F_t(t) = k\, x_{pf}(t) + c\, v_p(t)\,, \qquad (11)$$

with $x_p(t)$ and $v_p(t)$ from Eq. (7) and Eq. (10).

The magnitude of the total transmitted force (F_t) is given by

$$F_t = \frac{F_o\sqrt{k^2+\omega^2 c^2}}{\sqrt{(k-m\omega^2)^2+c^2\omega^2}} - F_f\,, \qquad (12)$$

and the amplitude of the total excited force

$$F_e = F_o - F_f\,. \qquad (13)$$

The force transmissibility or force transmission ratio of the damper (T_d) is defined as the ratio of the magnitude of the force transmitted (F_t) to that of the total exciting force (F_e). Thus, with Eq. (12) and Eq. (13) is obtained

$$T_d = \frac{F_t}{F_e} = \frac{M_f - \mu_s\mu_a\delta_{rs}}{1-\mu_s\mu_a\delta_{rs}}\,, \qquad (14)$$

with

$$M_f = \frac{X_1}{\delta_{st}} = \sqrt{\frac{1+(2\alpha\, r)^2}{(1-r^2)^2+(2\alpha\, r)^2}}\,. \qquad (15)$$

This parameter (M_f) is called the *force magnification factor, force amplification factor*, or *force amplitude ratio without friction* [8, 9]. The parameter ŭ$_{rs}$ is the relative deflection of the radial and the axial deflection (ŭ$_{rs}$ = ŭ$_r$/ ŭ$_{st}$).

In order to achieve isolation, the force transmitted to the foundation needs to be less than the excitation force. It can be seen, from Figures 6, 7 and 8, that the forcing frequency (ɤ) has to be greater than $\sqrt{2}$ times the natural frequency of the system (p) in order to achieve isolation of vibration. The isolation will be performing when the transmissibility is smaller. Thus, the magnitude of the force transmitted to the foundation can be reduced by increasing the relative deflection of material of gasket (grater radial deflection of rubber) (Figure 6).

The magnitude of the force transmitted to the foundation can be reduced by decreasing the natural frequency of the system (*p*), or to increase the damping ratio (Ŭ) (fig. 7).

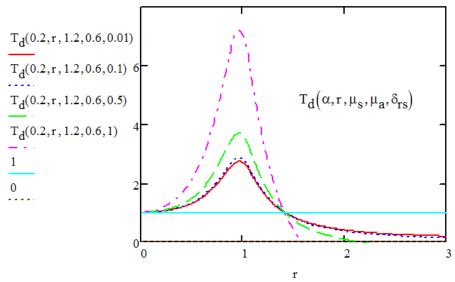


Fig. 6. *The effect of relative deflection (δ_{rs}) ~~about~~ on force transmissibility of damper (T_d)*

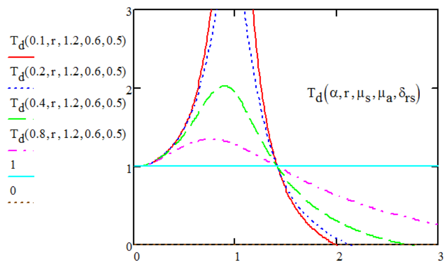


Fig. 7. *The effect of damping ratio (α) ~~about~~ on force transmissibility of damper (T_d)*

The effect of the friction (static friction - ε_s, form of Striebeck curve- v_o, v_m, ε_m, v_{cr}, c_h) is essential for the Coulombian damper. Thus, for example, the fig.8 shows the effect of the static friction coefficient about the transmissibility. The force transmitted to foundation decreases rapidly when the static friction increases.

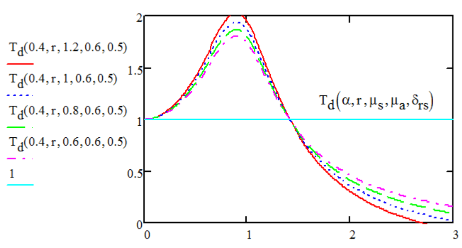


Fig. 8. *The effect of static friction (μ_s) ~~about~~ on force transmissibility of damper (T_d)*

In many applications, the isolation is required to reduce the motion of the mass under the applied force. The displacement amplitude of the mass *m* due to the force *F*(*t*), given by Eq. (8), can be expressed as

$$T_{dd} = \frac{kX}{F_o} = \frac{1}{\sqrt{(1-r^2)^2+(2\alpha\, r)^2}} - \mu_s\mu_a\delta_{rs}, \qquad (16)$$

where T_{dd} is called the *displacement transmissibility* or *amplitude ratio* and indicates the ratio of the amplitude of the mass, *X,* to the static deflection under the static force F_o ($\breve{u}_{st} = F_o/k$). The displacement transmissibility has maximum values (T_{ddmax}), when

$$\frac{dT_{dd}}{dr} = 0 \quad and \quad r = \sqrt{1-2\alpha^2}, \qquad (17)$$

and

$$T_{dd\,\max} = \frac{1}{2\alpha\sqrt{(1-\alpha^2)}} - \mu_s\mu_a\delta_{rs}. \qquad (18)$$

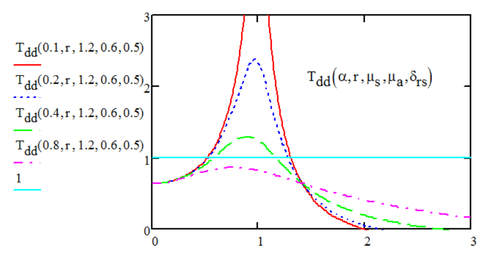


Fig. 9. *The effect of damping ratio (α) ~~about~~ on displacement transmissibility of damper (T_{dd})*

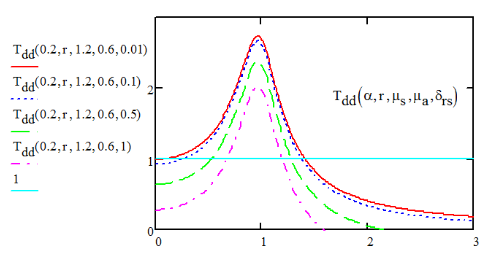


Fig. 10. *The effect of relative deflection* ($\tilde{u}_{rs}$) ~~*about*~~ *on displacement transmissibility of damper* (T_{dd})

For example, Figures 9, 10 and 11 show the effect of damping ratio, relative displacement and static friction coefficient about the displacement transmissibility.

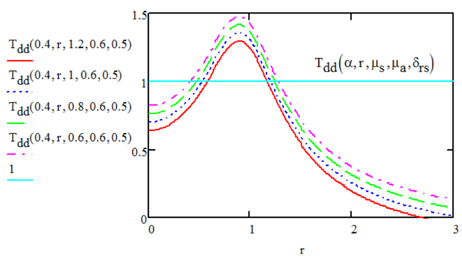


Fig. 11. *The effect of static friction* (μ_s) ~~*about*~~ *on displacement transmissibility of damper* (T_{dd})

The following observations can be made from Figures 9-11:

The displacement transmissibility increases to a maximum value at (Eq. (17)). Equation (17) shows that, for small values of damping ratio the displacement transmissibility (or the amplitude of the mass) will be maximum at $r \approx 1 \quad or \quad \omega \approx p$. Thus the value of is to be avoided in practice. If the excitation frequency is fixed, hence we can avoid $r \approx 1 \quad or \quad \omega \approx p$ by altering the value of the natural frequency p which can be accomplished by changing the value of either or both of m and k.

The amplitude of the mass, X, approaches zero as r increases to a large value. The reason is that at large values of r, the applied force $F(t)$ varies very rapidly and the inertia of the mass prevents it from following the fluctuating force.

The displacement transmissibility decreases rapidly when the static friction of material combination is greater.

An application of this theoretical model is the Coulombian damper used to wash automatic machine. The speed of this machine (forcing frequency) varies (increases and decreases). It is necessary one compromise in choosing the amount of damping to minimize the force transmitted. The amount of damping should be sufficient to limit the amplitude X_1 and the force transmitted while passing through the resonance (ɤ = p), but not so much to increase unnecessarily the force transmitted at the operating speed.

b) *The second type of isolation is used when a mass to be protected against the motion or excitation of its base or foundation* [8].

When the base is subjected to vibration, the mass m will experience not only a displacement $x(t)$ but also a force. The displacement of the mass $x(t)$ is expected to be smaller than the displacement of the base $y(t)$. For example, a delicate instrument or equipment is to be protected from the motion of its container or package (as when the vehicle carrying the package experiences vibration while moving on a rough road). The force transmitted to the mass also needs to be reduced. For example, the package or container is to be designed properly to avoid transmission of large forces to the delicate instrument inside to avoid damage. The force experienced by the mass m (same as the force transmitted to mass m) is given by [8].

$$F_t(t) = m\frac{d^2x}{dt^2} = k\{x(t) - y(t)\} + \\ + c\left\{\frac{dx}{dt} - \frac{dy}{dt}\right\}, \qquad (19)$$

here $y(t)$ is the displacement of the base.

If the displacement of the base is harmonic

$$y(t) = Y\sin(\omega t). \qquad (20)$$

By using the solution indicated by Eq. (7), the steady-state response of the mass, can be expressed as

$$x_{py}(t) = X_{1y}\sin(\omega\, t - \phi) - \\ - X_{2y}\ sign(dx_{py}/dt). \quad (21)$$

The ratio of the amplitude of the response $x_{py}(t)$ to that of the base motion $y(t)$, X/ Y, is called the *displacement transmissibility by friction.* The results are similarly to Eq. (14).

3. Conclusions

The Coulombian damper can be used when a mass is subjected to a force or excitation (*first type of isolation*) or when a mass to be protected against the motion or excitation of its base or foundation (*second type of isolation).*

The analytical dynamic model of damper can explain the effect of the damping ratio, the static and kinetic friction coefficient and the geometry of damper about the transmissibility.

The force transmissibility and the displacement transmissibility can be used to optimization of Coulombian damper design.

The friction coefficient in damper has the Striebeck curve type. The stick-slip phenomenon appears in the transitory period when the friction coefficient decreases and the vibration velocity increases.

References

1. Draghici, I., Ivan M., et.al.: *Suspensii si amortizoare*, (*Suspensions and dampers*) Ed. Tehnica, Bucuresti, 1979.
2. Ye Shao Chun, Williams Keith A.: *Torsional friction damper optimization*, Journal of Sound and Vibration 294 (2006) 529ï 546.
3. Bratu, P.: *Izolarea si amortizarea vibratiilor la utilajele de constructii* (*Isolation and damping vibrations from construction building*) Edit. INCERC, Bucuresti, 1982.
4. Wahad, H.S., Tudor, A., et. al.: *The effect of friction in Coulombian damper,* ROTRIB 16, Galati, 22-24 sept. 2016.
5. Al Sayed Baraa, Chatelet, E.et. al.: *Dissipated energy and boundary condition effects associated to dry friction on the dynamics of vibrating structures*, Mechanism and Machine Theory 46 (2011) 479ï 491.
6. Chengwu, D., Rajendra, S.: *Dynamics of a 3d of torsional system with a dry friction controlled path*, Journal of Sound and Vibration 289 (2006) 657ï 688.
7. Wu Ziying, Liu Hongzhao et.al.: *Identification of nonlinear viscous damping and Coulomb friction from the free response data*, Journal of Sound and Vibration 304 (2007) 407ï 414.
8. Rao, S.,S.: *Mechanical Vibrations*. Fift Edit. Prentice Hall, 2011.
9. Voinea, R., Voiculescu, D. et. al.: *Mecanica (Mechanics)*, Ed. Didactica si Pedagogica, Bucuresti, 1983.
10. Buzdugan, Gh., Fetcu, L. et al: , *Vibratii mecanice (Mechanical Vibrations)*. Ed. Did. Pedag. Bucuresti, 1982.