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NONCIRCULAR GEARS WITH TRANSMISSION RATIO AS HYBRID FUNCTION

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Abstract: *The paper deals with the design of noncircular gears whose output motion is a piecewise of several continuously variable motions. In order to achieve the desired gears kinematics, multiple parameters are used to define the transmission ratio sub-functions. The AutoCAD/ AutoLISP applications are used to perform the calculus and graphics. The data base enables to design and generate a noncircular gears train that would modify the kinematics of the crank-slider mechanism from a nail machine, in order to improve the nail head forming phase.*

Keywords: *variable transmission ratio, noncircular centrodes, noncircular gears, crank-slider mechanism.*

1. Introduction

Mechanisms that perform variable rotational motions are required in many industrial applications. One alternative of providing variable motion, by simple and compact mechanical devices, is the use of noncircular gears; most of them are elliptical gears [1] - [3], eccentric spur gears [4], oval gears [5] - [7], gears with lobes [8] whose transmission ratio is defined by a single variation law. There are also applications within the output motion should be modified only during a phase of the functioning cycle or, in order to fulfill the technological requirements, a single variation law of the output motion is not enough.

Several industrial applications that are using crank-slider mechanisms to generate motion require modifications of the slider kinematics that are usually achieved by using electric motors and additional mechanisms. The modification of the crank-slider mechanism kinematics has been approached by few researches. Doege et al. optimize the design of the noncircular gears used in the press drives, in order to reduce the pressure dwell time and to increase the cooling and handling times [9]. Mundo et. al. propose a modified crank-slider mechanism of a pressing machine, whose ram is driven, according to an optimized law motion, by a pair of noncircular gears [10]. Quintero et. al. present a novel modified crank-slider mechanism of an internal combustion engine, by introducing a noncircular gear pair [11]. Yokoyahama et. al. use a pair of noncircular gears into a powder compacting press to get a longer powder feeding time and a longer time for compacting process [12].

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2. Research Objectives

Usually applied for continuously variable motion outputs, noncircular gears are also designed for applications that require a combination of variable and uniform motions and even stop and dwell; in these cases, the transmission ratio could be a real analytic function or a hybrid function whose sub-functions defines the piecewise motions during the functioning cycle.

The authors are interested in the design of a noncircular gears train that would modify the kinematics of the nail forming process, in case of a traditional nail machine. By analyzing similar researches and considering the hypothesis of the noncircular centrodes modelling, the gear transmission ratio variation is identified as the starting point of the research itinerary. Therefore, the main objectives of the paper are as follows:

- to define a noncircular gears transmission ratio as a hybrid function, with multiple parameters, to be applied in industrial applications;
- to analyze the influence the transmission ratio definition has on the mechanism kinematics and centrodes geometry;
- to model the noncircular centrodes, based on the predefined transmission ratio;
- to generate a noncircular gears train, with the transmission ratio varying as previously mentioned, in order to modify the kinematics of the crank-slider mechanism from a MCC337 nail machine and to improve the nail head forming phase, respectively.

3. Defining Hybrid Transmission Ratio Function

Let us consider a gears train coupled to a conventional crank-slider mechanism, functioning as a motion generator. In case of using standard gears, with constant transmission ratio i_{21} = 1, the slider displacement and velocity, as functions of the driving gear rotational angle, have symmetric graphs. Trying to modify the slider stroke variation, Doege et. al. [9] proposed the use of a noncircular gear train with the transmission ratio varying as in Figure 1.

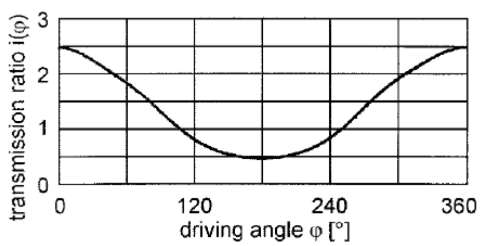


Fig. 1. *Noncircular gears transmission ratio proposed by Doege et. al. [9]*

Trying to get a similar variation for the transmission ratio that could be further manipulated in order to meet different requirements, the authors introduce a hybrid multi-parameters function to define the transmission ratio:

$$i_{21}(\varphi_1) = \begin{cases} c_1 \cdot \varphi_1^3 + c_2 \cdot \varphi_1^2 + c_3 \cdot \varphi_1 + c_4, \varphi_1 \in [0, \varphi_0] \\ c_5 \cdot \varphi_1^3 + c_6 \cdot \varphi_1^2 + c_7 \cdot \varphi_1 + c_8, \varphi_1 \in [\varphi_0, 2\pi] \end{cases}. \quad (1)$$

where φ_1 is the driving centrode/gear rotational angle; φ_0 - a variable angle that splits the functioning cycle into two phases; c_1é c_8 ï constants to determine as required by the theory of gearing and machine technical parameters.

The transmission ratio should be a positive function, varying within imposed limits, $i_{21} \in [a, b]$, where a and b are the chosen minimum and maximum values; it should also be a continuous, derivative and periodical function.

Considering the transmission ratio defined by equation (1), the driven centrode/gear and crank rotational angle, respectively, is determined as:

$$\varphi_2(\varphi_1)=\int_0^{\varphi_1} i_{21}(\varphi)d\varphi=\begin{cases}\frac{c_1}{4}\cdot\varphi_1^4+\frac{c_2}{3}\cdot\varphi_1^3+\frac{c_3}{2}\cdot\varphi_1^2+c_4\cdot\varphi_1+ct_1, \varphi_1\in[0,\varphi_0] \\ \frac{c_5}{4}\cdot\varphi_1^4+\frac{c_6}{3}\cdot\varphi_1^3+\frac{c_7}{2}\cdot\varphi_1^2+c_8\cdot\varphi_1+ct_2, \varphi_1\in[\varphi_0,2\pi]\end{cases}. \tag{2}$$

Constants defining the noncircular centrodes kinematics

Table 1

$c_1=\frac{2(b-a)}{\varphi_0^3}$	$c_2=\frac{-3(b-a)}{\varphi_0^2}$	$c_3=0$	$c_4=0$
$c_5=\frac{-2(b-a)}{(2\pi-\varphi_0)^3}$	$c_6=\frac{3(b-a)(2\pi+\varphi_0)}{(2\pi-\varphi_0)^3}$	$c_7=\frac{-12\pi(b-a)\varphi_0}{(2\pi-\varphi_0)^3}$	$ct_1=0$
$c_8=\frac{8a\pi^3-12a\pi\varphi_0+6b\pi\varphi_0^2-b\varphi_0^3}{(2\pi-\varphi_0)^3}$		$ct_2=\frac{\pi(b-a)\varphi_0(4\pi^2-6\pi\varphi_0+\varphi_0^2}{(2\pi-\varphi_0)^3}$	

where ct_1, ct_2 are new constants to be determined. The φ_2 function should be a continuous, monotonically increasing and derivative function.

Considering all the above mentioned requirements for the transmission ratio and driven centrode rotational angle, the $c_1..c_8$, ct_1, ct_2 constants are expressed in relation with the extreme values of the transmission ratio and splitting angle (Table 1).

In order to get closed centrodes, it is necessary that $\varphi_2(2\pi) = 2\pi$, leading to a new restriction:

$$a+b=2. \tag{3}$$

As seen in Table 1 and considering the restriction from relation (3), the variation of the transmission ratio depends on two parameters, i.e. one of the extreme ratio value, a or b, and the splitting angle, φ_0.

4. Noncircular Centrodes Modelling

Since the transmission ratio was defined, the noncircular centrodes that generate the desired variable motion can be easily modelled as polar curves, based on the mating centrodes theory [13]:

$$r_1(\varphi_1)=\frac{A}{1+i_{21}(\varphi_1)}. \tag{4}$$

$$r_2(\varphi_2(\varphi_1))=\frac{A\cdot i_{21}}{1+i_{21}(\varphi_1)}. \tag{5}$$

It is obvious that the variation of the transmission ratio defining parameters, firstly chosen to produce a desired kinematics, influences the centrodes geometry. An analysis of the influence of the transmission ratio parameters on the mating centrodes geometry is required, to confirm that the curves shapes and curvatures allow the further teeth process generation.

5. Modifying the Kinematics of the Crank-Slider Mechanism

In case of transmitting motion from the noncircular gears, with the transmission ratio defined by equation (1), to a crank-slider mechanism, the crank rotational motion variation leads to a slider stroke (Figure 2) expressed as:

$$s(\varphi_2) = -r\cos\varphi_2 + \sqrt{l^2 - r^2\sin^2\varphi_2}. \quad (6)$$

where r and l are the crank and connection rod lengths, respectively.

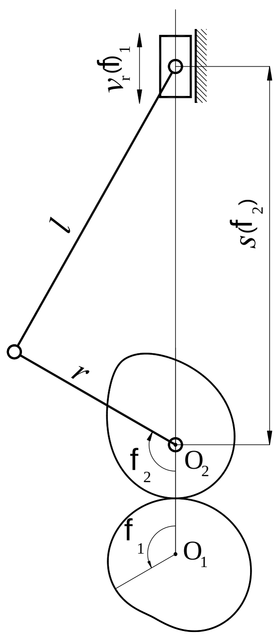


Fig. 2. *Noncircular gears as motion generator for the crank-slider mechanism*

Considering the rotational angle of the driving gear, φ_1, as input data, the slider relative velocity is expressed as:

$$v_r(\varphi_1) = \frac{ds(\varphi_2)}{d\varphi_1} = i_{21} \cdot [r\sin\varphi_2 \cdot (1 - \frac{r\sin\varphi_2}{\sqrt{l^2 - r^2\sin^2\varphi_2}})]. \quad (7)$$

Both the slider displacement and relative velocity are functions of the driven noncircular gear rotational motion and the input defining parameters, the minimum transmission ratio and the splitting angle, respectively. Therefore, a modification of the crank-slider kinematics is achieved by the proper choice of the input data, assuming that their influence on the slider displacement and velocity are well understood.

6. Results and Discussions

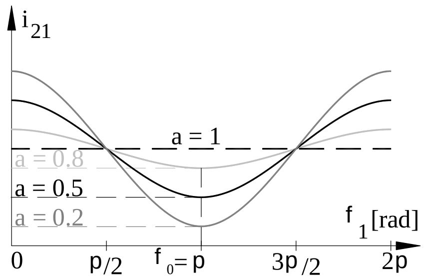
Using a transmission ratio that is defined by equation (1), the main objective of the authors is to design a noncircular gears train that would modify the kinematics of the crank-slider mechanism in order to improve the nail forming process, especially the nail head forming phase, in case of the MCC337 nail machine. This assumes that the transmission ratio defining parameters should be properly chosen so as to meet both the technological and noncircular gears design requirements.

6.1. Varying the Transmission Ratio Defining Parameters

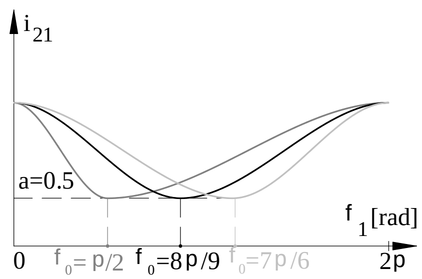
Figure 3 illustrates the influence of the defining parameters on the transmission ratio variation. It can be noticed that a similar variation is achieved, as proposed by Doege ([9], Figure 1); the hybrid multi-parameter function proposed in equation (1) enables modifications of both the amplitude of the transmission ratio and the angle of gear rotational motion/position within the functioning cycle where the minimum transmission ratio is achieved.

Figure 4 illustrates the mating centrodes with variable geometries, varying by the initial data, the minimum transmission ratio (Figure 4a) and the splitting angle (Figure 4b).

As illustrated, for reduced values of the minimum transmission ratio, the geometry of the driving centrode exhibits concave zone, with high curvature, as in case of reduced values of the splitting angle; the curves geometry losses its symmetry related to the center line O_1O_2, as the splitting angle is reduced or increased from the medium value $\varphi_0 = \pi$.

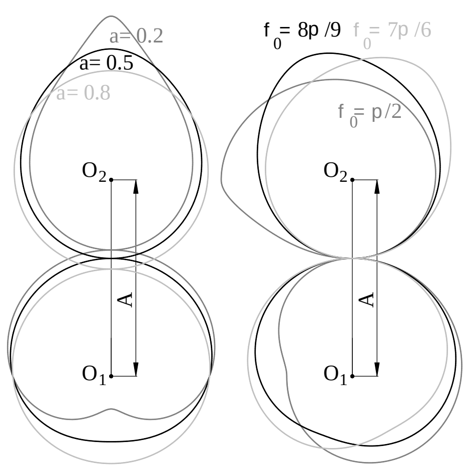


(*a*)



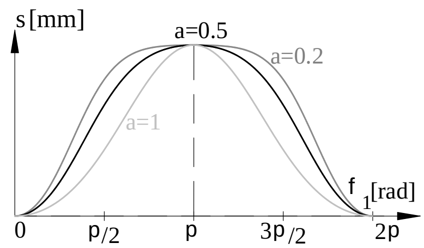
(*b*)

Fig. 3. *Transmission ratio as function of minimum value (a) and splitting angle (b)*

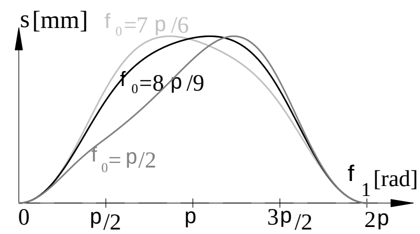


(a) $\varphi_0 = \pi$ *(b) a = 0.5*

Fig. 4. *Noncircular centrodes geometries as function of minimum transmission ratio (a) and splitting angle (b)*

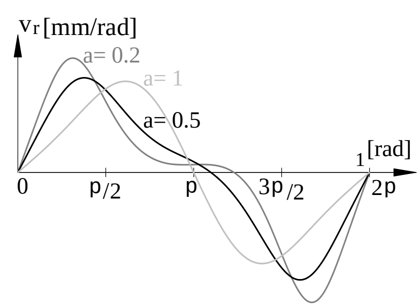


(a) $\varphi_0 = 8\pi/9$

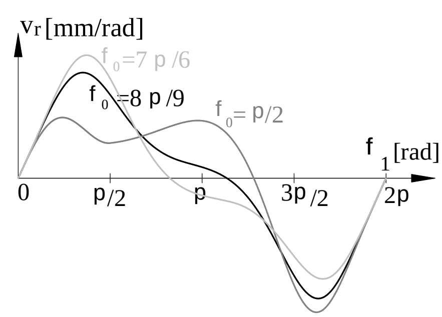


(b) a = 0.4

Fig. 5. *Slider displacement as function of minimum transmission ratio (a) and splitting angle (b)*



(a) $\varphi_0 = 8\pi/9$



(b) a = 0.4

Fig. 6. S*lider relative velocity as function of minimum transmission ratio (a) and splitting angle (b)*

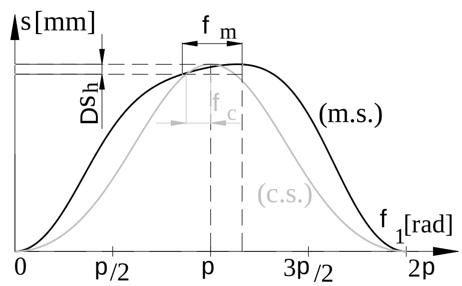
Figures 5 and 6 illustrate the influence the transmission ratio defining parameters have on the slider displacement and relative speed. As seen in figures, decreasing the minimum transmission ratio, the slider velocity is reduced in the vicinity of the middle stroke, the slider performing a near uniform motion, for a longer period of the functioning cycle. Comparing the results in case of splitting angle influence, it is noticed that the better slider behaviour is recorded as angle is increased to values close to π.

6.2. Noncircular Gears for the Crank-Slider Kinematics Modification

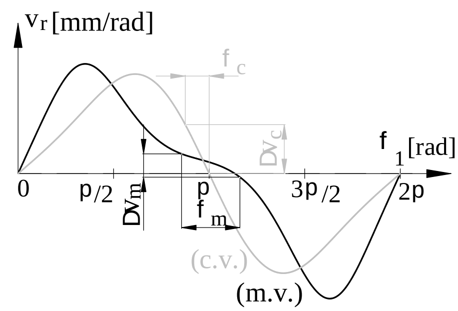
In case of the MCC37 nail machine, an improve of the nail forming process regards both the decrease of the process velocity during the nail head forming phase and the increased time of forming forces action; these would improve the final product quality, would enable production of nails with enlarged head, would increase the tools life and process stability. In order to achieve these advantages, a modification of the kinematics of the machine's crank-slider mechanism is proposed, by additional noncircular gears attached to the mechanism.

The paper considers a concrete application, the nail forming process for nails of □4 x 50 type, with the nail head diameter □13, respectively. The above study and the machine technological requirements lead to the proper choice of the transmission ratio defining data, as follows: a = 0.4 and $\varphi_0 = 8\pi/9$. For the machine crank-slider mechanism with the lengths of parts of r = 150 mm and l = 350 mm, the modifications of the slider displacement and relative velocity are shown in Figure 7.

Using the MCC37 nail machine, the nail head forming phase is performed along the slider displacement Δs_h = 16 mm, during the crank rotational angle $\varphi_c = 22°$ (curve (c.s.) from Figure 7a). Introducing a variable rotational motion for the crank, the head forming phase occurs during an enhanced rotational angle, $\varphi_m = 50°$ (curve (m.s.) from Figure 7a), while the relative speed variation is reduced from Δv_c = 80 mm/rad to Δv_m = 38.4 mm/rad (curves (c.v), (m.v) from Figure 7b).



(a)
(c.s.), (m.s) – *conventional and modified displacements*



(*b*)
(c.v.), (m.v) – *conventional and modified velocities*

Fig. 7. *Modification of the slider kinematics in case of the nail forming process*

The noncircular mating pitch curves, defined by the hybrid multi-parameters transmission ratio (equation (1)) and initial data as: a = 0.4, $\varphi_0 = 8\pi/9$, the center distance A = 174 mm, are illustrated in Figure 8.

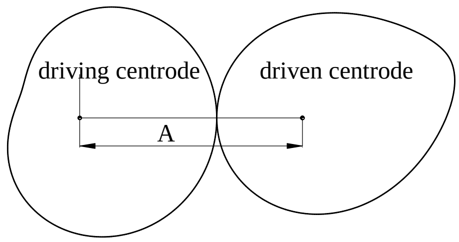


Fig. 8. *Noncircular mating pitch curves*

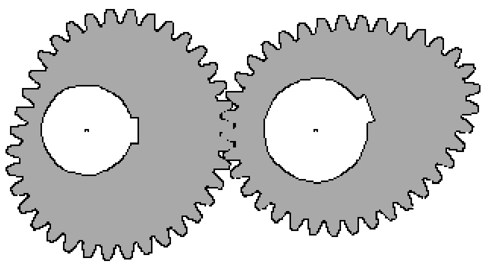


Fig. 9. *Noncircular gears*

The gears teeth are generated by combining an analytical procedure with the editing AutoCAD facilities. For the generation of the teeth flanks profiles, an analytical procedure is applied, based on the rolling method of a rack-cutter tooth [14, 15], with a constant pressure angle of 20°. The gear z_1 teeth are placed at a constant circular pitch:

$$p = \frac{\int_0^{2\pi} \sqrt{1 + [r_1^{'}(\varphi)]^2}\, d\varphi}{z_1}. \quad (8)$$

To complete the gears sections, the teeth flanks are trim between the addendum and dedendum curves, as offset curves to the gear pitch curve, at distances of 1*m* and 1.25*m*, and filleted by the root radius of 0.38*m*, where *m* is the gear modulus.

The virtual solid models of the noncircular gears are produced by simple extrusion of the gears sections, considering the face width B = 20 mm (Figure 9).

7. Conclusions

The authors proposed a modified kinematics of the crank-slider mechanism from a nail machine, in order to improve the nail head forming process. The modification of the mechanismős kinematics is performed by introducing a pair of noncircular gears, as a function generator, whose transmission ratio is predefined as a hybrid multi-parameters function. This provides a high flexibility in manipulating the nail generation process through dividing it into phases and varying the driven gear/crank rotational motions.

The analysis of the influence of the transmission ratio defining data on the mechanisms kinematics and centrodes geometry, respectively, enabled the proper choice of the design data in order to meet both gear generation and machine technological requirements.

The use of a multi-parameters hybrid function for the noncircular gears transmission ratio is a versatile solution that makes the gears generation process more difficult, but enables complex variable rotational motions to be achieved by using a single compact mechanism.

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