IMPROVING THE DYNAMIC BEHAVIOUR OF A VEHICLE SUSPENSION SYSTEM BY STATISTICAL TOOLS

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Abstract: *This work approaches the optimization of the dynamic behaviour of a vehicle suspension system in an innovative way, based on statistical tools (design of experiments and regression analysis). The goal is to improve the comfort and stability performance of the vehicle through the minimization of the roll, pitch, and yaw variations induced by the road disturbances. The optimization is conducted by using a half-car model, which corresponds to the guiding & suspension system of the front axle, with application for a single-seater race car. The virtual prototyping package ADAMS of MSC.Software is used for developing & optimizing the dynamic model.*

Key words: *optimization, regression function, vehicle, wheel suspension.*

1. Introduction

The evaluation and improvement of the kinematic and dynamic behaviour of the wheel suspension mechanisms is a permanent concern and challenge for the automotive research & development field, having in view the major influence of the suspension system on the stability and comfort performance of the motor vehicles.

Since the kinematics is usually approached by analytical methods and in-house made programs [1], [4], [8-10], the complexity of the dynamic model requires the implementation of automatic algorithms, such as those incorporated into the commercial MBS (Multi-Body Systems) software environments [2, 3], [5].

In this paper, the dynamic optimization of the suspension system used the front wheels of a race car is approached as a multi-criteria design problem, by using statistical tools based on design of experiments (DOE) and regression analysis. To the best of our knowledge, this is the first study that approaches the dynamic optimization of the wheel suspension systems in such a way.

The optimization goal is to minimize the pitch, roll, and yaw movements of the chassis (car body), whose variations have significant influence on the dynamic behaviour of the car (in terms of stability and comfort).

A half-car model corresponding to the front wheels suspension is developed, the study being performed by using specific modules from the MBS software package MSC.ADAMS, as follows: ADAMS/View - to develop and analyze the dynamic model of the suspension system; ADAMS/Insight - to configure and perform the optimization process.

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2. Suspension Design

The suspension system of the front wheels is symmetrically disposed relative to the longitudinal axis of the car. For each wheel, the suspension system contains a classical SLA (Short-Long Arm) four-bar mechanism, which controls the wheel travel (fig. 1). In addition, for transmitting the motion to the spring & damper assembly (which is mounted in transversal direction) a pusher & rocker group is used. The spring & damper group is mounted between rocker and chassis.
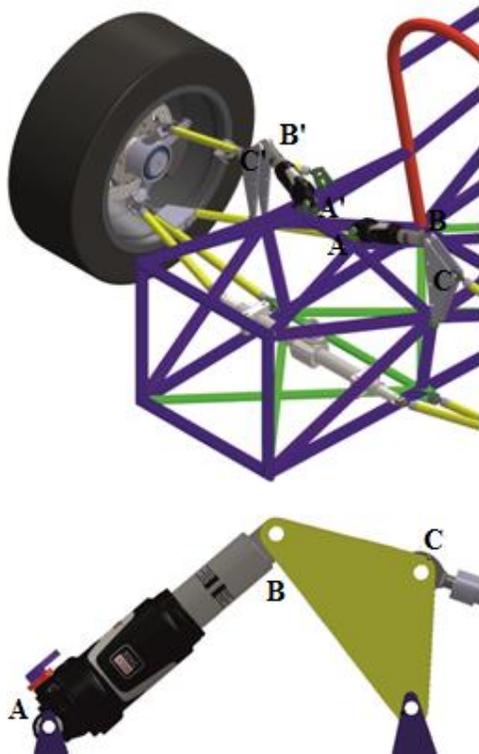

Fig. 1. *The wheel suspension system*

The mechanical structure of the front wheel suspension mechanism was designed and optimized from the kinematic point of view in [11], considering a quarter-car model, through a screening experimental design. The goal of the kinematic optimization was to minimize the variations of the wheelbase, wheel track, induced deflection and castor angle, considering as independent design variables the global coordinates of the spherical joints in the primary suspension loop (i.e. the four-bar suspension mechanism).

The dynamic optimization is performed for a half-car model, which corresponds to the suspension system of the front axle. In the lack of the rear suspension, modelling a fictive spherical joint between chassis and ground ensures the car equilibrium (fig. 2). The spherical joint is disposed at the rear axle level, in the vertical - longitudinal (YZ) plane. The dynamic model is analyzed in the passing over bumps regime. The wheels are anchored on driving actuators, which execute vertical motions (to simulate the road profile).
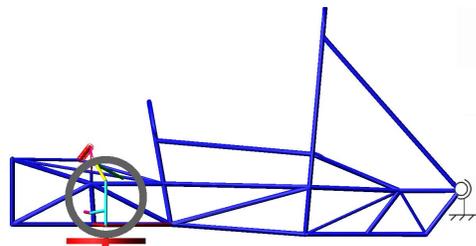

Fig. 2. *The half-car model*

The movement for the right actuator (wheel) corresponds to a sinusoidal profile bump with the amplitude of 50 mm (this being a rule requirement for most of the race cars), while the left actuator is not driven (it is kept fixed). The profile of the runway at the two wheels was materialized by the use of motion laws for the driving actuators.

As mentioned, for this work, the dynamic model of the suspension system was developed by using the MBS software environment ADAMS, which automatically formulates and solves the motion equations.

For the dynamic optimization, the design variables refer to interest points from the secondary loop, as follows: the global coordinates of the connection points of the spring & damper group on chassis (i.e. the points A - Aø), and respectively on rocker (B - Bø); the connection of the rocker on pusher (C - Cø).

Because the suspension system is symmetrical relative to the longitudinal axis of the vehicle, the vertical (Y) and longitudinal (Z) coordinates of the interest points for the left - right wheels are identical, while the transversal (X) coordinates are mirrored (positive / negative sign). Thus, 9 design variables are obtained, as shown in table 1.

The design variables for optimization

Table 1

Joint / point	X	Y	Z
A	DV_1	DV_2	DV_3
B	DV_4	DV_5	DV_6
C	DV_7	DV_8	DV_9
Aø	-DV_1	DV_2	DV_3
B¢	-DV_4	DV_5	DV_6
Cø	-DV_7	DV_8	DV_9

For the optimization process, each design variable is defined by a standard value (corresponding to the initial design) and a variation field ([-10, +10] mm relative to the standard value). In the initial design, the independent variables have the following values (in [mm]): DV_1 = 20; DV_2 = 512.6; DV_3 = -606.4; DV_4 = 210.27; DV_5 = 520.0; DV_6 = -637.87; DV_7 = 281.2; DV_8 = 484.08; DV_9 = -655.7.

The purpose of the dynamic optimization is to minimize the pitch, roll, and yaw oscillations of the chassis. The monitored value for each design objective is the root mean square (RMS) during the dynamic simulation.

3. Optimization Algorithm

The dynamic optimization is performed by using design of experiments (DOE) and regression techniques, in the following steps sequence:
- modelling the purpose of the optimization, in this case the minimization of the design objectives / responses (r_01 - yaw angle, r_02 - roll angle, and r_03 - pitch angle);
- choosing the set of design variables for the suspension mechanism that we are investigating, in accordance with table 1;
- setting the investigation strategy, and planning a set of trials in which the design variable value vary from one trial to another;
- executing the runs and recording the performance of the system at each run;
- fitting the results to a regression model (function);
- evaluating the soundness of the results;
- optimizing the suspension system.

The optimization study is performed with ADAMS/Insight, which provides a collection of statistical tools for analyzing the results of the experiments. For the suspension mechanism in study, there have been evaluated several DOE investigation strategies (Screening, Response Surface), and design types (Full Factorial, Plackett-Burman, D-Optimal, Latin Hypercube).

The best results, in terms of fit accuracy, have been obtained for the DOE Screening strategy with D-Optimal design and Interactions model. DOE Screening identifies the design variables and combinations of design variables that most affect the behaviour of the system, picking the high and low values of a setting range.

The D-Optimal design produces a model that minimizes the uncertainty of coefficients, consisting of a random collection of rows from a larger pool of candidates that are selected using minimization criteria [6].

Based on the design specifications, there have been created the design space and the work space of the experiment, considering the minimum number of runs/trials for the selected investigation strategy (in this case, 46 trials). The design space is a matrix with the rows representing the runs, and the columns representing the design variables settings, which are in a normalized representation. The work space is a matrix with the rows indicating the trials and the columns identifying the design variables settings and resulting objectives values. There are combinations with the minimum and maximum values of the design variables.

For each trial, a simulation will be performed; after ADAMS/View completes the runs, the simulation results appear in the work space. A selection from the work space is shown in table 2, for several representative trials. The analyses have been performed considering the passing over bumps dynamic regime.

Based on the work space of the experiment, an appropriate regression model (function) is developed, which is then used to establish a relationship between the design objectives (responses) and variables (factors).

Selection from the work space trials

Table 2

	Trial 1	Trial 24	Trial 46
DV_1	30	10	30
DV_2	522.6	502.6	522.6
DV_3	-616.4	-616.4	-616.4
DV_4	200.27	220.27	220.27
DV_5	510.0	510.0	530.0
DV_6	-647.87	-647.87	-647.87
DV_7	271.2	291.2	291.2
DV_8	474.08	474.08	494.08
DV_9	-665.7	-665.7	-645.7
r_01	0.758	0.737	0.753
r_02	0.385	0.367	0.451
r_03	0.090	0.084	0.078

The regression function captures the factors (variables) - response (objective) relationships to a specified order (linear, quadratic or cubic), the best results (in terms of fit accuracy) being obtained for a linear model with interactions (the interactions effects are captured through special terms that consist of products of design variables), which is defined by the following equation:

$$r\_0i = a_1 + a_2 \cdot DV\_1 + a_3 \cdot DV\_2 + a_4 \cdot DV\_3 + \ldots + a_{10} \cdot DV\_9 + a_{11} \cdot DV\_1 \cdot DV\_2 + \\ + a_{12} \cdot DV\_1 \cdot DV\_3 + \ldots + a_{45} \cdot DV\_7 \cdot DV\_9 + a_{46} \cdot DV\_8 \cdot DV\_9 + e, \tag{1}$$

where: r_0i - the design objective (r_01 - yaw angle, r_02 - roll angle, r_03 - pitch angle), DV_1 to DV_9 - the design variables, a_1 to a_{46} - the coefficients computed by the regression analysis (a1 being the constant term), e - the remaining error that is minimized by the regression analysis.

The fit accuracy is defined by the following statistical measures [7]: R-squared (R2), R-squared-adjusted (R2adj), regression significance (P), range-to-variance (R/V), and F-ratio (F). R-squared indicates the variance in the predicted results versus the real data. This is the proportion of total variability in the data which is explained by the regression model, a score of "1" indicating a perfect fit. R-squared-adjusted is similar to R-squared, but it is adjusted to account for the number of terms. Regression significance indicates the probability that the fitted model has no useful terms, small values (less than "0.01") indicating that the

fit does have useful terms. Range-to-variance ratio indicates how well the model predicts values at the data points. F-ratio is used in the regression to test the significance of the regression, high values suggesting that the regression is significant, and the model is useful.

ADAMS/Insight provides graphical symbols that indicate the soundness of the results. In accordance with the goodness of fit tables shown in figure 3 (a - pitch, b - roll, c - yaw), the three regression models for the selected strategy (DOE Screening, D-Optimal design, Interactions model) matches the test data very well (the green indicator specifies that the entity is likely appropriate), for all the responses.

	DOF	SS	MS	F	P
Model	45	0.000385	8.56e-06	1e+20	0 ●
Error	0	2.53e-29	0		
Total	45	0.000385			
R2	1 ●				
R2adj	1 ●				
R/V	1e+20 ●				

a.

	DOF	SS	MS	F	P
Model	45	0.387	0.00859	1e+20	0 ●
Error	0	1.8e-26	0		
Total	45	0.387			
R2	1 ●				
R2adj	1 ●				
R/V	1e+20 ●				

b.

	DOF	SS	MS	F	P
Model	45	0.00953	0.000212	1e+20	0 ●
Error	0	5.81e-28	0		
Total	45	0.00953			
R2	1 ●				
R2adj	1 ●				
R/V	1e+20 ●				

c.

Fig. 3. *The goodness of fit tables*

The goodness of fit tables contain also the number of independent variables that go into the estimation of a parameter (DOF), the sum of squares (SS), and the mean square (MS), for the three parts of the statistical model - regression (model), residual (error), and total [8].

These results prove the quality (accuracy) of the three regression models, which define the relationships between the design objectives and the design variables. Therefore, there is no need to refine the regression models

In the final step, the effective optimization of the suspension mechanism was performed for minimizing the root mean squares for the three design objectives. The method used in optimization is OptDes-GRG (Generalized Reduced Gradient), which is provided with ADAMS/Insight. This is a conventional gradient-based optimizer that uses finite differencing to compute partial derivatives of the overall cost with respect to the factor values.

The differencing method (for computing derivatives using finite differences) is Central, which perturbs above and below the nominal value and uses the average slope as the derivative. During optimization, the design variables are adjusted so that the resulting responses come as closely as possible to the specified target values.

The results obtained in this way show substantial improvements for all the interest parameters, by comparing the initial (before optimization) and optimal suspension configurations in terms of root mean squares (RMS) during the dynamic simulation, as follows: pitch - initial RMS=0.2251, optimized RMS=0.1499; roll - initial RMS=0.6068, optimized RMS=0.403; yaw - initial RMS=0.1099, optimized RMS=0.0548. These prove the viability (usefulness) of the adopted optimization strategy.

4. Final Remarks

The method presented in this paper allows the multi-criteria kinematic and dynamic optimal design of the suspension systems used for motor vehicles. In this research, three design objectives have been considered, the optimization goal being the minimization (as possible in terms of functional and constructive requirements) of the pitch, roll, and yaw movements (oscillations) of the chassis.

The modelling, simulation and optimization in virtual environment precede the realization and implementation of the physical prototype, targeting the evaluation and improvement of the dynamic behaviour of the suspension system. The proposed optimization strategy leads to a powerful suspension system, without developing expensive physical prototypes. The obtained suspension will be implemented on the race car of the *Transilvania* University, and the data sets achieved by measurements will be compared with the results of the virtual prototype analysis, for the reciprocal validation of the virtual and physical prototypes.

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