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SOME CONSIDERATIONS CONCERNING AXODES OUTLINING DURING THE MOTION OF BODIES WITH MULTIPLE CONTACTS

S. ALACI[1] F.C. CIORNEI[1] F. MIHAI[1]
D. AMARANDEI[1] S.T. SIRETEAN[1]

Abstract: *The rigid body kinematics uses in the studies the axodes of motion as very helpful instruments for motion characterization. In the case of mechanisms, the significance of these curves is increased as they allow mechanisms' classification as regards the shape of the relative motion axodes. The paper presents some aspects and consequences of establishing the axodes of relative motion for the relative motion of two rigid bodies between which several concentrated contacts are completed.*

Keywords: *contact point, spherical axodes, rolling friction*

1. Theoretical background

The velocity distribution for general motion of a rigid body is described by Eulerôs equation, [1]:

$$\mathbf{v} = \mathbf{v}_O + \omega \times \mathbf{r}' \qquad (1)$$

where, in Fig.1, $\mathbf{v}_O$ is the velocity of a point from rigid body and also the origin of the system $Ox'y'z'$ attached to the rigid, ω is the vector of angular velocity and $\mathbf{r}'$ is the position vector of a point of the rigid with respect to the reference system attached to the body. In literature, [2] it is proved that the locus of the solidary points with rigid for which is valid the condition:

$$\mathbf{v} = \mathbf{0} \qquad (2)$$

is the straight line Δ.

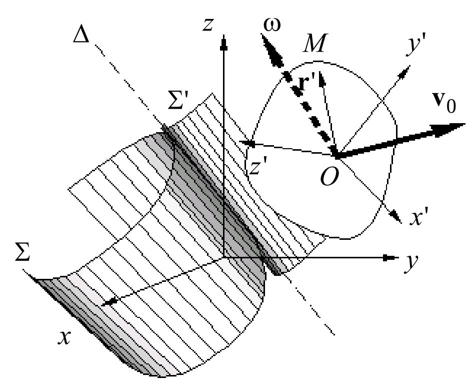

Fig. 1. *Kinematical parameters of first order*

$$\mathbf{r}' = \frac{\omega \times \mathbf{v}_0}{\omega^2} + \lambda\omega \qquad (3)$$

[1] ñStefan cel Mareò University of Suceava

where λ is a real parameter, named the *instantaneous axis of rotation*. During motion, the straight line Δ generates two ruled surfaces, one solidary with the rigid Σ', the mobile axode and another one fixed, Σ named the fixed axode. The two surfaces are permanently tangent, their compound motion being described by a rolling motion in a plane normal to the Δ axis and a sliding motion along this straight line. As it can be observed from relation 3, the instantaneous axis of rotation is fully established by the kinematical parameters of motion, of order 1, explicitly $\mathbf{v}_0, \omega$. The rigid motion can be described by the relative motion of the two axodes. In the case of relative motion, the relative velocity distribution is characterized by the linear relative velocity and by the angular relative velocity. By means of these velocities, there are defined, in a similar manner, the instantaneous axis of relative motion and the axodes of relative motion. Concerning the gear theory, the axodes of relative motion are basic characteristics of a gear mechanism, representing the rolling surfaces of the gear mechanism. Depending on the shape of relative axodes, the main classification of gear mechanisms is made, into cylindrical gears (axodes are cylinders with generators parallel to the axis of the wheels), conical gears (the axodes are cones with the apex in the point of axis intersection) and crossed gears (the axodes are hyperboloids with one sheet). When the transmission ratio is constant, the axodes turn into revolution surfaces.

2. Experimental evidences

For the dynamical study of a thrust bearing with four point contacts, the situations when between the rolling bodies and race, pure rolling motion or rolling motion exists, must be identified. A correct dynamical model for such a system can be created only after answering to this question. It is considered the case of two bodies, Fig. 2., between which a point contact is made, by bringing into contact two points, C_1, C_2 belonging to the two bodies. The notations used in Fig. 2 are: $\mathbf{n}$ the versor of common normal in the point of contact, $\mathbf{v}_{al}$ is the sliding velocity between the contacting points, contained into the common tangent plane, $\mathbf{T}$ is the friction force from the contact point, also enclosed by the tangent plane and $\mathbf{N}$, the normal reaction force.

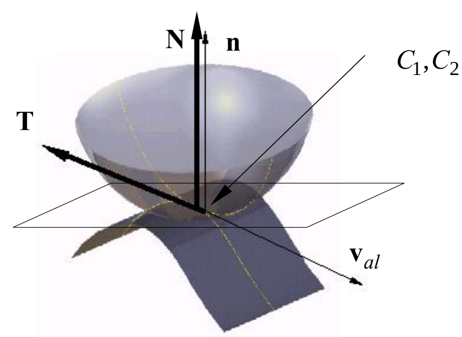

Fig. 2. *Concentrated contact with friction*

If rolling motion occurs between the two bodies, the number of independent kinematical parameters reduces due to the relations characteristic to rolling motion, generally expressed by the relation, [3]:

$$v_{al} = 0, \tag{4}$$

expression that articulates a relationship between relative angular velocity and relative linear velocity. In this situation, the magnitude of friction force $\mathbf{T}$ must be found from dynamical equations, which has both direction and size unknown. If sliding occurs between C_1 and C_2 points, the relation 4 is no longer valid and thus the relative linear velocity and relative angular velocity are independent. The friction force is found in this case using the relation:

$$\mathbf{T} = -\mu_d N \mathbf{u}_{al} \tag{5}$$

where μ_d is the dynamic friction coefficient and $\mathbf{u}_{al}$ is the unit vector of sliding velocity. It is considered a bearing ring 1, with contact in four points and the rolling body consisting in a ball, 2, As in Fig. 3. The race is made of two co-axial conical surfaces.

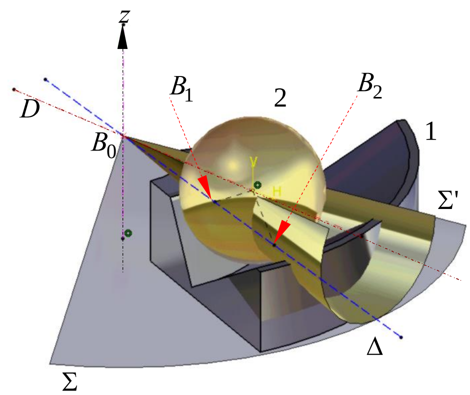

Fig. 3. *Rolling body-rolling race double contact*

It is assumed that in the theoretical points of contact between ball and track, B_1 and B_2, pure rolling occurs. According to relation 4, the sliding velocity in these pints is zero. Moreover, the contacts in the points B_1 and B_2 are accomplished between a fix point from the race and a point from the rolling body.

The rolling condition leads to the conclusion that in the contact points the *velocity of the points from the rolling body must be zero.* This condition shows that the contact points from the rolling body are placed on the instantaneous axis of rotation Δ. The intersection of Δ axis with the symmetry axis of rolling track is a fix point, B_0. Additionally, the angle between the instantaneous axis of rotation Δ and the straight line D - the axis of symmetry of rolling body, passing through the point B_0 is permanently the same. Thus, one can state that in the case when pure rolling motion exists in contacting points, the axodes of motion are two cones of revolution Σ and Σ', fact that ensures a spherical motion for the rolling body.

In the situation that between the two bodies another contact points occurs, to ensure pure rolling in this point too, it is required that the contact point should be collinear with the points B_1 and B_2, since the rolling condition necessitates satisfying condition 2 that is satisfied only on the points belonging to instantaneous axis of rotation. In Fig. 4 is presented a simple experimental set-up used in verifying that the straight line passing through the contact points is the instantaneous axis of rotation. A threaded hole was made into the ball and a thin aluminum rod which materializes the axis of symmetry was assembled. The ball makes contact with the race in two points whose position was obtained by placing an indigo paper between the ball and track. Placing the ball into a random position on the track followed by setting in motion the ball, it is noticed that the axis of the rod describes a conical ñwavyò surface.

Fig. 4. *Experimental set-up*

Repeating the test for different initial positions of the ball, it is observed that there is an arrangement for which the rod described a conical surface. The motion of the ball was video-captured and two frames from a movie are presented in Fig. 5. As it can be seen from the two frames, the initial position of the ball ensures permanently that the axis of the rod should be always in the vertical plane that contains the axis of symmetry of the track. This fact ensures constant angle between the axis of the rod and the straight line passing through the contact points.

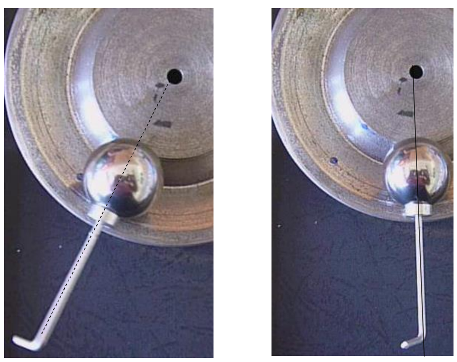

Fig. 5. *Video-frames from ball's motion*

3. Dynamical Simulation Of The System

Considering a mechanical system, it is desirable that in the joist, rolling friction occurs instead of sliding friction, with the purpose to ensure reduced wear and energy dissipation, [4], [5]. In order to guarantee pure rolling conditions, the parts of the system should obey certain geometrical and kinematical constrains. Referring to the analyzed system, to provide pure rolling conditions implies conformity to the following conditions: the race tracks should be revolution surfaces, the axis of the two surfaces accomplishing the contacts with the rolling body must coincide, and the instantaneous axis of rotation of rolling body must intersect the axis of symmetry of the ring. If any of these constraints is broken, at least in one of the contact points, sliding will occur instead of rolling. To illustrate the effect of sliding friction compared to rolling friction effect, the system from Fig. 6 was considered, and modeled using MSC.ADAMS. An extremely difficult task is modeling concentrated contacts, due to numerous parameters to be stipulated. The model consists of the ring with the race and two identical balls. A rod of negligible mass was fixes to one of the balls. Using the rod, the ball is obliged, via a spherical joint, to maintain a fix point on the axis of the ring. The immobile point differs from B_0 where the line passing through contact points that the ball makes with the ring, intersects the axis. In this manner, the free ball may present pure rolling with respect to the ring while for the ball with attached rod, as shown above, the occurrence of pure rolling is impossible due to non co-linearity between the contact points and the point from the axis of the ring.

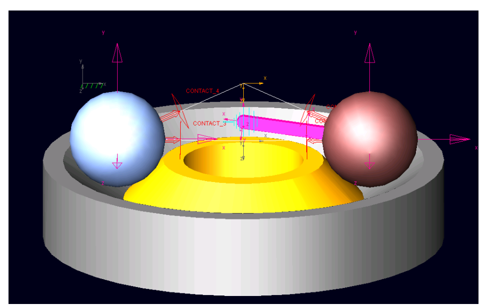

Fig. 6. *Model used in dynamical simulation*

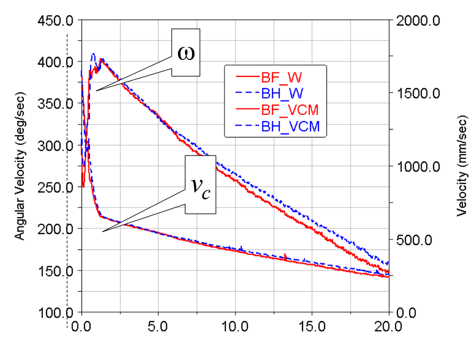

Fig. 7. *Variation of angular velocity and of velocity of mass centre*

A first aspect to be verified is the effect produced by attaching the rod to a ball. To this purpose, it was first considered that the initial relative motions of the two balls with respect to the ring are identical and the motion of the balls was simulated considering that the spherical joint is deactivated. In Fig. 7 there are represented the velocities of centers of mass and the angular velocities and the similarity between the motions of the two balls is confirmed. Using the data from Fig. 7, the dependencies of angular velocity as function of the velocity of mass centre of the ball are represented in Fig.8, where the point (1) indicates the initiation of motion and the point (2) represents the ending of motion. On the plot it is clearly visible that towards the end of simulation there is proportionality between the angular velocity and the velocity of mass centre, aspect that is characteristic to pure rolling. To reach pure rolling phase, a sliding period has to be undergone (the nonlinear section of the graphs).

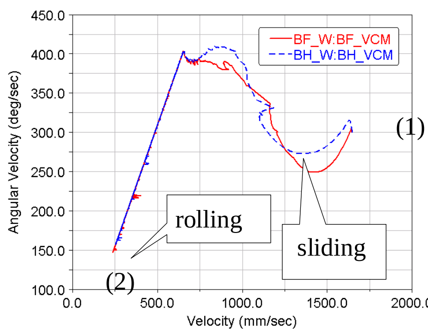

Fig. 8. *Angular velocity variation with respect to velocity of mass centre*

Next there will be presented the results of ballsô motion simulation, namely the case of the ball with rod attached for which the spherical joint is considered. The variations of angular velocity and velocity of mass centre of the balls are presented comparatively in Figs. 9 and 10. As expected, the sliding friction contributes to both angular and translating velocity rapidly diminishing.

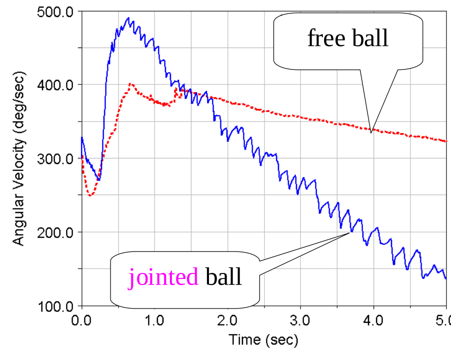

Fig. 9. *Balls' angular velocities variations*

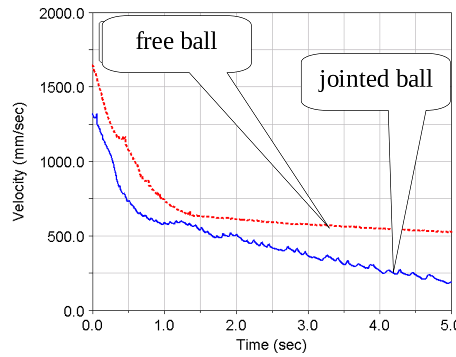

Fig. 10. *Balls' centers of mass velocity variations*

Concerning the friction type occurring between the balls and the ring Fig. 11 is highly suggestive.

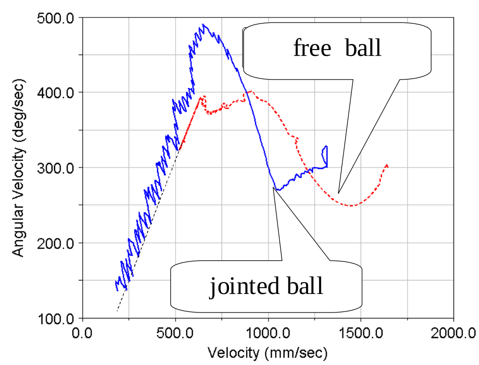

Fig. 11. *Angular velocity variation with the velocity of mass centre*

Fig. 11 presents the angular velocity variation with respect to velocity of mass centre. Similar to Fig. 8, the motion of free ball consists of a sliding phase followed by a pure rolling phase. For the ball with attached rod, however, one can notice the existence of an initial zone where sliding friction is present, followed by a region where there are only instants in which the characteristic point is situated on the straight line region.

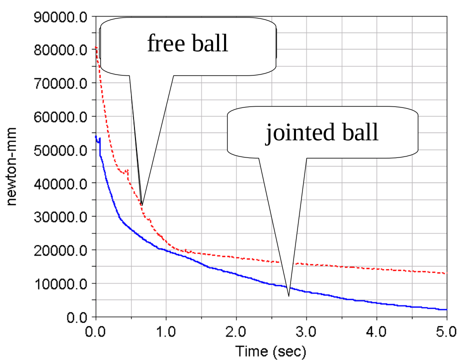

Fig. 12. Total kinetical energy variation

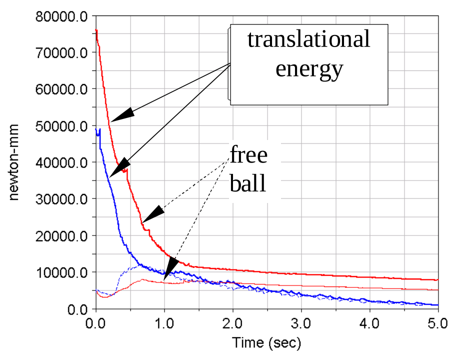

Fig. 13. *Translational and rotational kinetic energies*

The energy of the balls is discussed with reference to the next two figures. The ballsô total kinetical energy variation is presented in Fig. 12, and a rapid decrease is noticed as expected for the ball with attached rod. The variations of translational energy (mass centre) and rotation energy are shown in Fig.13. Regarding the rotational kinetic energies, after a short period they present the same variation and are practically equal. The translational kinetic energy of the free ball is always greater than the one of the jointed ball.

4. Conclusions

The paper presents some aspects regarding identification of instantaneous rotation axis and implicitly, the revealing of axodes of rigid bodies with multiple concentrated contacts. Imposing pure rolling condition for all contact points leads to the conclusion that all contact points be collinear. In technical applications it is desired that rolling motion exist in concentrated contacts in order to diminish wear and energy dissipation. As to multiple contact bodies, meeting the requirement of pure rolling in all contacts is a difficult task. For a simple example, explicitly the two point contact between a ball and a circular ring, there are shown kinematical and energetically effects produced when pure rolling conditions are not satisfied.

References

1. Ardema D.M., *Newton-Euler Dynamics,* Springer, 2005
2. Voinea, R., Ceauкu, V., Voiculescu, D.: *Mecanica*, EDP Bucuresti, 1983.
3. Mangeron, D., Irimiciuc,N.: *Mecanica rigidului cu aplicatii in inginerie, Vol. I,* Ed. Tehnica, 1978.
4. Stolarski, T.A., Tobe, S.: *Rolling Contacts*, Prof. Eng. Publishing Ltd, London and Bury St Edmunds, 2000.
5. Tabor, D.: *The mechanism of rolling friction.* Proc. R. Soc., A229, 1955.

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