Bulletin of the *Transilvania* University of Braşov • Vol. 9 (58) No. 2 - 2016
Series I: Engineering Sciences

FRACTIONAL ADAPTIVE CONTROL FOR A FRACTIONAL-ORDER DC ELECTRICAL DRIVE

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Abstract: *The major contribution of this paper is to introduce the fractional calculus used to develop a fractional-order model for the DC electrical drive and, based on it, to design a fractional adaptive controller. In such a context, two modifications of the conventional Model Reference Adaptive Control theory are presented. Based on simulation results obtained in Matlab/ Simulink, the benefits of fractional-order control are shown. Focus is on system performance increase.*

Key words: *fractional-order calculus, fractional-order DC electrical drive, fractional-order parameter adjustment rule, fractional-order reference model.*

1. Introduction

Fractional calculus is a mature topic applicable to many fields, such as physics (Parada et al., 2007), electrical engineering (Bode, 1949), control systems (Axtell and Bise, 1990), robotics (Marcos et al., 2008), bioengineering (Magin, 2006) and other [7]. These theory, allow us to describe and model real systems more accurately than the classical "integer" methods. The reasoning of introducing such theory in adaptive control is motivated by the very good proven performances [2].

In [8], two ideas are presented to extend the conventional Model Reference Adaptive Control (MRAC) method, by using fractional-order parameter adjustment rule and fractional-order reference model. As one step further, all those methods are proposed to control a process (DC electrical drive), which in classical way has an integer order. To increase the performance of the system, besides the fractional adaptive control low used, we also developed a new model for the DC electrical drive, a fractional model one. So the situation presented in control theory is fractional-order process with fractional-order controller. The simulations made in Matlab/Simulink using FOTF (fractional-order transfer function) blocks confirm the efficiency of fractional adaptive control.

2. Fractional Calculus Theory

Fractional calculus is a generalization of integration and differentiation to non-integer order fundamental operator ${}_aD_t^{\alpha}$, where a and t are the limits of the operation and α the fractional-order [3], [6].

The definition of integro-differential operator is:

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$$
{}_aD_t^{\alpha} = \begin{cases} \mathrm{d}^{\alpha}/\mathrm{d}t^{\alpha}, & R(\alpha) > 0, \\ 1, & R(\alpha) = 0, \\ \int_a^t (\mathrm{d}\tau)^{-\alpha}, & R(\alpha) < 0. \end{cases} \tag{1}
$$

In the current literature, two definitions are used for fractional differintegral, the Riemann-Liouville (RL) definition and the Grunwald-Letnikov (GL) [3], [5], which are presented below:

1) RL

$$
{}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} \mathrm{d}\tau, \tag{2}
$$

2) GL

$$
{}_aD_t^{\alpha} f(t) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t-jh), \tag{3}
$$

Just as example, when applying the Laplace transform on RL definition, the fractional derivative has the following form:

$$
\int_0^{\infty} e^{-st} {}_0D_t^{\alpha} f(t)\,\mathrm{d}t = s^{\alpha}F(s) - \sum_{k=0}^{n-1} s^k {}_0D_t^{\alpha-k-1} f(t)\big|_{t=0}. \tag{4}
$$

Starting from the aforementioned theory, the fractional calculus can be applied in the control system theory as well.

A fractional-order system that needs to be controlled can be described by a typical *n*-term linear fractional-order derivative equation in time domain [2]:

$$
a_n D_t^{\alpha_n} y(t) + \ldots + a_0 D_t^{\alpha_0} y(t) = 0, \tag{5}
$$

where a_i, $i = \overline{1,n}$ are real coefficient.

By Laplace transform, we can get a fractional-order transfer function:

$$
G(s) = \frac{Y(s)}{U(s)} = \frac{1}{a_n s^{\alpha_n} + \ldots + a_0 s^{\alpha_0}}. \tag{6}
$$

In general, a fractional-order dynamic system can be represented by a transfer function of the form:

$$
G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^{\beta_m} + \ldots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + \ldots + a_0 s^{\alpha_0}}. \tag{7}
$$

The theory mentioned in this chapter is going to be applied on a certain process (DC electrical drive) as well as for developing a proper controller.

3. Developing a Fractional-Order Model for the DC Electrical Drive

The DC electrical drive model was determined experimental as it can be seen in [4]. The obtained model is a first order one model with the following transfer function:

$$
G(s) = \frac{Y(s)}{U(s)} = \frac{b}{s+a} = \frac{0.79}{9.3s+1}. \tag{8}
$$

A model reference with the below depicted transfer function is selected:

$$
G_m(s) = \frac{Y_m(s)}{R(s)} = \frac{b_m}{s+a_m} = \frac{1}{4s+1}. \tag{9}
$$

Starting from this point, we developed a fractional-order model for the DC electrical drive (Figure 1).

Transfer function resulted is presented below [5]:

$$G_f(s)=\frac{Y(s)}{U(s)}=\frac{0.82}{6.7s^{0.88}+1}. \quad (10)$$

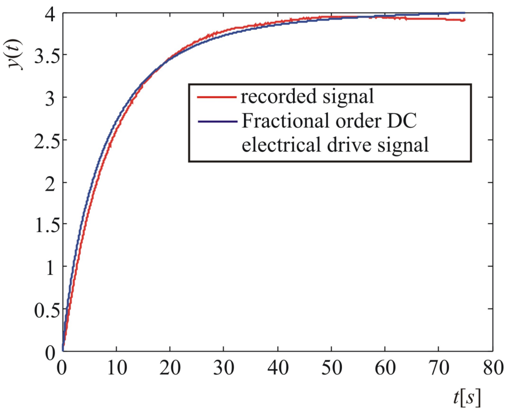

Fig. 1. *Signal's representation*

4. Designing a Fractional Controller for the DC Electrical Drive Fractional Model

All the DC electrical drive parameters are unknown, so an adaptive control must be performed [1].

The most known Model Reference Adaptive System (MRAS) scheme was developed by Whitaker and was introduced to control systems with unknown parameters or changing in time [1]. So the well-known MIT rule for MRAC is used to adjust or update the unknown parameter using gradient information.

To adjust the DC electrical drive fractional model parameters, fractional calculus is introduced into MRAC in two ways [8]:

1) by using fractional-order parameters adjustment rule;

2) by using a fractional-order reference model.

4.1. Controller Parameters Adjustment by Using Fractional-Order Adjustment Rule

Taken into account the fractional calculus, the MIT rule [1] can be written as:

$$\frac{d^{\alpha}\theta}{dt^{\alpha}}=-\gamma\frac{\partial J}{\partial\theta}=-\gamma e\frac{\partial e}{\partial\theta}, \quad (11)$$

where:

θ - is the controller parameter;

e - the error between the process and the model outputs;

γ - the adaptation gain;

$\frac{\partial e}{\partial\theta}$ -the sensitivity derivative of the system;

α - a real number denoting the fractional-order derivative.

The theory mentioned above can be applied more easily on the DC electrical drive fractional-order - see Equation (10) - when applying the fractional derivatives. Thus, the Equation (10) becomes:

$$D_t^{\alpha}y(t)=-ay(t)+bu(t). \quad (12)$$

The reference model from the Equation (9) is described by the following differential equation:

$$\frac{d\,y_m(t)}{dt}=-ay_m(t)+br(t). \quad (13)$$

A perfect following of the reference model is achieved with a P adaptation low [1]: $u(t)=t_0r(t)-s_0y(t)$, and the error of the system is: $e(t)=y(t)-y_m(t)$.

The sensitivity derivatives required by the adjustment mechanism are obtained by taking the partial derivatives of the error variable [4]. Eventually, after the approximation is done [4], the adjustment equations for the controller's parameters can be obtained:

$$\frac{d^{\alpha}t_0(t)}{dt}=-\gamma 1\left(\frac{1}{p+a_m}r(t)\right)e(t),$$
$$\frac{d^{\alpha}s_0(t)}{dt}=\gamma 2\left(\frac{1}{p+a_m}y(t)\right)e(t), \quad (14)$$

where, the parameter b is introduced in the adaptation gain γ and $p = \frac{\mathrm{d}}{\mathrm{d}t}$. Laplace transform can be applied on the fractional-order derivatives from the Equation (14) [3], [5], resulting in:

$$t_0 = -\frac{\gamma 1}{s^{\alpha}}\left(\frac{1}{s + a_m} R(s)\right) E(s),$$
$$s_0 = \frac{\gamma 2}{s^{\alpha}}\left(\frac{1}{s + a_m} Y(s)\right) E(s). \tag{15}$$

The benefit of using this method is clearly demonstrated in Figure 2, with $\alpha = 0.75$. For the adaptation gain, two values are chosen: $\gamma 1 = 25$ and $\gamma 2 = 1$.

The error signal, which represents the difference between the output process and the output reference model, is reduced to zero (Figure 3). The stability is achieved more faster than the case when classic adjustment rule is used even if some small oscillations are present (observe the Figure 2 and Figure 3) [2], [4]. The block diagram for the MRAC scheme for adjusting the unknown parameters is shown in Figure 4.

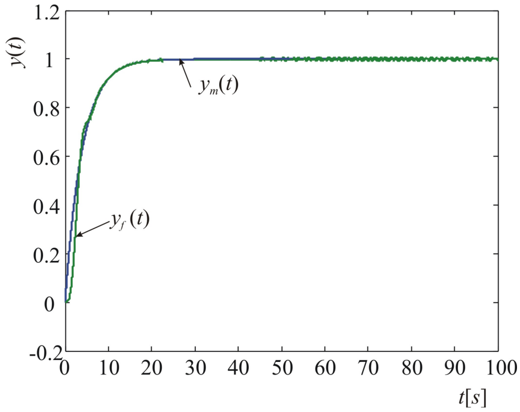

Fig. 2. *The output signals using the fractional-order adjustment rule*

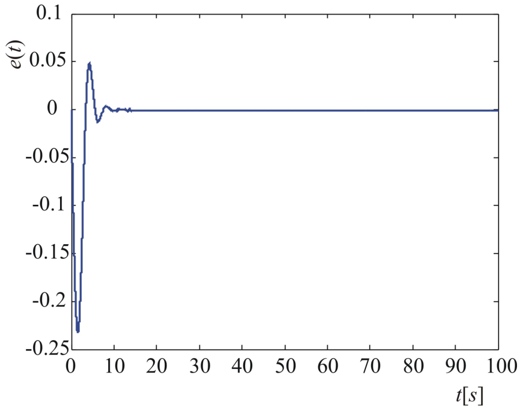

Fig. 3. *The error* $y_f(t) - y_m(t)$

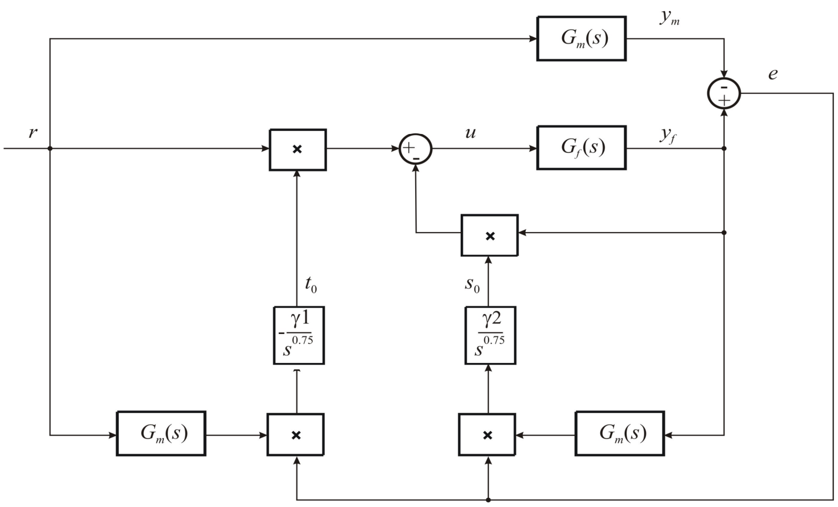

Fig. 4. *Block diagram for a MRAC with a fractional-order adjustment rule*

4.2. Controller Parameters Adjustment by Using Fractional Reference Model

We introduce another modification to the classic MIT rule, by using a fractional-order reference model besides the classic reference model [8]. With this method, the system performance is improved.

The fractional-order reference model proposed is:

$$G_{mf}(s) = \frac{Y_m(s)}{R(s)} = \frac{1}{4s^{0.85}+1}. \qquad (16)$$

The adaptation low with the rate change of the parameters t_0 and s_0 depending only of the adaptation gain is going to be used in this case as well. Proceeding with the same steps like we did in the chapter above, the sensitive functions are:

$$\begin{aligned} t_0 &= -\frac{\gamma 1}{s}\left(G_{mf}(s)R(s)\right)E(s), \\ s_0 &= \frac{\gamma 2}{s}\left(G_{mf}(s)Y(s)\right)E(s). \end{aligned} \qquad (17)$$

Again, the benefit of this method is presented in Figure 5, where by using the same values of the adaptation gain (see Chapter above) error is reduced to zero (Figure 6).

A block diagram for this method was implemented in Matlab/Simulink also (Figure 7).

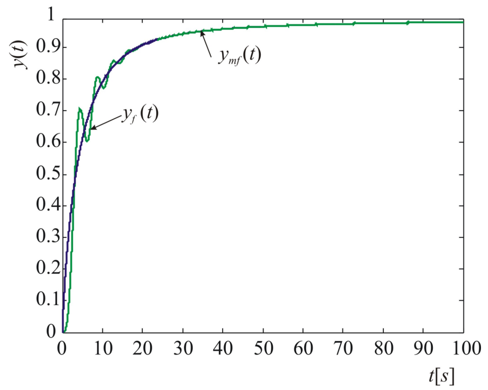

Fig. 5. *The output signals using the fractional-order reference model*

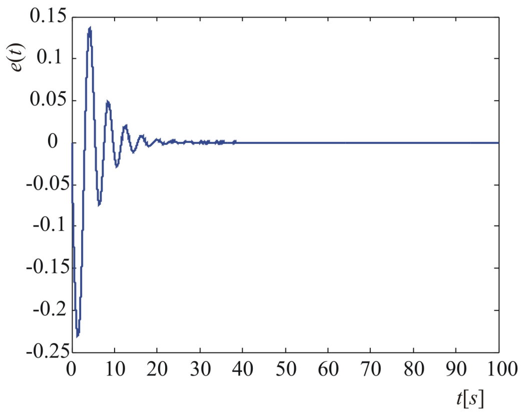

Fig. 6. *The error* $y_f(t) - y_{mf}(t)$

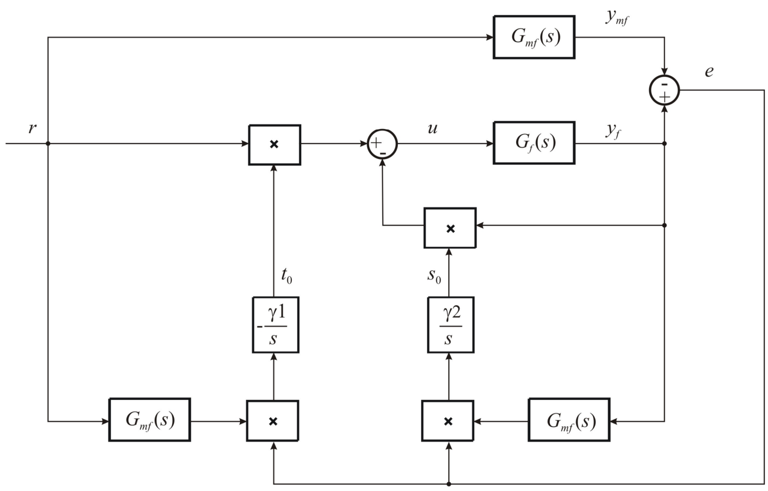

Fig. 7. *Block diagram for a MRAC with a fractional-order reference model*

5. Conclusions

In this paper, by finding out the control law which takes the system to desired performance with improved behavior, our objectives are fulfilled. Simulation results confirmed the benefits of the proposed methods. However, when a fractional adaptive law using fractional-order adjustment is used, the tracking performance is almost perfect. Future work will consist in a stability analysis in the frequency domain.

Acknowledgements

We hereby acknowledge the structural founds project PRO-DD (POS-CCE, 0.2.2.1., ID 123, SMIS 2637, ctr. No. 11/2009) for providing the infrastructure used in this work.

References

1. Astrom, K., Wittenmark, B.: *Adaptive Control*. USA, Ed. Addison-Wesley, 1989.
2. Bensafia, Y., Ladaci, S.: *Adaptive Control with Fractional Order Reference Model*. In: International Journal of Science and Techniques of Automatic Control and Computer Engineering IJ-STA **5** (2011) No. 2, p. 1614-1623.
3. Chen, Y., Petras, I..: *Fractional Order Control - A Tutorial*. American Control Conference, 2009, p. 1397-1411.
4. Coman, S., Boldisor, Cr.: *Model Reference Adaptive Control for a DC Electrical Drive*. In: Bulletin of the *Transilvania* University of Brașov, Vol. 6 (55) No. 1, Series I, 2013, p. 33-39.
5. Li, Z.: *Fractional Order Modeling and Control of Multi-Input-Multi-Output Processes*. In: PhD. Thesis, University of California, USA, 2015.
6. Maiti, D., Konar, A.: *Approximation of a Fractional Order System by an Integer Order Model Using Particle Swarm Optimization Technique*. In: IEEE Conf. on Computational Intelligence, Control and Computer Vision in Robotics & Automation, 2008, p. 149-152.
7. Petras, I.: *Fractional-Order Nonlinear Systems. Modeling, Analysis and Simulation*. Beijing, Ed. Springer Heidelberg Dordrecht, London, New York, 2011.
8. Vinagre, B.M., Petras, I., Chen, Y.: *Using Fractional Order Adjustment Rules and Fractional Order Reference Models in Model-Reference Adaptive Control*. In: Nonlinear Dynamics **29** (2002) No. 1, p. 269-279.