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MODEL REFERENCE ADAPTIVE CONTROL FOR MULTI-INPUT-MULTI-OUTPUT FIRST ORDER SYSTEM

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Abstract: *In this paper, the authors propose to illustrate how the model reference adaptive control theory related to SISO (single-input-single-output) systems can be extended for a class of MIMO (multi-input-multi-output) systems. More precisely, two methods, the MIT (Massachusetts Institute of Technology) rule and Lyapunov stability are applied on a MIMO system consisting of first order transfer functions with variable parameters. A few cases are considered. The benefits of the proposed adaptive control law are shown through Matlab/Simulink examples.*

Key words: *MIMO systems, MIT rule, Lyapunov stability, reference model, adaptive control law.*

1. Introduction

Many automatic control systems have plants where more than one variable must be controlled. Moreover, the variables interact with each other. As a direct consequence, the systems' stability and performance are hard to be achieved. Based on this assertion, a question of how such systems (i.e. MIMO) need to be controlled can be addressed. An appropriate solution to such question is to find a control technique which can control the variables. Such control should ensure that the effect of interaction is minimized or reduced [5-7]. As a remark, most of the control techniques that are applied on SISO systems can be applied on MIMO systems as well.

So, in this paper, an adaptive control theory is applied to control a MIMO system. The MIMO system is based on the first order plants. Further on, a few situations are presented where the system's performance imposed by the reference model are achieved.

2. Theoretical Aspects Related to MIMO Systems

The proposed multivariable system has two inputs and two outputs. The easiest way to control a multivariable system is to apply a mono-variable solution. As can be seen in the Figure 1, as well in many papers [1-2], [4], [8], [9], for each input a structure with a single control law is adopted. Each adaptive control law is developed considering the system decoupled (meaning that when a control law is developed for a certain input, the other input is equal to zero).

The two outputs can be written in the following form:

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$$Y_1(s) = G_{11}(s)U_1(s) + G_{12}(s)U_2(s), \quad Y_2(s) = G_{21}(s)U_1(s) + G_{22}(s)U_2(s). \tag{1}$$

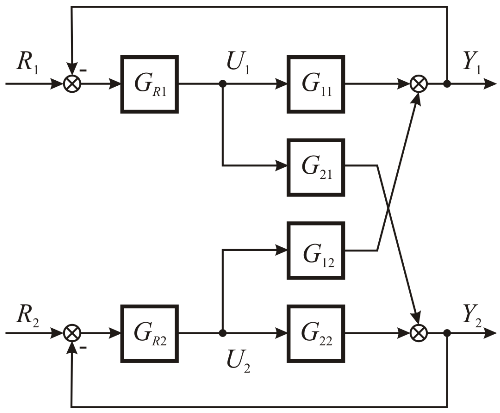

Fig. 1. *Multi-input-multi-output control system*

Starting from them, the MIMO system can be characterized by the following plant matrix:

$$G_p(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}, \tag{2}$$

where, $G_{11}(s)$ and $G_{22}(s)$ are called principal transfer function. The others, $G_{12}(s)$ and $G_{21}(s)$ are called coupling transfer function and they are influencing the outputs [5-7].

A very important element is the coupling factor, in the steady-state domain:

$$K_0 = \frac{G_{12}(0) \cdot G_{21}(0)}{G_{11}(0) \cdot G_{22}(0)} = \frac{K_{12} \cdot K_{21}}{K_{11} \cdot K_{22}} < 1, \tag{3}$$

where, the K_{ij} represents the gain factors for each input-output [5-7].

A downside of this solution (i.e. mono-variable) is that it can be applied to MIMO systems only if the coupling factor is very small, less than one. To overcome this negative aspect, for each input an adaptive control law is developed based on the most known MRAS (Model Reference Adaptive System) scheme which was developed by Whitaker [3], for SISO systems. Per Whitaker, the inner loop which contains the plant and the controller, and the outer loop which adjusts the controller's parameters ensure the system's performance given by reference model. For the adaptation mechanism, the two methods commonly used in the literature are applied here. These methods are the MIT rule and Lyapunov stability (for more details please refer to [3]).

3. The MIT Rule and Lyapunov Stability for MIMO System with First Order Plants

In the MIMO system with first order transfer functions, the parameters which can be time varying are gain factor and time constant. An adaptive control law for each input, which offers a perfect tracking of the reference model, must be developed.

For all situations that are presented from this point on, the adaptive control law is:

$$u(t) = t_0 r(t) - s_0 y(t), \tag{4}$$

with the adjustment parameter $\theta = [t_0 \; s_0]$.

As starting point is the supposition that the only parameter varying in time is the gain factor. Thus, the plant matrix is chosen as:

$$G_p(s) = \begin{bmatrix} \dfrac{2}{0.5s+1} & \dfrac{0.1}{0.5s+1} \\ \dfrac{0.2}{0.5s+1} & \dfrac{4}{0.5s+1} \end{bmatrix}, \tag{5}$$

with $K_{11}=2$, $K_{12}=0.1$, $K_{21}=0.2$, $K_{21}=4$ (the values for the gain factors satisfy the requirement from Equation (3)) and $T_{11}=T_{12}=T_{21}=T_{22}=0.5$. The reference model which describes the behavior of the system is:

$$G_m(s) = \frac{1}{0.5s+1}. \tag{6}$$

First, the MIT rule is used considering that the error of the system is:

$$e(t) = y(t) - y_m(t). \tag{7}$$

The goal is to find out the controller's parameters. Applying the MIT mechanism on Equation (4) the sensitivity derivatives are calculated ($\partial e / \partial t_0$, $\partial e / \partial s_0$) [2-4]. Finally, the controller's parameters are obtained:

$$\begin{aligned} \frac{dt_0(t)}{dt} &= -\gamma \left(\frac{1}{p+a_m} r(t) \right) e(t), \\ \frac{ds_0(t)}{dt} &= \gamma \left(\frac{1}{p+a_m} r(t) \right) e(t), \end{aligned} \tag{8}$$

where, p is the is the differential operator, γ the adaptation gain, and a_m is the model reference pole.

The same goal, namely to determine controller's parameters, is achieved when applying the Lyapuov stability to Equation (4). In this case, a function V must be chosen accordingly, as [3]:

$$V(e, t_0, s_0) = \frac{1}{2}\left(e^2(t) + \frac{1}{b\gamma}(bs_0 + a - a_m)^2 + \frac{1}{b\gamma}(bt_0 - b_m)^2\right). \tag{9}$$

If the error is zero and the controller's parameters are set up as:

$$\frac{dt_0(t)}{dt} = -\gamma r(t)e(t),$$
$$\frac{ds_0(t)}{dt} = \gamma y(t)e(t), \tag{10}$$

then the function V derivative is negative [3].

By considering a square wave as input signal with amplitude 1 and $\gamma = 1$, the simulations reflected in Figure 2 show the results of applying the MIT rule for the first input and Lyapunov stability for the second one. In this case, for the closed loop MIMO system, the required performance is achieved.

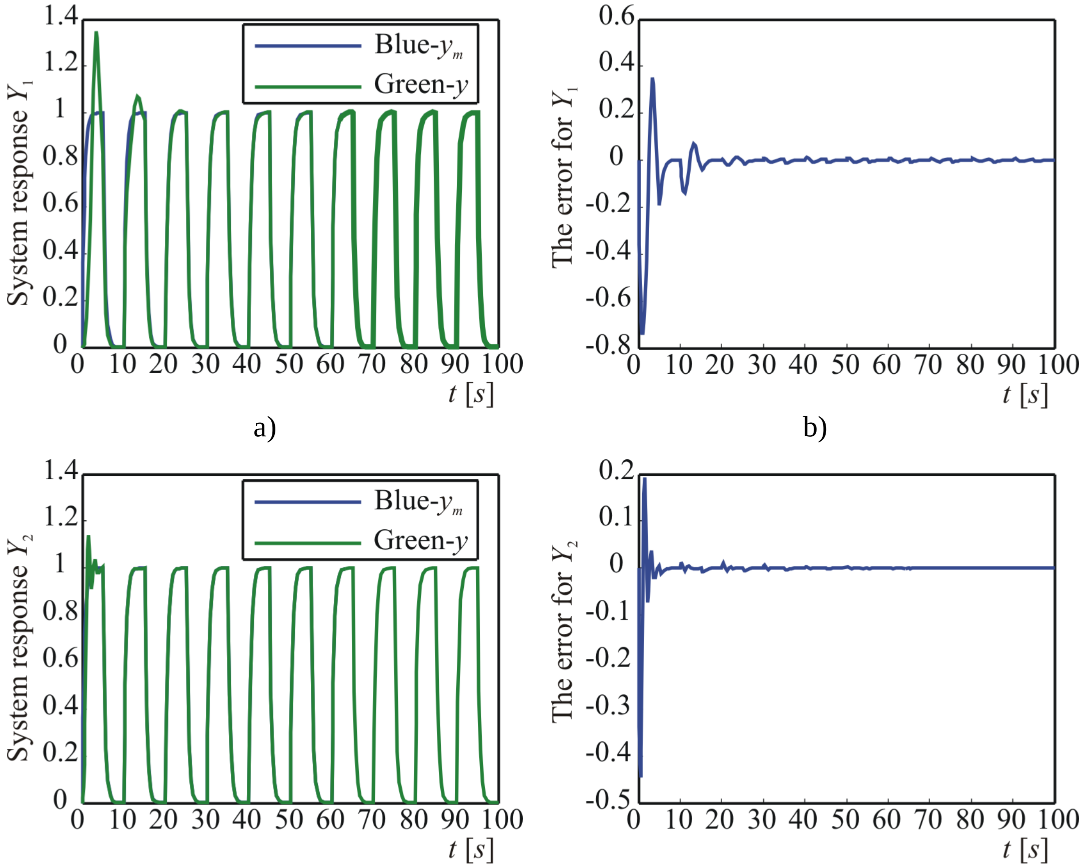

Fig. 2. a) *System response*, Y_1; b) *The error for* Y_1;
c) *System response*, Y_2; d) *The error for* Y_2

As it can be seen in the Figure 2, the outputs follow the model reference output and the errors are reduced to zero. By keeping the same value for γ, the values for the gain factors can increase/decrease, ensuring the convergence of the error to zero.

If the two parameters are time varying (gain factor and time constant), the adaptive control low from Equation (4) performs well. The entire concept from Equations (7)-(10) is applied again (MIT rule for the first input and Lyapunov stability for the second one). The reference model transfer function from Equation (6) is kept. In this case, the plants from the matrix plant have the following form:

$$G_p(s) = \begin{bmatrix} \dfrac{2}{0.6s+1} & \dfrac{0.1}{0.7s+1} \\ \dfrac{0.4}{0.9s+1} & \dfrac{5}{1.5s+1} \end{bmatrix}, \tag{11}$$

such, that the condition from Equation (3) is fulfill.

Simulations from Figure 3 are done for the same input signal (for each input) and $\gamma = 1$.

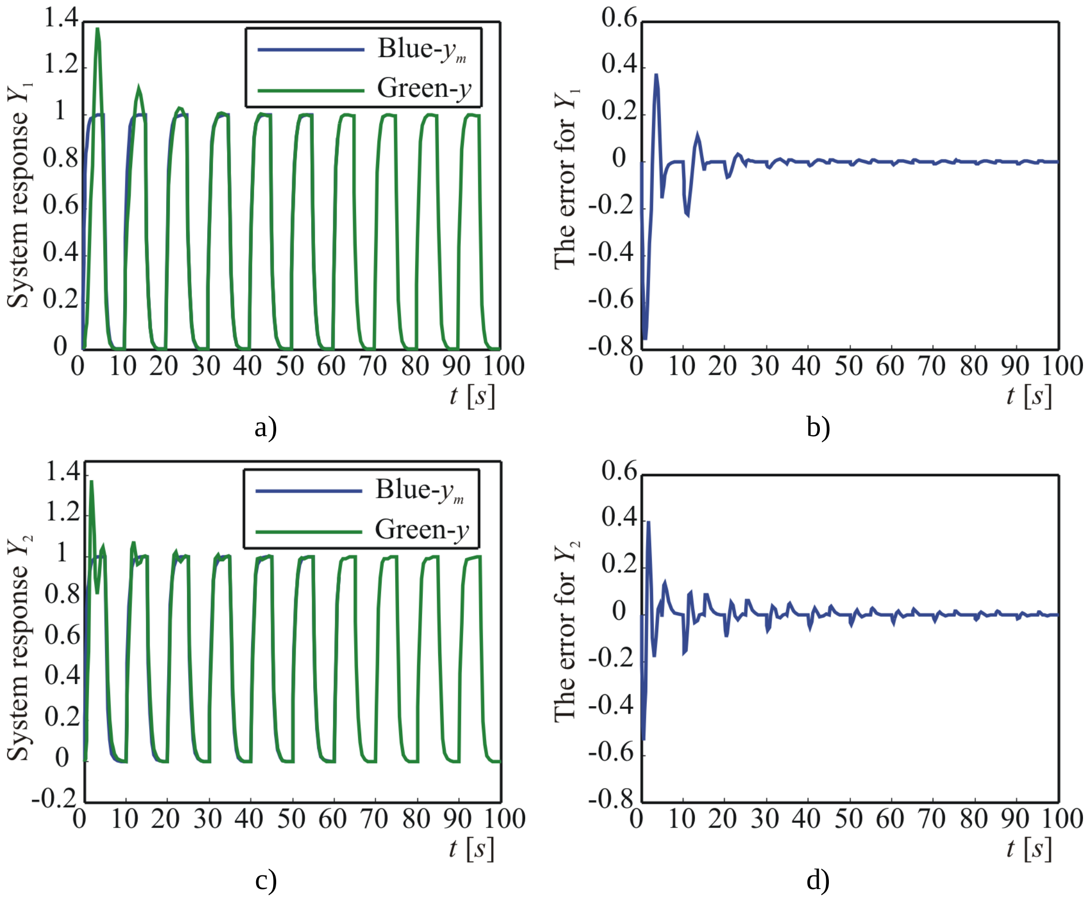

Fig. 3. a) *System response, Y_1, with two parameters time varying*;
b) *The error for Y_1 with two parameters time varying*;
c) *System response, Y_2, with two parameters time varying*;
d) *The error for Y_2 with two parameters time varying*

By changing the gain values or time constants, the system's performance and dynamics are achieved very well. Similar performance can be achieved in case of using again of the same combination already proposed (when the gain factor was the only parameter time varying).

4. Conclusion

In this paper is presented a study related to how the adaptive control techniques can be applied on MIMO system with first order plants. The idea is to apply the same adaptive control law using different methods (MIT rule for the first input and Lyapunov stability for the second one) and good results are obtained. It goes without saying that, depending on desired performance, the vice-versa can be done as well.

This study can be extended in a future work where MIMO systems with unstable plants or second order plants are considered.

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