

LOAD- AND STRAIN-DEPENDENT STIFFNESS MODULUS $E_s(\sigma, \delta)$ FOR COHESIVE SOILS

W. REITMEIER¹

Abstract: *Proposal for an extension of the power stress-strain approach by Ohde under consideration of the magnitude of the load increase based on conventional compression test results with reduced load increase.*

In Germany, the estimation of the settling behavior is, according to DIN 4019, in general done by integrating the strains along the depth z . For this purpose, the stiffness modulus E_s is required, which can be determined by using simple compression tests.

The standard procedure foresees to double the load stages after 24 hours in order to keep the ratio of the measured variables to the error value small.

For cohesive soils, this is, however, associated with very high elongations. These elongations are significantly greater than the elongations under a building foundation. DIN 4019 deals with this uncertainty with the notification that because of the simplified assumptions only an approximation of the settling is possible. These approximations are often inaccurate (i.e., they often overestimate the settling by 50% and, in exceptional cases, underestimate the settling). One of the influencing factors is the effect of the magnitude of the load increase on the stiffness modulus, which is neglected in conventional compression tests.

These relationships are also referred to as small-strain effects in the literature.

Therefore, for a better prediction of the settlement, the magnitude of the stress increase needs to be adjusted in every individual layer in order to have a better estimation of the stiffness modulus.

This article shows how the magnitude of the load increase could influence the stiffness modulus by using a mathematical approach to extend the stress-strain model of Ohde.

Key words: *stiffness modulus, strain, load increase factor, load increment, small-strain, compression tests, power law.*

¹ Fakultät Bauingenieurwesen Lehrgebiet Geotechnik, University of Applied Sciences Konstanz

1. Introduction

The experimental investigation of the consolidation behavior of a cohesive soil can be carried out in a compression test, both under disturbed (treated soil with a water content at the liquid limit) and undisturbed conditions (usage at the removal state of the soil).

In a standard approach, the individual load levels are doubled in each case.

With this experimental approach it is possible to obtain an estimation of the stiffness modulus and accordingly settlement behavior, which, however, can differ considerably from the in situ measured building deformations.

One reason for this deviation is the magnitude of the load increase steps (load doubling) which is used in the standard test procedure. Especially with increasing depth under the foundation this doubling takes not place and the load change is significantly lower.

With conventional compression tests but with adapted load increments, a more precise estimation of the stiffness moduli $E_{s(\sigma,\delta)}$ is possible, so that a more realistic approximations of the deformation behavior in practice are possible with stress and strain-dependent values.

The article focuses exclusively on the influence of the amplitude of the load increment, i.e. the influence of the load increase on the stiffness module in the load ranges relevant for settlement calculations.

First of all, the derivation of the stress-strain relation for the standard test, i.e. with load doubling and account and the magnitude of the load increase, are compared using examples.

Definitions in this article

Strain $\epsilon = \frac{h_i - h_{i+1}}{h_i}$ h_i initial height h_{i+1} height after stress change

Stress increment $\Delta\sigma$

Stress increase factor $\delta_i = \frac{\Delta\sigma}{\sigma_i}$ $\frac{\Delta\sigma}{\sigma_i} = 0 \Rightarrow$ no load increase

$\frac{\Delta\sigma}{\sigma_i} = 1 \Rightarrow$ load doubling

Stiffness modulus from the Oedometer experiment E_{oed} sample height at placement into the Oedometer h_a relative settlement $s' = \Delta h/h_a$.

2. Instructions for Editing

For the evaluation of a compression test with disturbed sample, which requires a soil to be applied close to the liquid limit, the moisture mass m_f at installation and the water content $W_{nRemoval}$ at removal need to be determined.

Starting with a load of e.g. 12,5 kN/m², the load is doubled after 24 hours for each additional load stage and held constant for 24 hours.

For evaluation of the test results the relative settlement $s' = \Delta h/h_{a_a}$ at removal,

and the water content $W_{nRemoval}$ is determined. By drawing these characteristics in a graph, a simple proportionality of the stress state to the corresponding water content of the soil could be seen.

Hence, the stiffness modulus E_{oed} can be taken from the graph for the in situ relevant stress state, both, depending on the magnitude of the stress as well as the water content or the voids ratio.

Example of measurement data from a conventional compression test (soil 2) Table 1

σ [kN/m ²]	12,5	25	50	100	50	25	12,5
Δs [mm]	1,00	1,45	2,01	2,57	2,54	2,50	2,46
s'	5,28	7,62	10,55	13,49			

σ [kN/m ²]	25	50	100	200	400	200	100
Δs [mm]	2,47	2,52	2,60	3,21	3,86	3,82	3,75
s'				16,81	20,21		

σ	50	25	12,5	25	50	100	200
Δs	3,69	3,62	3,55	3,57	3,61	3,68	3,77
s'							

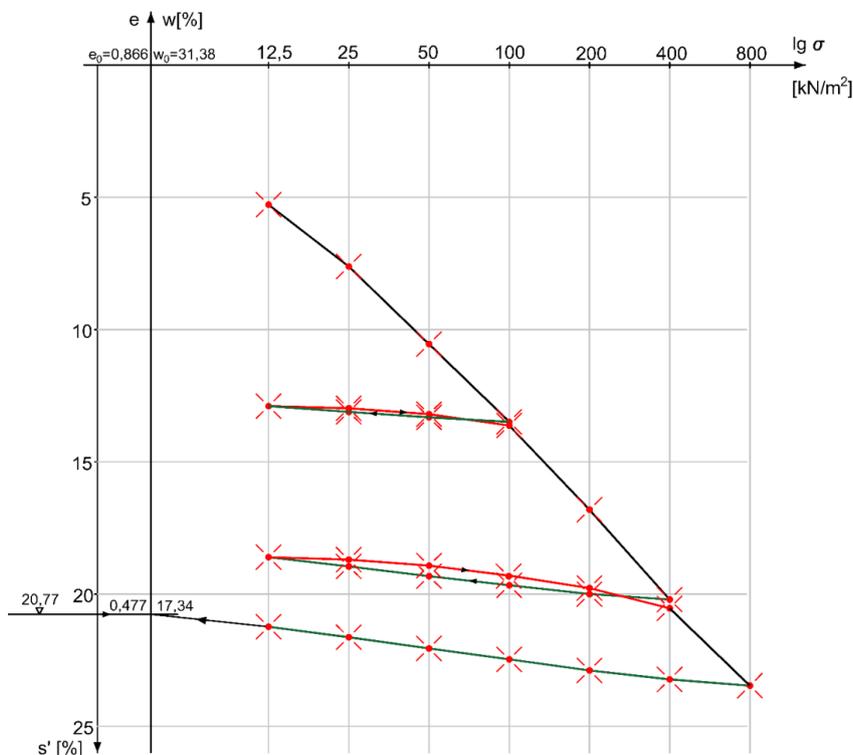


Fig. 1. Compression test (conventional)

3. Compression Test with Reduced Load Increment

All tests to determine the influence of a load increase on the stiffness modulus were performed incorporating a homogenized soil near the liquid limit in order to ensure uniform initial conditions.

For each soil, four compression tests with reduced load increases and additionally a conventional test were carried out to determine the lines shown in Figures 5 and 6.

The reduced load increases were realized in seven individual steps with a constant load increase factor of $\delta_i = 0,1041$.

In order to ensure uniform initial conditions, each sample was first loaded until the corresponding initial stress (50 kN/m² in the example), relieved and stressed again up to the initial stress with conventional load increments and a load duration of 24 hours at each increment.

Table 2 exemplifies these test results for a load increase from 50 to 100 kN/m².

Table 2
Sample data of a compression test with reduced load increment (starting from 50 kN/m²)

σ [kN/m ²]	12,5	25	50	25	12,5	25
Δs [mm]	0.93	1.38	1.96	1.94	1.90	1.92

σ [kN/m ²]	50	55,21	60,95	67,30	74,30	82,04	90,58	100
Δs [mm]	1.98	2.00	2.02	2.08	2.14	2.22	2.31	2.41

σ	200	400	800	400	200	100	50	25	12,5
Δs	3.256	3.897	4.531	4.481	4.413	4.336	4.255	4.168	4.072

Figure 2 shows that with a reduced load increment in a stress region of 50 to 100 kN/m² the deformations of the soil sample are smaller compared to the standard test.

This confirms experience gained in practice in which in addition to the magnitude of the load increment and thus implicitly also the load speed has an influence on the deformation behavior.

If the load is increased, e.g. from 50 to 55.2 kN/m², a significantly larger stiffness modulus results for this load increment than for any higher load increment.

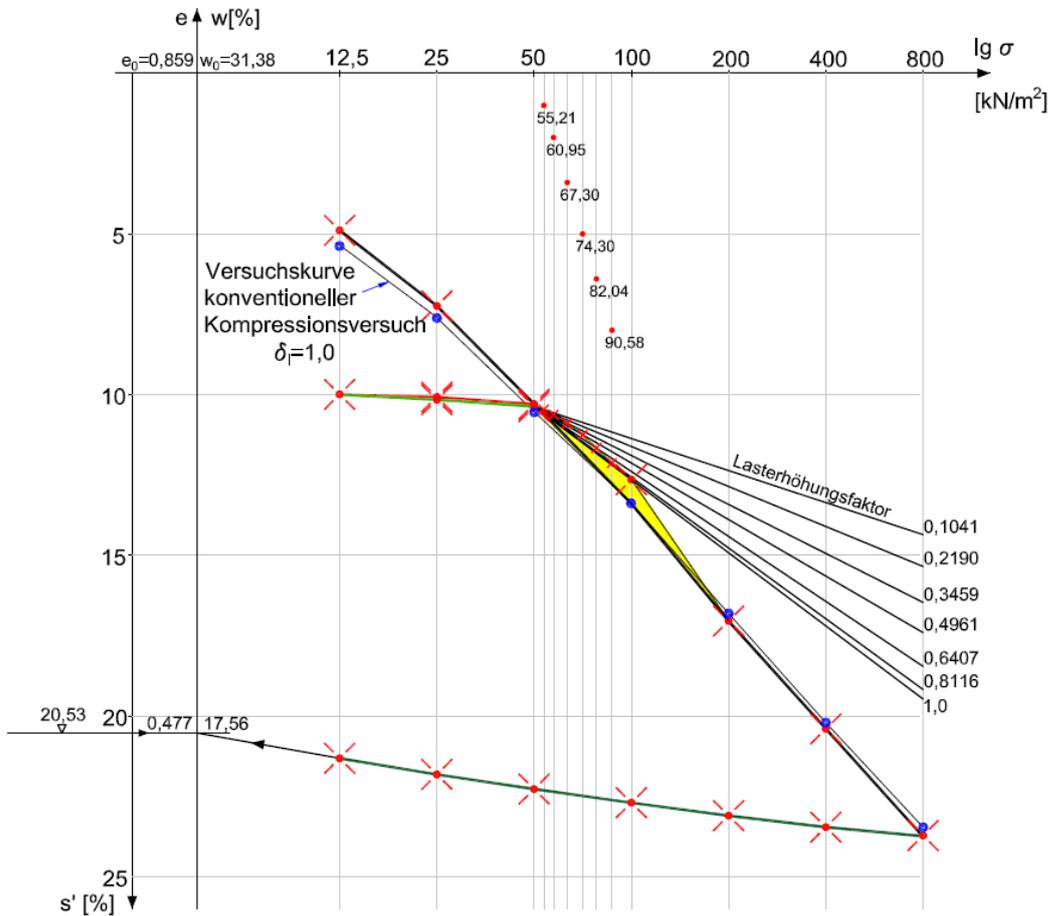


Fig. 2. Compression test with reduced load increment

For the further steps, the secant modulus E_s is calculated corresponding to Equation 1 for each intermediate step.

$$E_s = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{\Delta\sigma}{\Delta s'} (1 - s'_1) \quad (1)$$

4. Derivation of a Stress Independent Stiffness Modulus E_{oed} with the Deformation Measured in the Compression Test

When drawing the secant modulus $\ln\left(\frac{E_s}{\sigma_{at}}\right)$ in a twice logarithmic scale diagram $\ln\left(\frac{\sigma_m}{\sigma_{at}}\right)$, the secant moduli are approximately on a line. As parameters for a function describing this line, the stiffness coefficient v and the stiffness exponent w can be derived:

$$\text{stiffness coefficient } v(0) = e^{\ln \frac{E_s}{\sigma_{at}}} \quad (2)$$

$$\text{stiffness exponent } w = \tan \alpha = \frac{\Delta \ln \frac{E_s}{\sigma_{at}}}{\Delta \ln \frac{\sigma_m}{\sigma_{at}}} \quad (3)$$

With $\sigma_{at} = 100 \text{ kN/m}^2$ the by Ohde proposed stress-strain relation corresponding to Equation 4 can be derived.

As an association of the derived secant modulus to the stress interval $(\sigma_i - \sigma_{i+1})$, the geometric average $\sigma_m = \sqrt{\sigma_i \cdot \sigma_{i+1}}$ of both stress levels is proposed.

$$E_s = \sigma_{at} \cdot v \left(\frac{\sigma}{\sigma_{at}} \right)^w \quad (4)$$

Sample data of a compression test with reduced load increment (starting from 50 kN/m²)

Table 3

σ [kN/m ²]	12.5		25		50		100		200		400		800
s' [%]	5.27		7.61		10.55		13.49		16.81		20.20		23.46
E_{oed}		506		788		1520		2606		4898		9799	
σ_m		17.68		35.36		70.71		141.42		282.84		565.69	
$\ln \frac{\sigma_m}{100}$		-1.73		-1.04		-0.35		0.35		1.04		1.73	
$\ln \frac{E_{oed}}{100}$		1.62		2.06		2.72		3.26		3.89		4.58	

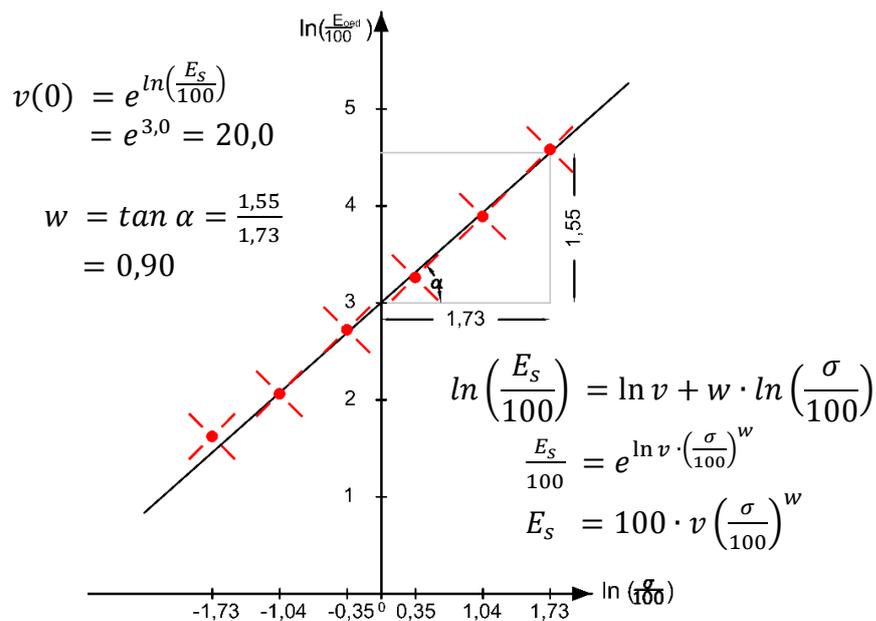


Fig.3. Derivation of the stress strain relation of Ohde

5. Extension of the Stress Dependent Stiffness Modulus E_s with Consideration of the Load Increment Magnitude

For all load levels, the same load duration of 24 hours was selected. The evaluation of the time-settlement curve showed a negligible influence of secondary deformations (creep) in relation to the primary deformations in all samples.

In Figure 2, in addition to the results of the standard experiment (blue dots), the corresponding stress levels and gradients for seven successive intermediate steps (load increase factor constant with $\delta_i = \frac{\Delta\sigma}{\sigma_i} = 0.1041$) are shown.

By determining the gradients, it must be considered that the, corresponding to Equation 1, computed secant moduli of the specific initial stress (in the example $\sigma = 50 \text{ kN/m}^2$) after load removal and reapplication of load must be applied. This ensures accurate data gathering even at small deformation increases in the subsequent first load activation.

In contrast to the deformation which results from only one load step, with a gradual load increase the final measured deformations are measured are significantly smaller.

This is more remarkable, since the load duration, corresponding to the number of intermediate steps, is much longer.

In order to be able to access each load increase factor in the determination of the stiffness modulus E_s in practical application, the gradients of the stepwise load increase must be adapted to the standard curve with $\delta(1)$.

Alternatively, it is possible to determine directly the test results adapted to the standard curve with seven different load increase factors, then in seven individual tests. However, this increases the number of experiments for each soil from 5 to at least 25 individual ones.

The simpler option is to use the test results, determined with incremental loads, based on the standard curve.

For this purpose, the procedure described below is proposed.

The gradient for $\delta_a(0.1041)$ and $\delta_a(1.0)$ at incremental load increase is known. Similarly, the gradient $\delta_b(1)$ from the standard test is known.

Indices a indicate an incremental load, indices b indicate the standard test and c the correction or the adaptation to the standard curve respectively.

The gradient $\delta_a(0.1041)$ has been determined with only a single load increment and can therefore be taken as a base value for the angle adaptation. All gradients of the corrected values from $\delta_c(0,2190)$ until $\delta_c(0.8116)$ are computed with Equation 5.

$$\phi_{ic} = \frac{\phi_{ia}}{\phi_{6a}} \cdot \phi_{6b} \quad (5)$$

The correction of the gradients corresponding to Equation 5 and therefore the adaptation and reduction of the stiffness moduli has been validated through several compression tests in different stress regions with different load increase factors in which a load increase with only a single increment, e.g. for a load increase factor of

$$\delta_i = \frac{\Delta\sigma}{\sigma_i} = \frac{74,30-50,0}{50,0} = 0.486, \text{ was realized.}$$

The difference between computation and experimental determination is negligible.

Figure 4 depicts the stress interval from 50 to 100 kN/m² and shows how to adjust the second to seventh load increment from a stepwise load in comparison to the standard curve for a load increase without intermediate steps using Equation 5.

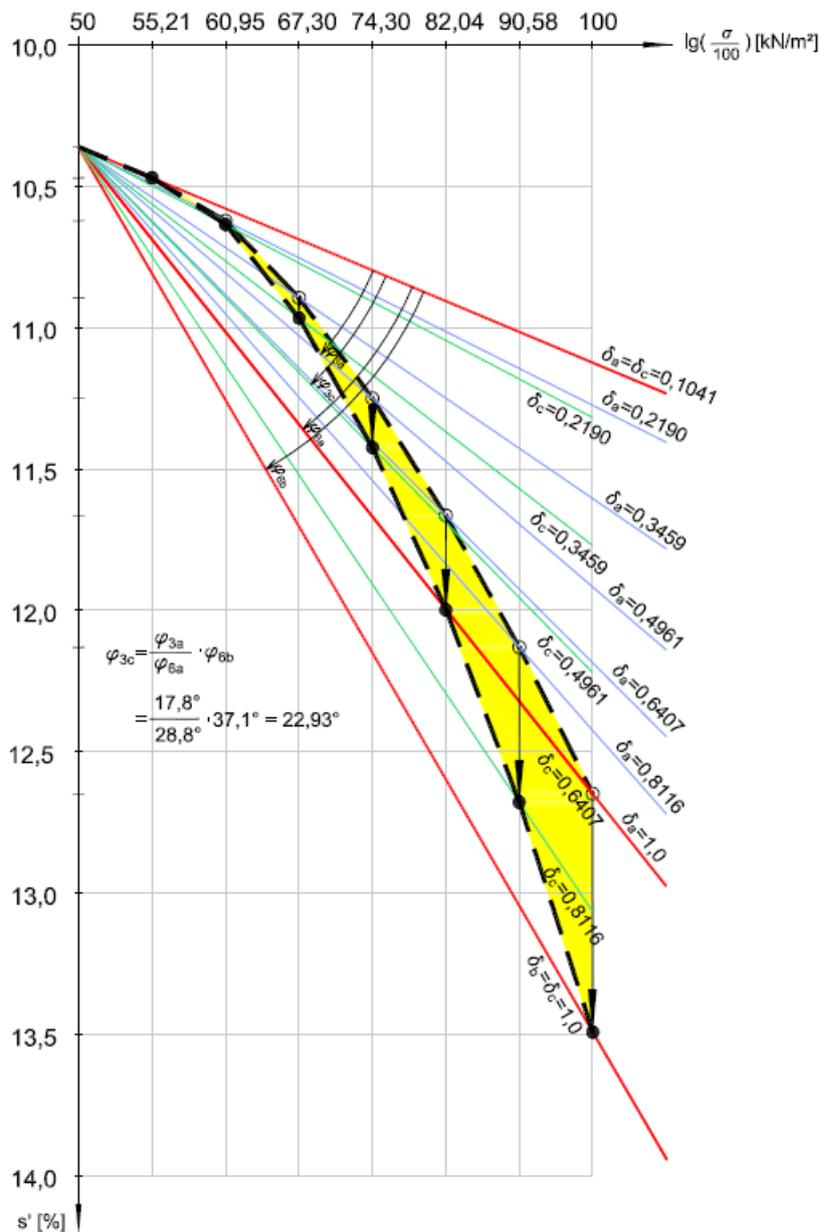


Fig.4. Correction of the incline of the secant for a gradual increase in stress with seven load increments for adaption to the standard curve

Figure 5 shows the standard line and the associated test data with seven single load increases for four different initial loads σ (25, 50, 100 und 200 kN/m^2).

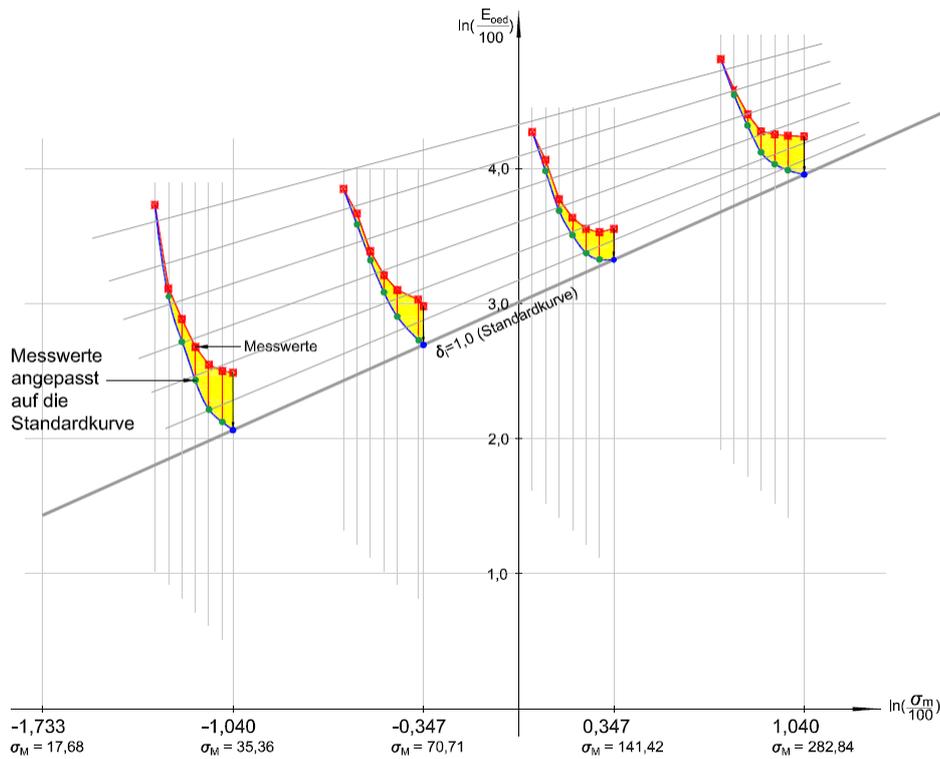


Fig. 5. Curves of the same amount of stress according to the approach of Ohde with the load increment dependent stiffness moduli $\ln \left(\frac{E_{oed}}{\sigma_{at}} \right)$
 red \equiv measured data, green \equiv corrected measurement data, according to Fig.4., shown for the selected four stress regions (25, 50, 100 and 200 kN/m^2)

All experimental results are subject to statistical variance as well as variance due to uncertainty in measurements. With a simple linear regression, the data for the derivation of the stiffness coefficient $v(\delta)$ and the stiffness exponent $w(\delta)$ is smoothed.

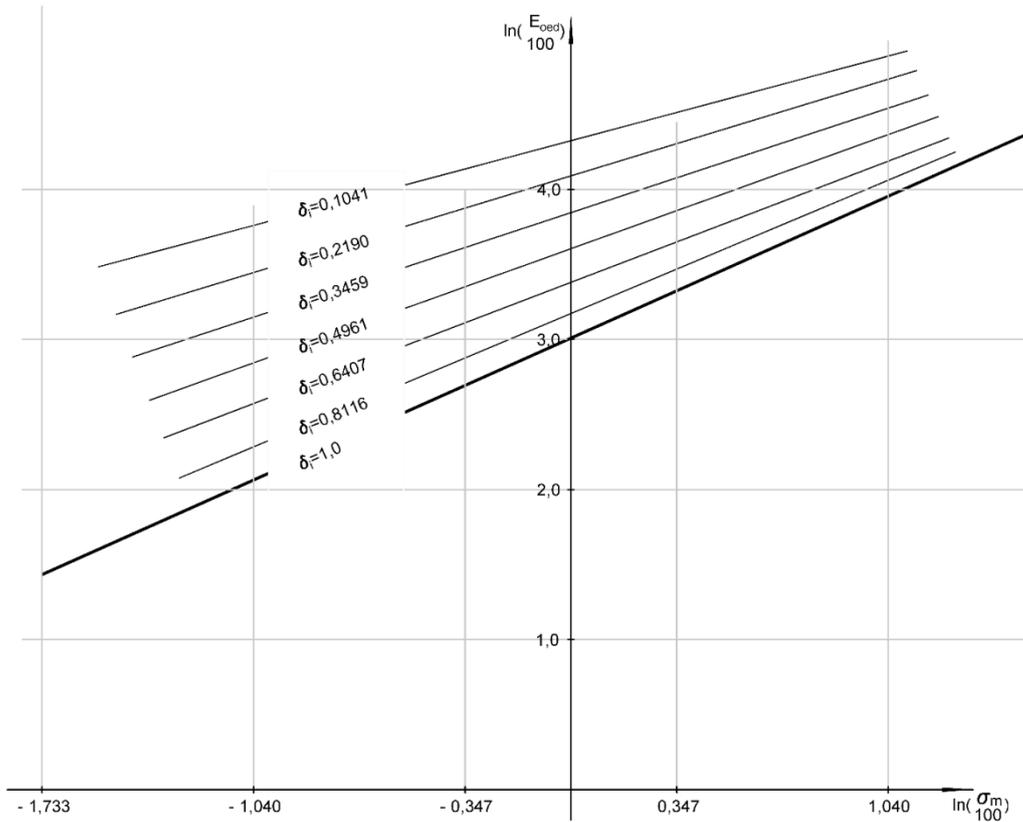


Fig.6. Load increase curves for seven individual steps

For each line independent from the load increment, the stiffness coefficient $v(\delta)$ and the stiffness exponent $w(\delta)$ can be computed.

By drawing $v(\delta)$ and $w(\delta)$ as a function of the load increment, the in Figure 7 and 8 showed test curves results.

Figures 7 and 8 additionally contain the extrapolated trend until a decreasing load increment of $\Delta\sigma \rightarrow 0$, i.e. for a load increment of zero.

When doubling the load, the curves ends in the standard values $v(1)$ and $w(1)$ of the compression experiment and therefore at a load increment of one.

The mechanical properties of the in Figures 7 and 8 showed six test results are summarized in Table 4.

Characteristics of the used soil

Table 4

1	TL (TM)	$w_L = 34,6 \%$	$w_P = 18,5 \%$	$I_p = 16,1 \%$	Seeton Konstanz
2	TL (TM)	$w_L = 34,1 \%$	$w_P = 19,7 \%$	$I_p = 14,3 \%$	Eggenfelden Bayern.
3	TA	$w_L = 56,8 \%$	$w_P = 27,9 \%$	$I_p = 28,9 \%$	Essenrode Niedersachen
4	OT/UA	$w_L = 68,0 \%$	$w_P = 41,2 \%$	$I_p = 26,9 \%$	Wehr Baden- Württemberg
5	TM	$w_L = 42,3 \%$	$w_P = 18,7 \%$	$I_p = 23,6 \%$	Überlingen B.-W.
6	TL	$w_L = 30,5 \%$	$w_P = 20,5 \%$	$I_p = 10,0 \%$	Friedrichshafen B.-W.

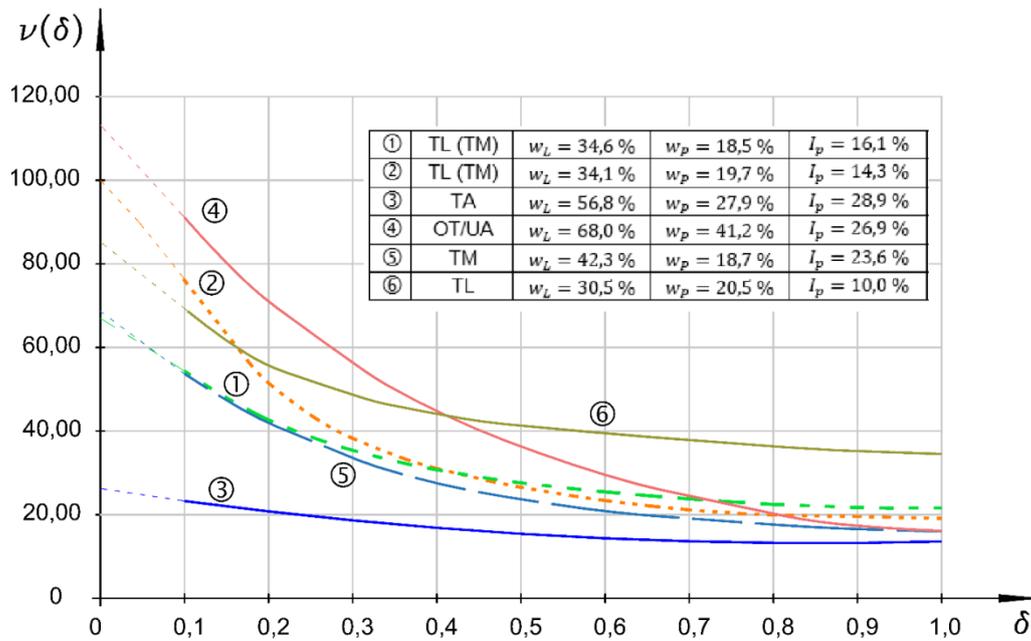


Fig.7. Stiffness coefficient $\nu(\delta)$ as a function of the magnitude of the load increment

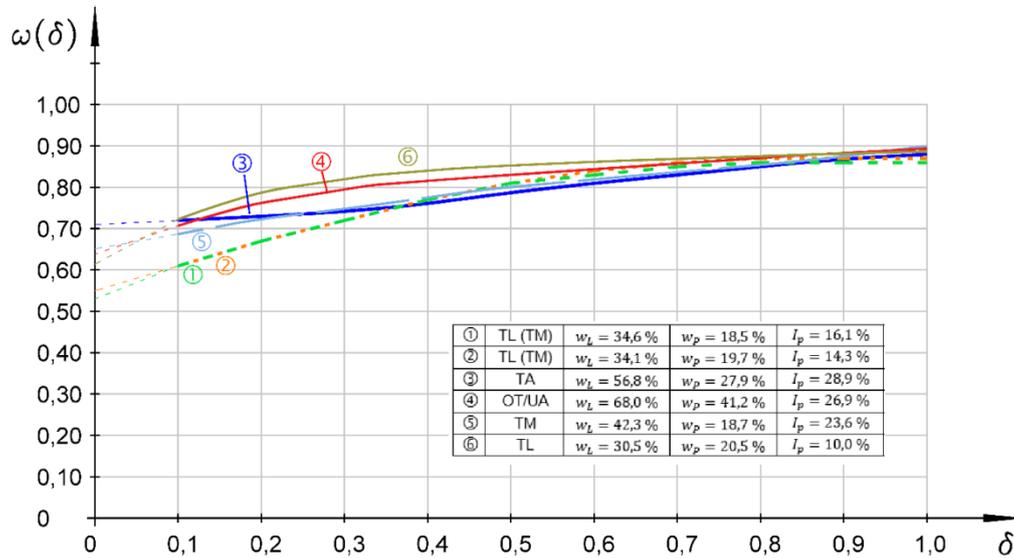


Fig.8. Stiffness exponent $w(\delta)$ as a function of the magnitude of the load increment

6. Applicable Approach for Using the Results of The Study in Practice

For the practical calculation of a settlement the stiffness modulus $E_s(\sigma, \delta_i)$, which depends on the load increase factor δ_i , can be determined from the oedometer values E_{oed} , as it is described in the following.

6.1. Estimation of the oedometer value E_{oed} from e.g. penetration tests (CPT), compression tests or other empirical values for each single soil layer $i \rightarrow$ hence, E_{oed} is known.

6.2. Calculation of the with E_{oed} associated stress σ_i in the compression test with a load increase factor of $\delta_i = 1 \rightarrow v(1)$ and $w(1)$ can be retrieved.

$$\begin{aligned} \text{from } E_{oed}(\sigma_i) &= \sigma_{at} v(1) \left(\frac{\sigma_i}{\sigma_{at}} \right)^{w(1)} \text{ follows for the stress level} \\ \sigma_i &= \sigma_{at} \left(\frac{E_{oed}(\sigma_i)}{\sigma_{at} \cdot v(1)} \right)^{\frac{1}{w(1)}} \end{aligned} \tag{6}$$

6.3. Determination of the in layer i resulting stress increase $\Delta\sigma_w$ due to the load of the construction

$$\Delta\sigma_w: f(\sigma_o, a, b, z)$$

6.4. Determination of the in the soil layer i resulting stress increase factor

$$\delta_i = \frac{\Delta\sigma_w}{\sigma_i}$$

6.5. Determination of the stiffness coefficient $v(\delta_i)$ and the stiffness exponent $w(\delta_i)$ for the in section 6.4 computed load increase factor δ_i using Figure 7 and Figure 8

6.6. Determination of the corrected stiffness modulus $E_{si}(\sigma_i, \delta_i)$ for the deformation calculation in layer i .

$$E_{si}(\sigma_i, \delta_i) = \sigma_{at} \cdot v(\delta_i) \cdot \left[\frac{\sigma_i}{\sigma_{at}} \right]^{w(\delta_i)} \quad (7)$$

6.7. Integration of the deformations in each layer i from $i=0$ to the required depth.

$$\Delta s = \int_{i=0}^{t_{gr}} \frac{\sigma_{wi}}{E_{si}(\sigma_i, \delta_i)} dz \quad (8)$$

In comparison to a settlement prediction without consideration of the influence of the load increase factor, the integral can be reduced to

$$\Delta s = \int_{i=0}^{t_{gr}} \frac{\sigma_{wi}}{E_{si}(\sigma_i, \delta=1)} dz \quad (9)$$

7. Discussion of the Study Results

The strain behavior of cohesive soils is also influenced by the structural resistance and the initial hydraulic gradient, especially with small load increments.

Structural resistance of the soil is understood to be a load magnitude, which must be exceeded in order to overcome the resistances in the contact points of the soil particles to allow deformation of the soil. The initial gradient denotes the hydraulic gradient up to which more water can be expressed from the pores of the soil, i.e. there is no consolidation on water-saturated soil. Only after exceeding this threshold the consolidation process of the soil will start. The process ends with the reduction of the additional pore water pressures caused by the load increment.

For clay minerals, which can be considered as layered silicate, the deformation behavior is significantly influenced by the number and valence of the cations within the silicate layers. Depending on the distance of the cations from the surface, the adhering water molecules are bound to different degrees.

Flow processes are then only triggered when a certain threshold is exceeded.

However, an accurate estimation of the deformation behavior, with the stiffness moduli determined in the compression test, presupposes that the creep deformations occurring after completion of the primary consolidation are negligibly small. Therefore, the evaluation of compression tests of soils which have considerable creep deformations is not entirely clear.

For the soils investigated in this article, the measured influence of creep on the load increase depending stiffness modulus, is negligible.

8. Examples for Practical Application

Example 1: Soil Nr. 1 - Soil group based DIN 18196 is TL close to TM.

From a characteristic soil profile in the region of Konstanz, Germany, in a depth of -15.0 m under the ground surface and for a 5-storey building project the following specifications are known:

$$\begin{aligned} q_c &= 0,6 \text{ MN/m}^2 \text{ (Peak resistance from a penetration test CPT)} \\ \sigma_{\ddot{u}}(z = 15,0\text{m}) &= 153 \text{ kN/m}^2 \text{ (soilstress)} \\ \Delta\sigma_z(z = 15,0\text{m}) &= 30 \text{ kN/m}^2 \text{ (stress change due to building load)} \end{aligned}$$

Estimation of the stress dependent stiffness modulus $E_s(\sigma)$ from the peak resistance of a penetration test according to DIN 4094:

$$v = 15,2 \cdot q_c + 50 = 15,2 \cdot 0,6 + 50 = 59,12 \quad w = 0,70$$

$$E_s = v \cdot p_a \left[\frac{\sigma_{\ddot{u}} + 0,5 \cdot \Delta\sigma_z}{p_a} \right]^w = 59,12 \cdot 100 \left[\frac{153 + 0,5 \cdot 30}{100} \right]^{0,70} = 8.500 \text{ kN/m}^2$$

$$\text{initial value for } \delta_i = 1 : E_{\text{oed}}(z = 15,0\text{m}) = 8.500 \text{ kN/m}^2$$

$$v(1,0) = 21,5 \quad w(1,0) = 0,86$$

$$\text{associated stress corresponding to section 6.2} \quad \sigma_i = 100 \left(\frac{8500}{100 \cdot 21,5} \right)^{\frac{1}{0,86}} = 494,5 \text{ kN/m}^2$$

with $\delta_i = \frac{\Delta\sigma}{\sigma_i} = \frac{30}{494,5} = 0,06$ follows for $v(0,06)$ and $w(0,06)$ from Figure 7 and Figure 8

$$v(0,06) = 60,0 \text{ and } w(0,06) = 0,59$$

therefore follows corresponding to section 6.5 for $E_{si}(\sigma_i, \delta_i)$

$$E_{si}(494,5 ; 0,06) = 100 \cdot 60,0 \cdot \left[\frac{494,5}{100} \right]^{0,59} = 15.406 \text{ kN/m}^2$$

Example 2: Soil Nr. 2 – Soil group TL bordering to TM according DIN 18196

The stiffness moduli with consideration of the corresponding stress increment δ_i in depths from -1.5 m to -5.0 m under the ground surface are computed.

Under a fixed foundation with the dimension of 5.0 m x 2.5 m (load effective layer = ground surface) the normal stress σ_w increases due to a soil pressure of $\sigma_0 = 100 \text{ kN/m}^2$ in the characteristic point at a depth of -1.5 m by $\Delta\sigma = 38,0 \text{ kN/m}^2$ and in a depth of -5.0 m by $\Delta\sigma = 15,0 \text{ kN/m}^2$.

The water contents in -1.5 m and -5.0 m depth are known to be 22.6 % and 24.0 %.

The water contents in the compression test can be associated with a stress level of $\sigma = 87 \text{ kN/m}^2$ (corresponds to a consistency of $lc = 0.8$) and $\sigma = 53 \text{ kN/m}^2$ ($lc = 0.7$) using the in Figure 1 presented experimental data.

In -1,5 m depth the following values could be determined:

$$w_n = 22.6 \% \quad \sigma(w_n = 22,6 \%) \approx 87.0 \text{ kN/m}^2$$

→ in Figure 7 and 8 values for $v(1.0) = 19.0$ and $w(1.0) = 0.87$ can be retrieved. Hence, the stiffness modulus for a load increase of 100 % can be calculated with Equation 4:

$$E_{\text{oed}} = \sigma_{\text{at}} \cdot v(1) \left(\frac{\sigma}{\sigma_{\text{at}}} \right)^{w(1)} = 100 \cdot 19,0 \cdot \left(\frac{87,0}{100} \right)^{0,87} = 1.683 \text{ kN/m}^2$$

$$\text{and } \delta_i = \frac{\Delta\sigma}{\sigma_i} = \frac{38.0}{87.0} = 0.436 \text{ follows from Figure 7 and 8:}$$

$$\rightarrow v(0,436) = 29.3 \text{ and } w(0,436) = 0.79 \text{ therefore } E_s \text{ is}$$

$$\begin{aligned} E_{\text{si}}(\sigma_i = 87.0; \delta_i = 0.436) &= \sigma_{\text{at}} \cdot v(\delta_i) \cdot \left[\frac{\sigma_i}{\sigma_{\text{at}}} \right]^{w(\delta_i)} \\ &= 100 \cdot 29.3 \cdot \left[\frac{87.0}{100} \right]^{0.79} = 2.625 \text{ kN/m}^2 \end{aligned}$$

In -5,0 m depth the following values could be determined:

$$w_n = 24.0 \% \quad \sigma(w_n = 24.0 \%) \approx 53.0 \text{ kN/m}^2$$

→ in Figure 7 and 8 values for $v(1.0) = 19.0$ and $w(1.0) = 0.87$ can be retrieved. Hence, the stiffness modulus for a load increase of 100 % can be calculated with Equation 4:

$$E_{\text{oed}} = \sigma_{\text{at}} \cdot v(1) \left(\frac{\sigma}{\sigma_{\text{at}}} \right)^{w(1)} = 100 \cdot 19,0 \cdot \left(\frac{53.0}{100} \right)^{0,87} = 1.094 \text{ kN/m}^2$$

$$\text{with } \delta_i = \frac{\Delta\sigma}{\sigma_i} = \frac{15.0}{53.0} = 0.283 \text{ follows from Figure 7 and 8:}$$

$$\rightarrow v(0.283) = 40.0 \text{ und } w(0.283) = 0.72 \text{ therefore } E_s \text{ is}$$

$$\begin{aligned} E_{\text{si}}(\sigma_i = 53,0; \delta_i = 0,283) &= \sigma_{\text{at}} \cdot v(\delta_i) \cdot \left[\frac{\sigma_i}{\sigma_{\text{at}}} \right]^{w(\delta_i)} \\ &= 100 \cdot 40,0 \cdot \left[\frac{53,0}{100} \right]^{0,72} = \text{kN/m}^2 \end{aligned}$$

9. Appendix

9.1 Characteristics of the compression test of soil 2:

yield point $w_L = 34,1\%$ plastic limit $w_P = 19,7\%$ grain density $\rho_s = 2,662 \text{ g/cm}^3$
 sample diameter $\varnothing = 7,0 \text{ cm}$ installation height $h_a = 1,910 \text{ cm}$
 moisture mass at installation $m_f = 137,92 \text{ g}$
 water content at installation $w_{\text{installation}} = 31,38 \%$
 water content at removal $w_{\text{removal}} = 17,34 \%$

With these characteristics, the state at installation is as follows:

$$\text{moist density } \rho_f = \frac{m_f}{r^2 \cdot \pi \cdot h_a} = \frac{137,92}{3,5^2 \cdot \pi \cdot 1,91} = 1,876 \text{ g/cm}^3$$

$$\text{dry density } \rho_d = \frac{\rho_f}{1+w} = \frac{1,876}{1+0,3138} = 1,428 \text{ g/cm}^3$$

$$\text{porosity } n = 1 - \frac{\rho_d}{\rho_s} = 1 - \frac{1,428}{2,662} = 0,464 \quad e_{\text{installation}} = \frac{n}{1-n} = \frac{0,464}{1-0,464} = 0,866$$

$$\text{saturation level } S_r = \frac{w \cdot \rho_d}{n \cdot \rho_w} = \frac{0,3138 \cdot 1,428}{0,464 \cdot 1,0} = 0,966$$

with $\Delta s_{\text{removal}} = 4,056 \text{ mm}$ follows for $\Delta m_w = \frac{\varnothing^2}{4} \pi \cdot \Delta s \cdot \rho_w = \frac{7,0^2}{4} \pi \cdot 0,4056 \cdot 1,0 = 15,609 \text{ g}$
 $m_{f(\text{removal})} = 137,92 - 15,609 = 122,311 \text{ g}$
 $h_1 = h_a - \Delta s_{\text{removal}} = 19,10 - 4,056 = 15,044 \text{ mm}$

Removal state:

$$\rho_f = \frac{122,311}{3,5^2 \cdot \pi \cdot 1,5044} = 2,113 \text{ g/cm}^3$$

$$\rho_d = \frac{2,113}{1 + 0,1734} = 1,801 \text{ g/cm}^3$$

$$n = 1 - \frac{1,801}{2,662} = 0,323 \quad e_{\text{removal}} = \frac{n}{1-n} = \frac{0,323}{1-0,323} = 0,477$$

$$S_r = \frac{0,1734 \cdot 1,801}{0,323 \cdot 1,0} = 0,967$$

Calculated relative settlement at removal state s'_{removal}

with $s' = \frac{\Delta h}{h_a} = \frac{e_0 - e_1}{1 + e_0}$ and $e = \frac{w \cdot \rho_s}{S_r \cdot \rho_w}$ follows for the relative settlement at removal:

$$s'_{\text{removal}} = \frac{\frac{W_{\text{installation}}}{S_{r(\text{installation})}} - \frac{W_{\text{removal}}}{S_{r(\text{removal})}}}{\frac{\rho_w}{\rho_s} + \frac{W_{\text{installation}}}{S_{r(\text{installation})}}} = \frac{\frac{0,3138}{0,966} - \frac{0,1734}{0,967}}{\frac{1,0}{2,662} + \frac{0,3138}{0,966}} = 0,2077$$

Using the calculated relative settlement s'_{removal} , the ordinate in a graph can present the relative settlements with the values of the associated water contents as well as the

void ratios. Hence, with a specific water content associated stress level from a compression test can simply be retrieved from the graph.

9.2 Derivation of Equation 1 at the unity cube:

h_a - initial sample height

h_1 - sample height after load of $\Delta\sigma_1$

h_2 - sample height after load of $\Delta\sigma_1 + \Delta\sigma_2$



$$s'_1 = \frac{\Delta h}{h_a} = \Delta \varepsilon_1 \quad \Delta s'_1 = s'_2 - s'_1 = \frac{\Delta h + \Delta \Delta h}{h_a} - \frac{\Delta h}{h_a} = \frac{\Delta \Delta h}{h_a}$$

$$\Delta \varepsilon_2 = \frac{\Delta \Delta h}{h_1} = \frac{\Delta \Delta h}{h_a - \Delta h} = \frac{\Delta \Delta h / h_a}{1 - \Delta h / h_a} = \frac{\Delta s'_1}{1 - s'_1}$$

$$E_s = \frac{\Delta \sigma_2}{\Delta \varepsilon_2} = \frac{\Delta \sigma_2}{\Delta s'_1} (1 - s'_1)$$

References

1. De Simone, V. (2017): Master thesis at the HTWG Konstanz.
2. Ibrahim, K. (2017): Master thesis at the HTWG Konstanz.
3. Francese, M. (2016): Bachelor thesis at the HTWK Konstanz.
4. Schmidt, Ph. (2016): Bachelor thesis at the HTWG Konstanz.
5. De Simone, V. (2015): Bachelor thesis at the HTWG Konstanz.
6. Salzer, I. (2015): Bachelor thesis at the HTWG Konstanz.
7. Ohde (1939): Zur Theorie der Druckverteilung im Baugrund, Bauingenieur 20, S.551.
8. Alters, J. (1976): Die Grenztiefe bei Setzungsberechnungen, Bauingenieur 51 S.93 -96
9. Vardanega, P.J., ASCE, M., Bolton, M.D. (2013): Stiffness of Clays and Silt: Normalizing Shear Modulus and Shear Strain, J.Geotech. Geoenviron. Eng. 2013.139:1575-1589.
10. Reitmeier, W. (1989): Quantifizierung von Setzungsdifferenzen mit Hilfe einer stochastischen Betrachtungsweise, Diss. Heft 13, Schriftenreihe Lehrstuhl und Prüfamf für Grundbau, Bodenmechanik und Felsmechanik, Techn. Universität München .
11. Engels, J. (2002): Verfahren zur Festlegung von Kennwerten für bodenmechanische Nachweise, Institut für Geotechnik, Technische Universität Dresden, Heft 10.
12. Bassem S. (2005): Setzungsverhalten von Flachgründungen in normalkonsolidierten Böden. Schriftenreihe Geotechnik Universität Kassel, Heft 16 .
13. Soumaya B; Kempfert H.-G. (2010): Verformungsverhalten weicher Böden im spannungsgesteuerten Kompressionsversuch, Bautechnik 87(2010), Heft 2.

