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# FLEXURAL STRENGTHENING OF REINFORCED CONCRETE ELEMENTS USING EXTERNAL TIE-RODS

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***Abstract:*** *The present study analyses the setup and computational methods of flexural linear reinforced concrete elements strengthening systems. Behaviour particularities under exploitation of the reinforced concrete beam-tie-rods in trapezoidal system assembly are defined.*

*The computational algorithm is supposed to start with the preliminary design of the trapezoidal tie-rods considering a criterion that assures the nomogram system's optimal conformation. The preliminary design consists of Mathlab codes which provide the system's optimal characteristics.*

*The stresses determination in the resulted system is undertaken throughout statically indeterminate computational procedures and the solution is given by mathematical relationships in Mathlab or by finite element modelling.*

***Key words:*** *consolidation, tie-rods system, optimization.*

## 1. Introduction

The article submits attention to the way in which the consolidation systems of flexural linear reinforced concrete elements are composed and calculated. The consolidation solution involving exterior tie rods is analyzed, these systems having a contribution in increasing the capable moment and shear force. Exterior tie rods can be used with or without prestressing stress. The criteria for choosing the optimal variant are established. The general case is presented – consolidation with pretensioned exterior tie rods.

## 2. Defining the Solution

As a solution principle, the consolidation of reinforced concrete beams by external prestressing with exterior tie rod consists in the provision of some steel tie rods made of high strength steel, symmetrically disposed in the transverse section, anchored on the

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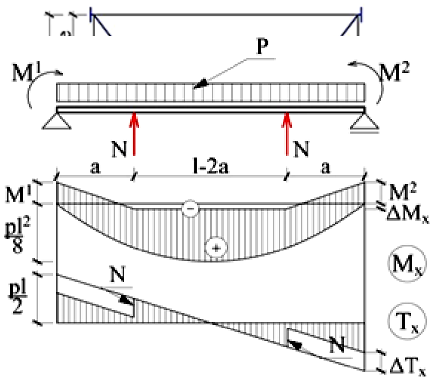
supports of the beam and following a polygonal path forced by the metallic partsfor deviation. The transfer of the initial stresses and the effect of the variation of the operating loads is done through the anchoring parts and the deviators of the tie rod. In Figure 1 the consolidation solution through prestressing with exterior tie rod and the equivalent load with the bending moments associated to the tension of the tie rod are schematically presented [2]. The favorable aspect of the prestressing on the reinforced concrete beam is to be noted (Fig. 2).



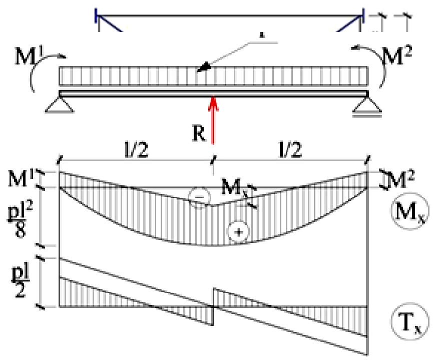




Fig.1 *The transfer of the initial stressesand the variation from operation loads*

Fig.2. *The favorable effect of prestressing on the reinforced concrete beam*

## 3. Computation for Consolidation Systems of Flexural Linear Elements

### 3.1. Behavior particularities under loads of the ensemble reinforced concrete beam - exterior tie rod type consolidation system

After the assembly of the exterior tie rod on the element which is consolidated a new system from the statical point of view is obtained, formed by the reinforced concrete beam and the exterior tie rod. The system in statically undetermined. The stresses are produced in the systemunder the actionof any exterior load disposed after assembly and the beginning of work of the exterior tie rod. The efficiency of the exterior tie rod typeconsolidation system is dependent on thetie rod deformability.It decreases with increasing eccentricity of the compression force given by the tie rod. The additional load which can be disposed on the beam after its consolidation with exterior tie rod passive system is conditioned by the ratio: *The effective moment of the beam when installing the consolidation systemMef / The capable moment of the beam Mcap*and the characteristics of the exterior tie rod ($A_t$, $E_t$, geometrical characteristics). In the case in which the ratio $M_{ef}/M_{cap}$ is close to 1 or the additional load is big compared to the initial one, taking over the stress cannot be done by a exterior tie rod passive system, but an active one – by the prestressing of the tie rod.

The conditions required by the consolidation system impose the verification of the possibility of undertaking the additional loads by passive systems or by active ones, a preliminary design of the tie rod area respectively [1, 3].

### 3.2. Preliminary design of the consolidation exterior tie rods

3.2.1. Stress calculation in the exterior tie rod

The expressions which result after the calculation are [1]:
The stress in the tie rod

$$X_1 = \frac{p\cdot(a^3+l^3-2a^2l)}{4\cdot a\cdot tg\alpha(3l-4a)}\cdot\frac{1}{1+\gamma} \quad \text{where} \quad \gamma = \frac{3EI}{E_t\cdot A_t}\cdot\frac{\frac{2a}{cos^3\alpha}+l-2a}{a^2\cdot tg^2\alpha(3l-4a)} \qquad (1)$$

3.2.2. The efficiency of the consolidation system is highlighted with the help of the **efficiency coefficient d** defined as:

$$d = \frac{\text{The value with which the moment from the load } p_c \text{ decreases,as an effect of the consolidation}}{\text{The moment which would appear under the action of the load } p_c \text{ ,in absence of the consolidation}}$$

Being moments on the reinforced concrete beam which is consolidated. $p_c$ represents the load which acts on the beam after the consolidation.

$$d = \frac{2(a^3+l^3-2a^2l)}{l^2(3l-4a)}\cdot\frac{1}{1+\gamma} \text{ denoting } \frac{2(a^3+l^3-2a^2l)}{l^2(3l-4a)} = d_{max} \text{ becomes } d = d_{max}\cdot\frac{1}{1+\gamma} \qquad (2)$$

The efficiency coefficient of the consolidation system d varies with the configuration of the tie rod path and the area $A_t$. For the given load case the minimum necessary value of the effieciency coefficient $d_{nec}$ is determined.

$$d \geq \frac{M_{nec}-M_{cap}}{M_{nec}-M_{ef}} = d_{nec} \qquad (3)$$

By adopting different areas $A_t$ for the tie rod ,which would lead to a value of the coefficient d smaller than the $d_{nec}$ value, the efficiency of the consolidation system is unsatisfactory (the reinforced concrete beam is destroyed under the load $p_c$ by exceeding $M_{cap}$). If $d_{nec}> d_{max}$ , the passive system cannot undertake the additional load and the adoption of an active system is necessary.

The active system undertakes the applied load after the consolidation system starts working, by a mechanism which can be schematically presented as follows:

• a part of the additional load is undertaken by passive effect, corresponding to the effective area of the tie rod and calculated accordingly;

• the rest of the additional load (until the one corresponding to $M_{nec}$ ) is equilibrated by the active effect, similar to a system of exterior forces, which manifest on the consolidated beam by reactions disposed at the point of change of the tie rod path, having the value:

$$V_{1\ activ} = X_{1\ activ} \cdot tg\alpha \quad (4)$$

### 3.3. The use of the efficiency coefficient $d_{nec}$ for the preliminary design of the area of the tie rod

The preliminary design of the area of the tie rod according to the efficiency coefficient $d_{nec}$ is done by using nomograms [1],[4] or in a computer program using the Mathlab programming language. In the program the following data was introduced:

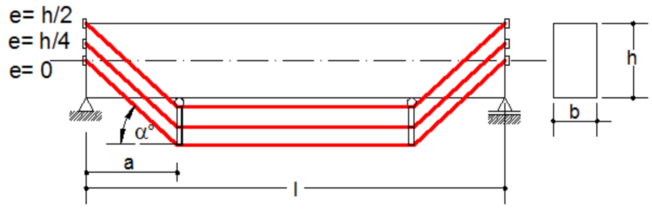



Fig.3. *The scheme with the disposal mode of the tie rods*

- the necessary efficiency coefficient ($d_{nec}$ );
- the mechanical and geometrical characteristics of the reinforced concrete beam: b,h,l,$E_b$;
- the exterior tie rod characteristics: a, α, e, $E_t$;

The a value is kept, regardless of the eccentricity with which the tie rod is fixed.
For this analized case, the programming code is detailed in previous research [1].
Simulations were done for different characteristics of the consolidation system. The results were compared to the ones obtained in the nomograms, similar values being obtained. For exemplification, the results of the program run for an analyzed case are presented.

```
>> Atdv001(0.14,20,50,450,245000,12.5,20,2100000);
```

Introduced data:
dnec: **0.14**;b [cm]: 20;   h [cm]: 50;   l [cm]: 450;  Eb [daN/cm2]: 245000
Et [daN/cm2]: 2100000;   e [cm]: 12.5;   alfa1 [grad]: 20
Results: Area of the tie rod [$cm^2$]:   **7.498**

The analysis was done for the uniform loading case. For other types of loads the process is similar .

**As a synthesis, the computational algorithm** of the consolidation of a reinforced concrete beam having exterior tie rods consists in the following stages:

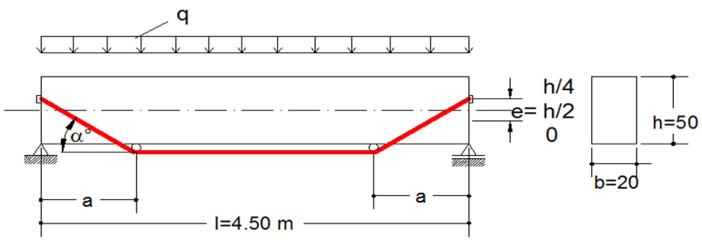
- The determination of the stress state of the beam ($M_{nec,}$ $M_{ef}$) and of its characteristics ($M_{cap,}$ $E_{bet}$).
- Establishing the geometrical parameters  (α , a, e) and the quality of the tie rod.
- The determination of the necessary efficiency coefficient $d_{nec}$.
- Establishing the adequate consolidation type: (i) passive system, (ii) active system.
- Preliminary design:

$A_t$for the passive system,  $A_t$ and the prestressing force $X_{activ}$for the active system.

- The calculation of the state of stresses in the elements of the ensemble consolidatedbeam – exterior tie rod.
- The verification:$M_{nec} - a \cdot tg\alpha \cdot \left(X_{1\,pasiv} + X_{1\,activ}\right) + e \cdot \left(X_{1\,pasiv} + X_{1\,activ}\right) \leq M^{*}_{cap}$

**4. Exemple of Computation**

The exterior tie rod consolidation type for a reinforced concrete beam was dimensioned.




A computation was done using the finite element method, using the computation program Robot. The element is linear,beingthe reinforced concrete beam.

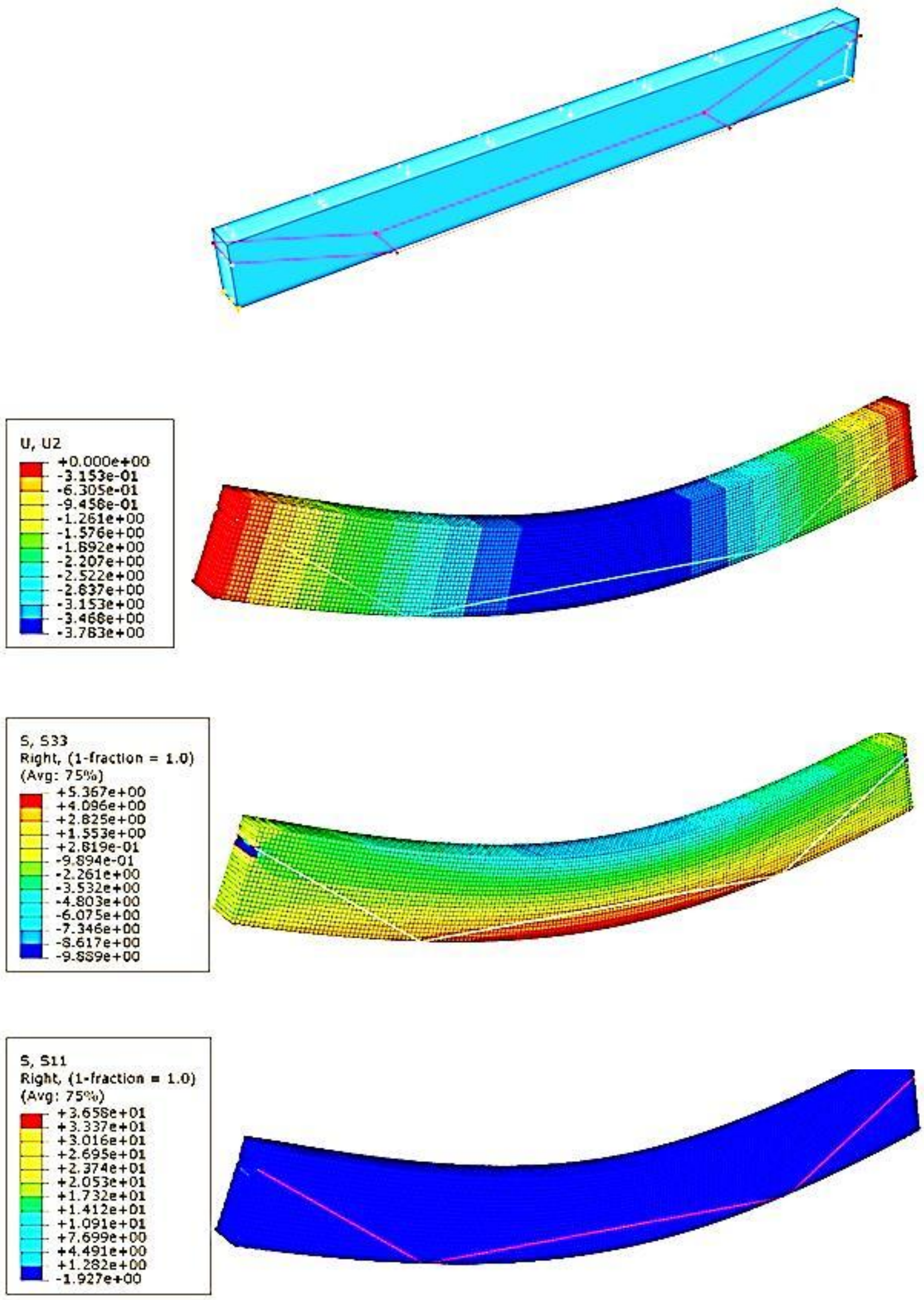
The following parameters for the reinforced concrete beam are considered:

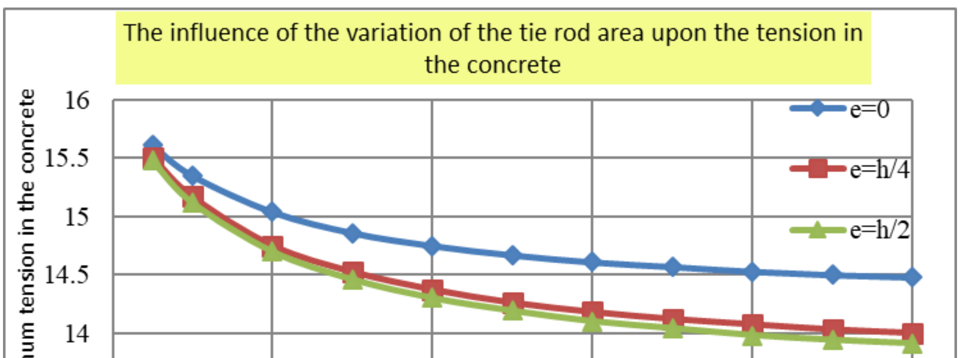
- in the section: b=200 mm; h = 500 mm; l=4500 mm; e=h/4, a=103 mm;
- hinged end in the left part and simply supported to the right;
- concrete class: BC 15, E=14700 N/mm$^2$, Poisson`s coefficient=0,2;
- reinforcement: steel grade PC 52 E=210000 N/mm$^2$, Poisson`s coefficient =0,3;
- the section area of the tie rod elements $A_{em}$= 2000 mm$^2$;
- the load q=0.138 N/mm$^2$.

The computation, whose results are presented below, aims to highlight the efficiency of the consolidation system type adopted. The displacements of the beam, the stresses in the concrete, respectively the stresses in the consolidation exterior tie rod are highlighted. The values presented in the Fig. 6-8 relate to the particular analized case.

The graphs obtained by finite element method analysis on the previously analyzed beam are presented below for different values of the tie rod area and different eccentricities. The graphs present the efficiency of the tie rods through the influence of the tie rod area upon the displacement, the stress in the concrete and the stress in the tie rod.

The computation has been carried out for the following values for eccentricities : e=0, e=h/4, e=h/2. In Tab.1 it is highlighted the assessment for e=h/4. [1]


U, U2
+0.000e+00
-3.153e-01
-6.305e-01
-9.458e-01
-1.261e+00
-1.576e+00
-1.892e+00
-2.207e+00
-2.522e+00
-2.837e+00
-3.153e+00
-3.468e+00
-3.783e+00
S, S33
Right, (1-fraction = 1.0)
(Avg: 75%)
+5.367e+00
+4.096e+00
+2.825e+00
+1.553e+00
+2.819e-01
-9.894e-01
-2.261e+00
-3.532e+00
-4.803e+00
-6.075e+00
-7.346e+00
-8.617e+00
-9.889e+00
S, S11
Right, (1-fraction = 1.0)
(Avg: 75%)
+3.658e+01
+3.337e+01
+3.016e+01
+2.695e+01
+2.374e+01
+2.053e+01
+1.732e+01
+1.412e+01
+1.091e+01
+7.699e+00
+4.491e+00
+1.282e+00
-1.927e+00


The influence of the variation of the tie rod area upon the tension in the concrete
16
15.5
15
14.5
14
tension in the concrete
e=0
e=h/4
e=h/2

.

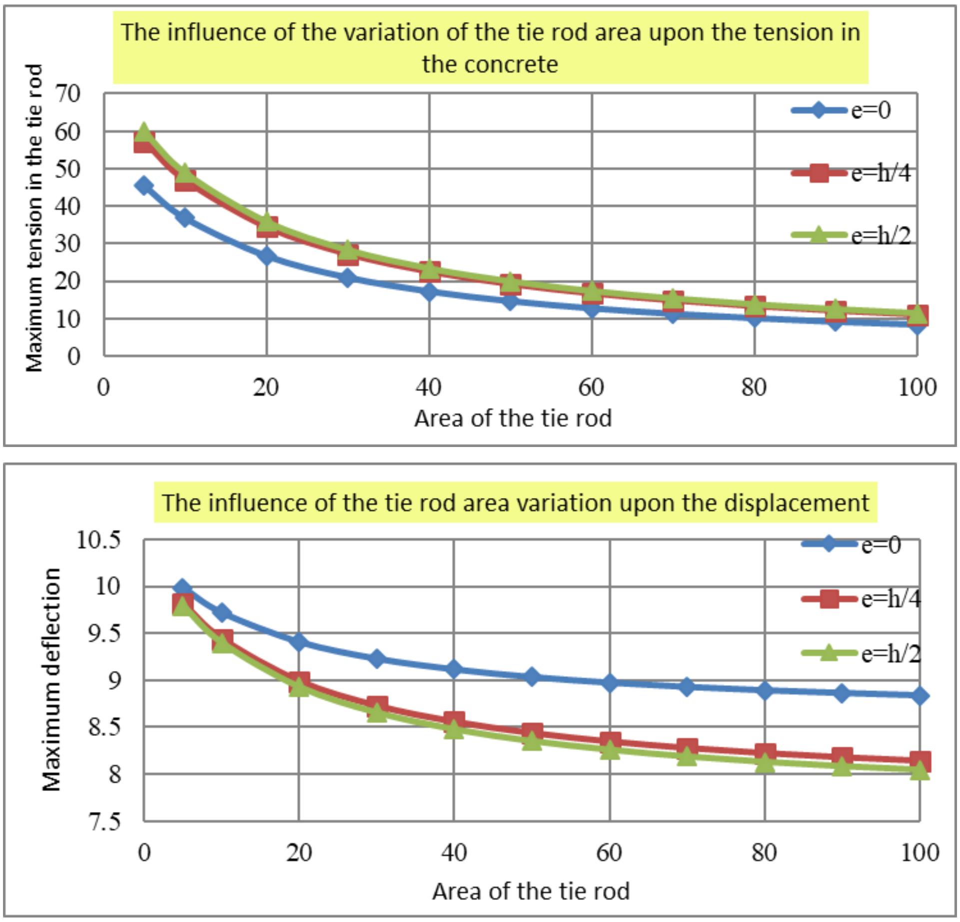



Fig.9.*Influence of the tie rod area upon the efficiency of the consolidation system*

Variation of the stresses and deformations with respect to the tie rod area for **e =h/4**

Table 1

| | Tie rod area [cm$^2$] | Maximum deflection [mm] | Maximum tension in concrete [N/mm$^2$] | Maximum tension in the exterior tie rod [N/mm$^2$] |
|---|---|---|---|---|
| Initial load | 0 | 5.204 | 8.02 | 0 |
| Value for initial load + the additional load | 0 | 10.408 | 16.04 | 0 |
| | 5 | 9.817 | 15.509 | 57.25 |
| | 10 | 9.44 | 15.17 | 46.88 |
| | 20 | 8.987 | 14.75 | 34.4 |
| | 30 | 8.725 | 14.525 | 27.16 |
| | 40 | 8.554 | 14.372 | 22.44 |
| | 50 | 8.434 | 14.26 | 19.12 |
| | 60 | 8.344 | 14.18 | 16.65 |
| | 70 | 8.275 | 14.12 | 14.75 |
| | 80 | 8.22 | 14.074 | 13.24 |
| | 90 | 8.176 | 14.03 | 12.01 |
| | 100 | 8.139 | 14 | 10.98 |

The analysis is also presented in Tab. 1 in which the reference elements used are the values of the stresses and deformations for the un-strengthened beam with exterior tie rod. The comparison allows highlighting the efficiency grade of the exterior tie rod as a function of its mechanical and geometrical characteristics.

## 5. Conclusions

Computational algorithm confirms the performance of the consolidation technique. Both the theoretical approach and the practical one are embodied in the present study, with preliminary design forms and optimization techniques of strengthening system’s characteristics.

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