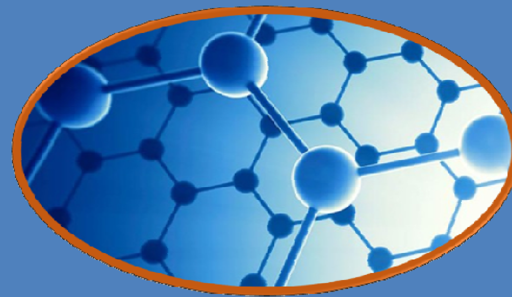

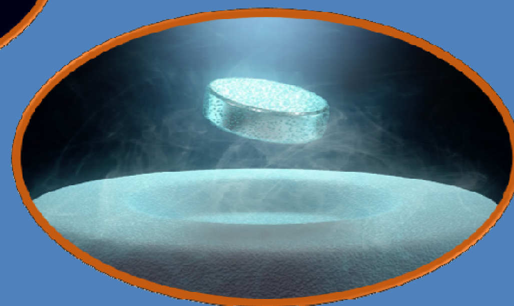

MATERIALS FOR ELECTRICAL ENGINEERING

- Handbook -

Elena HELEREA
Marius Daniel CĂLIN
Cristian Leonard MUŞUROI

Editura Universității Transilvania din Brașov

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Transilvania University of Brasov
Faculty of Electrical Engineering and Computer Science

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FOREWORD

This course support is intended for students from the bachelor's program "Electrical Engineering and Computers", taught in English, a program that was organized at the Faculty of Electrical Engineering and Computing Science at Transilvania University in Brasov, since the year 2004. But, this course support is also useful for students from other study programs as well as graduates who want to know new aspects related to the development and use of new categories of materials required by modern technologies.

The proposed handbook covers the subjects for Materials in Electrical Engineering Course is addressed for second year of study, and contains elements necessary for acquiring cognitive and practical and communication skills, which will allow students to explain the basic properties of materials used in electrical engineering, in correlation with intrinsic and extrinsic factors which influence their behaviour under various conditions: electrical, thermal, mechanical, environmental.

After following this course, students will be capable of understanding and assimilate the methodology and measurement techniques of the main parameters of conductive, semiconductive and electroinsulating materials, dielectrics and magnetic materials for optimal choice of materials used in technical applications, in association with their designing, fabrication, testing, exploitation and maintenance processes.

The course benefits from the experience of the authors in their research carried out in the field of materials used in electrical and electronic engineering, included in previously published works:

- Helerea E., Țiganea D., Stănulet D.: Materiale electrotehnice, Universitatea din Braşov, 1984.
- Helerea E.: Materiale electrotehnice. Dielectrici. Editura Universităţii Transilvania, Braşov, 1998.
- Helerea E., Ţică R., Dumitrescu L.: Materialeelectroconductoare, Materiale electroizolante. Interferenţe cu mediul, Universitatea Transilvania, p.250, 2003.
- Helerea E., Oltean I.D., Munteanu A.: Materials for electrical and electronic engineering, Lux Libris Press House, Braşov, p.158, 2004.
- Călin M.D., Helerea E.: Materiale magnetice pentru maşini electrice utilizate în transport. Metode şi procedure avansate de caracterizare şi modelare, EdituraUniversităţii Transilvania din Braşov, Romania, p.120, 2012.
- Călin M.D., Georgescu M.C., Helerea E., Magnetic materials for electrical machines used in transportation, Editura LAP-Lambert Academic Publishing, Germany, p.132, 2015.
- Helerea E., Călin M.D.: Materials in Electrical Engineering, Editura Universităţii Transilvania din Braşov, Romania, p.377, 2015.

New to this course support is the approach mode. Each proposed subject includes a table of contents, a short summary, followed by a detailed presentation of the proposed subject and at the end a set of self-assessment tests is proposed to evaluate the acquired knowledge.

Lecture 1, *Introduction in Science of Electrical Materials,* includes data on the trends in advanced material development, the objectives of electrical materials course, the structure, properties, parameters and some classification of materials.

Lecture 2, *Electromagnetic Theory and Material Laws,* includes an approach on the macroscopic and microscopic theories of electromagnetism, laws of materials in science, and laws of materials in electrical engineering.

Lecture 3, *Electric Conduction in Metals*, the technique of experimental determination of electrical resistivity of materials is presented, and for the case of metals, the mechanism of the electrical conduction and the expression of electric conductivity are detailed.

Lecture 4, *Energy-Band Model in Crystals and Superconducting State*, includes the energy-bands theory in crystals, electrical conduction in the energy-bands model, and an introduction regarding the superconducting state, characteristics, justification and applications.

Lecture 5, *Electrical Conduction in Semiconductive Materials,* is about the characteristics of semiconductive materials, and debates the electrical conduction in intrinsic and extrinsic semiconductors, and effect of temperature on semiconductor materials.

Lecture 6, *Applications of Semiconductive Materials*, describes the controlled conduction and p-n junction, applications of semiconductive materials for sensors and other devices.

Lecture 7, *Dielectrics in Electrical Engineering*, introduces a new subject regarding the dielectrics, namely, characterization, classification, particularities of electrical conduction and some aspects on electrical breakdown in dielectrics.

Lecture 8, *Dielectric Polarization in Constant Electric Fields*, treats the electronic, ionic and orientation polarizations in constant fields, with examples on each category of dielectrics.

Lecture 9, *Dielectric Polarization in Harmonic Electric Fields*, proposes a model to calculate the complex permittivity of dielectrics, and also the losses in dielectrics.

Lecture 10, *Magnetic Materials in Electrical Engineering*, proposes the characterization and classifications of magnetic materials and it details the characteristics of the materials with magnetic order.

Lecture 11, *Behavior of Materials in Constant Magnetic Fields*, treats the mechanism of diamagnetism, paramagnetism and super-paramagnetism of materials, and applications.

Lecture 12, *Characteristics and Losses in Soft Ferromagnetic Materials*, addresses the characteristics of soft magnetic materials and losses in magnetic materials.

Lecture 13, *Soft Ferromagnetic Materials - Magnetic Domain Structures and Applications,* includes the theory of magnetic domains in ferromagnetic materials, structure of the domains in magnetization process, and applications of soft magnetic materials.

Lecture 14, *Hard Magnetic Materials - Properties and Applications*, treats properties of hard magnetic materials, it describes the permanent magnet operation point and design selection criteria, and evolution and applications of permanent magnetic materials.

We believe that the final chapter, *Homework in Materials for Electrical Engineering*, will be a useful tool for students in applying the acquired knowledge to other disciplines they will follow in the bachelor's cycle, the master's cycle and in later engineering practice.

Authors,
Brașov, February 2023

CONTENT

1. Introduction in Science of Electrical Materials

Contents

1.1. Trends in Advanced Material Development

Section Summary: *In this section, a systematic investigation of materials is done, in which are pointed: new techniques for material properties measurement, new technologies for material processing, new materials and new applications of advanced materials in electrical and electronic engineering.*

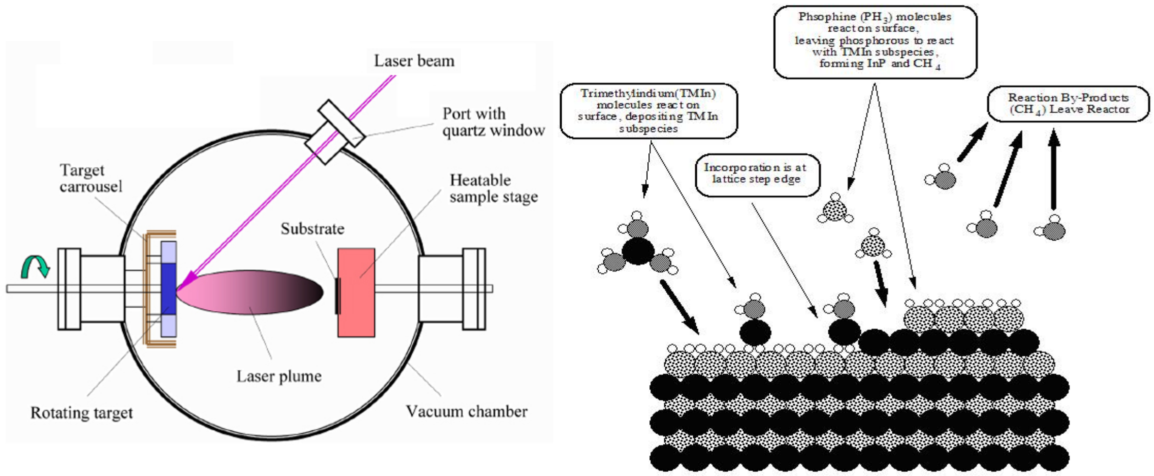
Investigation of the materials used in electrical and electronic technologies is made with tools of different branches of science:

- Physics & Chemistry – for define the composition and the structure of materials,
- Material Science – for describe the composition, structure and properties of materials
- Material Engineering – this includes the methods of manufacturing and processing the materials and semi-products utilized in electrical and electronic devices and equipment.

Today, new techniques for material properties measurement are developed, starting from laser techniques; electronic microscopy; electronic spectroscopy and continuing with special technics, as vibrating sample magnetometry (Fig. 1.1), and others.

Fig. 1.1. New types of magnetometers for magnetic material investigation

Also, new technologies for material processing are applied, as Pulse Laser Deposition (Fig. 1.2.a) and Liquid/ Vapor/Metal-organic Phase Epitaxy (Fig. 1.2.b)



a) b)

Fig. 1.2. New technologies for material processing:
a) Pulse Laser Deposition; b) Liquid/ Vapor/Metal-organic Phase Epitaxy

The actual trends in advanced material development is the introduction on the large scale of the new types of materials, as carbon nanotubes based on graphene, nanoferrites, smart composite materials as shape memory alloys, metamaterials, etc.

Carbon nanotubes are made of carbon, with single-wall or multi-wall carbon structures, having the diameters of order of nanometers. The length typically is much larger than its diameter. The carbon nanotubes havespecific properties: high electric conductivity, high thermal conductivity and high mechanical strength, which directed these materials for electronics, optics, and biomechanics. A synthetic view of the applications of carbon nanotubes is electrical engineering is given in Fig. 1.3.

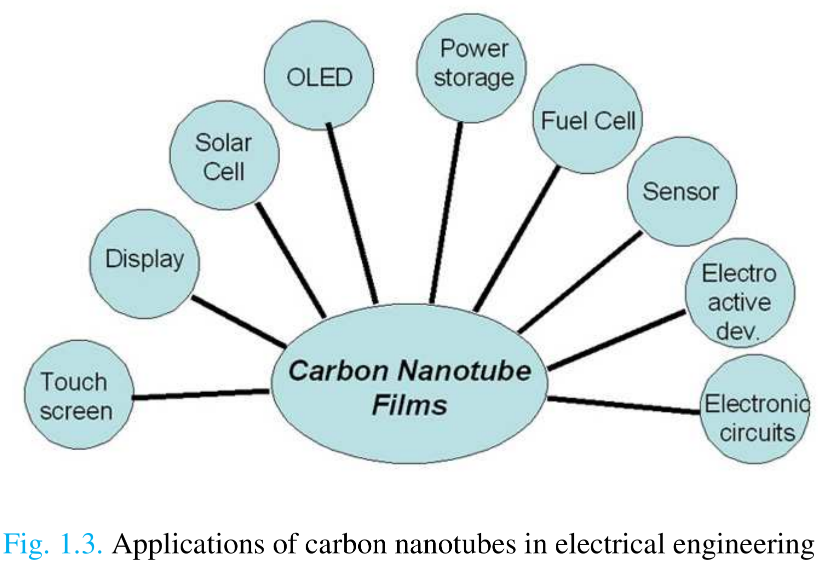
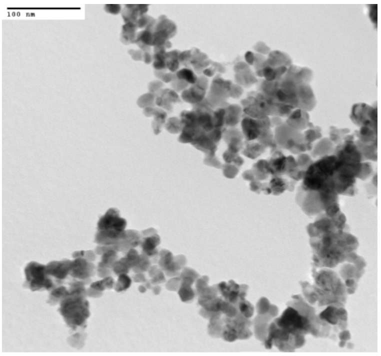
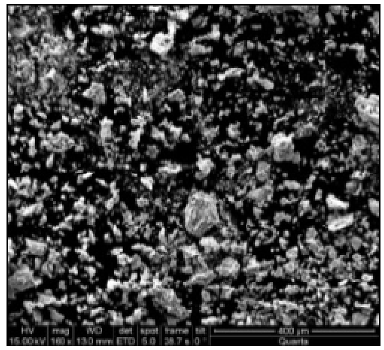


Fig. 1.3. Applications of carbon nanotubes in electrical engineering

Nanoferrites are new materials applied in electrical and electronic devices. Different structures of ferrites (combinations of metal oxides of Cu, Co, Zn with Fe_2O_3) with grains of order of nanometers are proposed (Fig. 1.4).



a)



b)

Fig. 1.4. TEM microscope images of some nanoferrites: a) $Co_{0.6}Zn_{0.4}Fe_2O_4$; b) $CuFe_2O_4$

Specific magnetic and electric properties are obtained by modifying the dimensions of grains and the weight of the components. An example is shown in Fig. 1.5: the electric resistivity of the nanoferrite of $Co_{(1-x)}Zn_xFe_2O_4$varies substantially when the concentration of zinc (Zn) is modified.

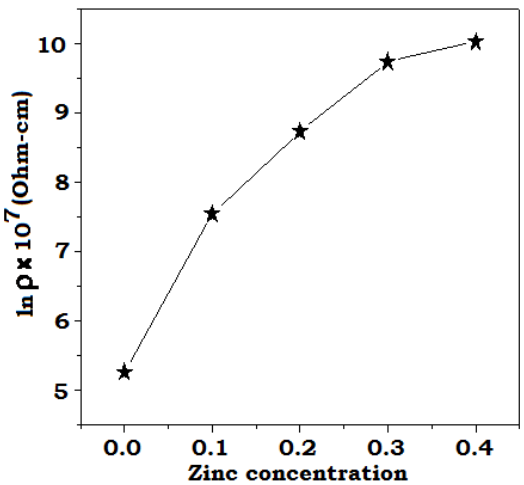
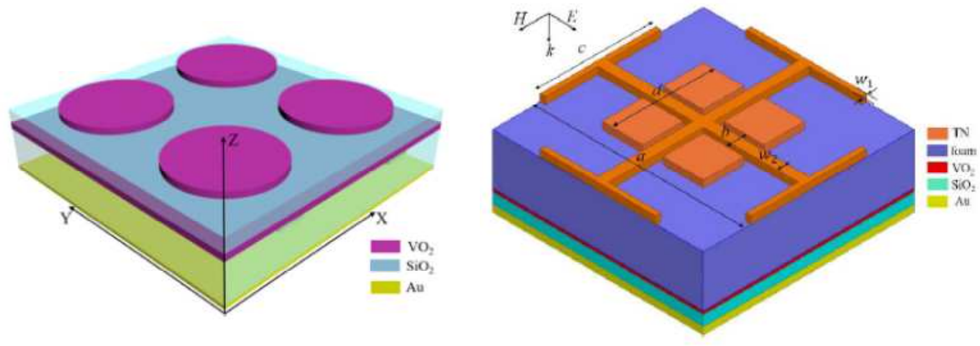


Fig. 1.5. Variations of DC resistivity of $Co_{(1-x)}Zn_xFe_2O_4$nanoferrites with zinc content

Smart materials are definedas any material that is capable of being controlled such that its response and properties change under a stimulus in a reproducible manner. Example: shape memory alloys for aerospace applications and piezoelectric materials.

Metamaterials provide unusual characteristics, such as negative refraction, perfect absorption or perfect transmission of electromagnetic waves, sub-wavelength resolution imaging, inverse Doppler effect, and backwards propagation.

These characteristics are obtained through the periodic arrangement of unit cells, called "meta-atoms", which have the size smaller than the incident electromagnetic wavelength. An example is given in Fig. 1.6.



a) b)

Fig. 1.6. Structures of unit-cells (meta-atoms) in a multilayer hybridmetamaterial based on vanadium dioxide (VO2): a) layers structure overview b) the composite resonant structure

In Fig. 1.6, the proposed structure of the layers includes: VO2 disks, silica (SiO2) layer, VO2 film, metallic strip, SiO2 layer, and a metallic film. The dimensions are of order of µm. The composite resonant structure (Fig. 1.6.b) consists of a criss-cross structure and four square patches of tantalum nitride (TN), with thickness of 21 µm, a multi-layer dielectric plate (foam layer, VO2 layer, SiO2 layer), and a gold ground plane.

Thus, in function of different structure and excitation frequencies, the metamaterials can be with one negative parameter or with double negative parameters. In Fig. 1.7, a classification of the conventional materials and metamaterials is shown, where relative electric permittivityε_r and relative magnetic permeability $\mu_r$take positive or negative values.

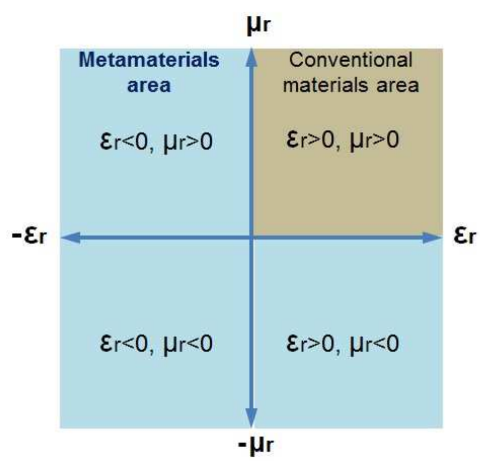


Fig. 1.7.Classification of electromagnetic materials into conventional materials and metamaterials with diagram of electric permittivity and magnetic permeability

Because their subwavelength, meta-atoms can be designed and customized in the desired ways, metamaterials have great capability and flexibility in controlling electromagnetic waves, being mainly used in electromagnetic devices to control the amplitude and phase of electromagnetic waves.

New applications are developed based on advanced materials, especially in the domains of electric and electronic devices, information storage systems, and medical devices.

An information recording/storage device is shown in Fig. 1.8.in which new magnetic shields and magnetization medium are used. For read current, the sensors based on Giant Magnetic Resonance (GMR) effect are used.

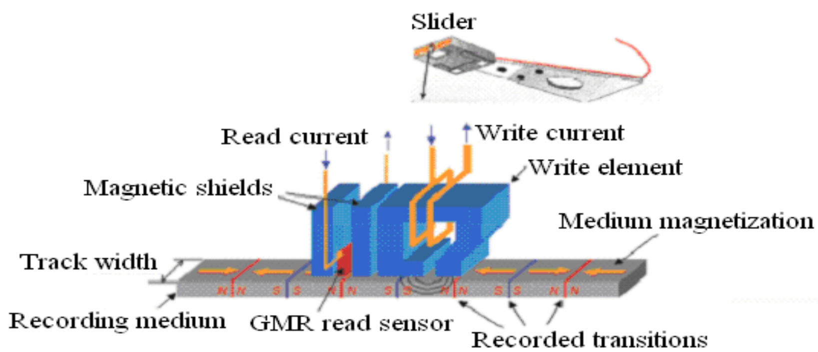
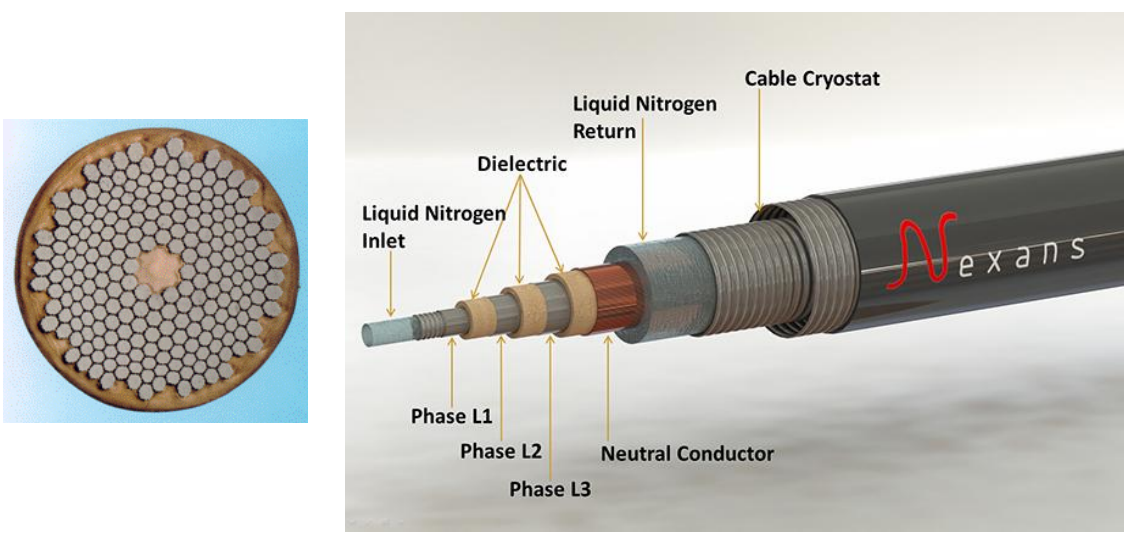


Fig. 1.8. Using advanced materials in information recording/storage device

Superconducting cables are based on property of superconducting effect in some materials. In Fig. 1.9.a, the structure of a superconductive cable is shown. The cable houses 36 strands of superconducting wires, each strand being of 0.825 mm in diameter. Each strand houses 6300 superconducting filaments of niobium-titanium (NbTi).



a) b)

Fig.1.9. Structure (a) and component parts (b) of a superconductive cable

The 15 cm diameter cable (Fig. 1.9.b), developed by Nexans, includes an inner and outer channel through which liquid nitrogen flows providing efficient cooling. The structure is insulated from its outer shell by a layer of vacuum to prevent thermal energy transfer to the surrounding environment.

1.2. Objectives of Electrical Materials Course

Section Summary: *In this section, an overview of the objectives of* ***Electrical Materials Course*** *is done. The role of professional organizations in electrical engineering is also pointed.*

The main objectives of the *Electrical Materials Course*are:

- Knowing and describing the phenomena which take place in materiales used in electrical and electronic engineering;
- Analysing and optimal choosing the materials for electrical and electronic systems;
- Knowing and applying the measurement techniques of the material parameters under different stresses for an optimal choice and design of electrical and electronic systems.

By completing this course, the students will acquire new competences:

- **Cognition competences**: the students can explain and justify the change of material properties in function of the nature of material and of the intensity of different stresses;
- **Practical skills**: at the end of course the students will know to distinguish the different classes of materials and to make the measurements for parameters determination;
- **Communication and relational skills -** at the end of course the students will know to use correctly the specific terms of the subject, they will have a team work capacity and a human, technical attitude characterized by creativity and innovation, with respect and interest for sustainable development in electrical engineering.

These skills can be acquired by reading and learning this text, participating in the practical applications in the laboratory hours and by preparing the appropriate homework.

In the formation of the competences and skills corresponding to an engineer in electrical engineering, a significant importance is implication in activities of professional organizations in the field, especially AGIR and IEEE.

AGIR – *AsociatiaGeneralaaInginerilor din Romania* – is a professional association which brings together engineers from different professions in Romania (www.agir.ro). AGIR disseminates the activity of engineers through various publications, organizes conferences, seminars and debates on current issues, and encourages the activity of young people through projects and contests.

www.agir.ro

IEEE - *Institute of Electrical and Electronics Engineers* - is the world largest professional association which brings together engineers from the domain of electrical and electronic engineering (www.ieee.ro). Under the motto "Advanced Technology for Humanity", IEEE and its members inspire a global community through highly-cited publications, conferences, technology standards, and professional and educational activities.

www.ieee.org

A component of IEEE is IEEE Romania, founded in July 1990. All the activities of the IEEE Romania Members Association are carried out in the spirit of IEEE regulations, respectively for all IEEE members in Romania, part of the formal group "IEEE Romania Section" (https://romania.ieeer8.org/).

1.3. Material Structure

Section Summary: *In this section, a general view on the atomic structure and bonding forces which are basis for the properties of materials in crystalline and amorphous states.*

Atoms, having dimensions of 10^{-10} m, and masses from $10^{-27}$to 10^{-30} kg, are basic units of the materials. Their main components, described in Fig. 1.10, are electrons and nucleus, consisting of protons and neutrons.

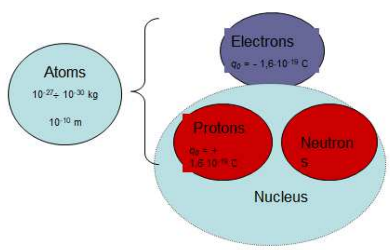


Fig. 1.10. Main components of the atom

From the electric point of view, electrons and protons are electric particles, having the charge of q_{oe}= 1.62 x 10^{-19} C, of negative character for electrons and positive character for protons.
In the case of a neutral atom, the number of electrons is equal to the number of protons and thus the total charge of the atom is zero.
The materials are formed by combining of different atoms which are kept in equilibrium by interaction forces – bonding forces.
Depending on the nature of the binding forces – metallic, ionic, covalent, hydrogen and van der Waals – the atoms are assembled in different structures, such as: metallic crystals, amorphous materials, polymers, liquid crystals, etc.
Temperature and pressure are the main factors which determine different aggregation states of materials:

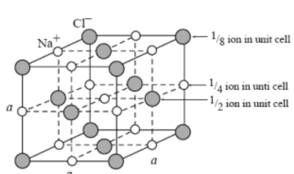
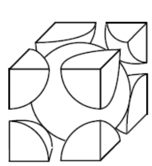
- **Gaseous state**, characterized by weak interactions; the bodies have no form and no proper volume;
- **Liquid state,** in which theinteractions forces are higher; the bodies have own volume, no proper form.
- **Solid state**, characterized by strong interaction forces; the bodies have proper volume and form.

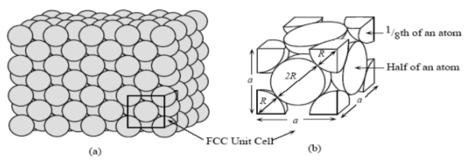
The solid state of materials is characterized by local or at distance order, corresponding for:

- crystalline structure – it is characterized by the order at long distances*:* unit cell, through translation, generates the whole crystal;
- *amorphous* or *glassy* structure – it corresponds to a local order, widely used in technique.

Crystals are a wade category of materials used in electrical engineering, as conductor, semiconductors and dielectrics.
In Table 1.1 some structures, characteristics and example of crystals applied in electrical engineering are given.

Table 1.1.Structures and characteristics of crystals used in electrical engineering

Crystalline Structure	**Number N of atoms per elementary cell (EC)**	**Examples of crystals**
Cubic Simple (CS) 	$N_{\mathrm{EC}} = \frac{0}{1}+\frac{0}{2}+\frac{8}{8} = 1$ atom /EC	NaCl, AgCl, LiF, MgO, CaO
Body/Volume Centered Cubic (BCC) 	$N_{\mathrm{EC}} = \frac{1}{1}+\frac{0}{2}+\frac{8}{8} = 2$ atoms/EC	Alkali metals (Li, Na, K, Rb), Cr, Mo, W, Mn, α-Fe (< 912 C), β-Ti (> 882^0).

Face Centered Cubic (FCC) 	$N_{\text{EC}} = \frac{0}{1} + \frac{6}{2} + \frac{8}{8}$ $= 4\ \text{atoms/EC}$	Ag, Al, Au, Cu, Ni, Pd, Pt, γ-Fe (> 912 °C)

The compactness of the elemental cell of a crystal is essential characteristic of crystalline structure and is characterized by N_{EC} which represents the number of atoms which belong to an elemental cell (EC) of the crystal. The relationship for calculation of N_{EC} is:

$$N_{\text{EC}} = \frac{Ni}{1} + \frac{Nf}{2} + \frac{Nn}{8} = 1\ \text{atom/EC} \qquad (1.1)$$

where

Ni is number of atoms located inside the elementary cell,
Nf is the number of atoms located on the faces of the elementary cell,
Nn is the number of atoms located in the nodes of the elementary cell.

Amorphous materials are characterized by alocal order of molecules/atoms.
Polymeric materials belong to the category of amorphous materials. Example of these structures are shown in Fig. 1.11.

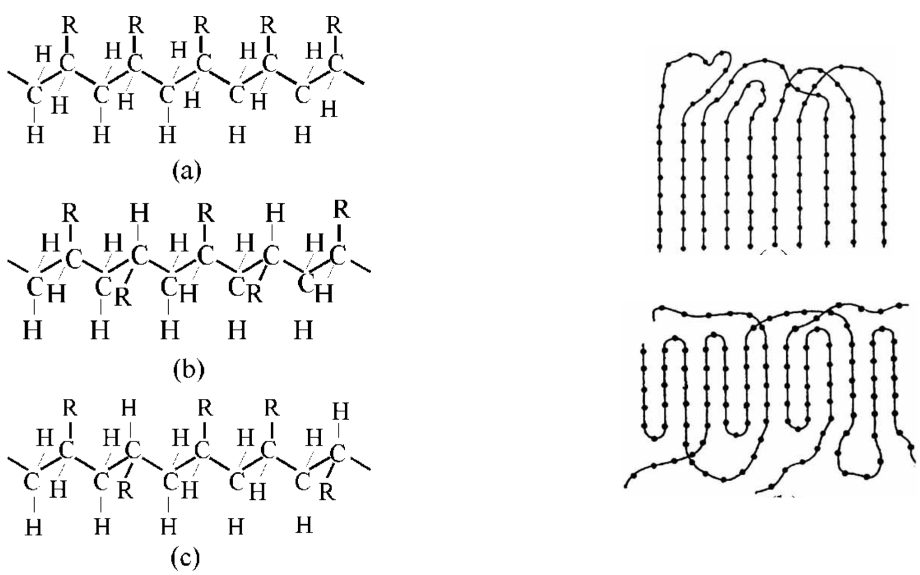


Fig. 1.11. Polymeric structures

A complex structure is found in the case of the quartz crystal (SiO_2). In Fig. 1.12 different structures of SiO_2 are shown.

SiO2 tetrahedrons with -O-bonds	SiO2 crystalline structure	Quartz glass of SiO_2

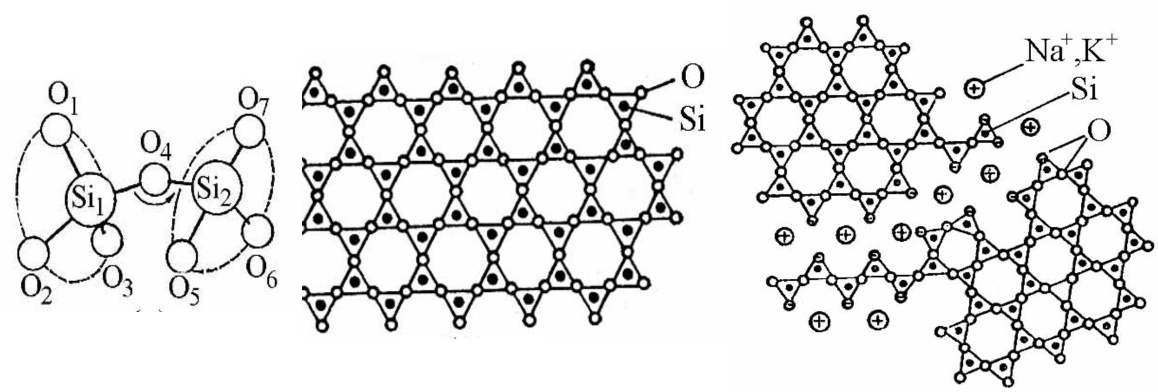


Fig. 1.12. Structures of SiO_2

1.4. Material Properties, Parameters and Classification

Section Summary: *In this section, the meaning of concepts of substance, material, material properties and material parameters are given. The criteria for classifying materials are discussed and examples with various criteria are given.*

In describing the properties of electrical materials, it is useful to know the meaning of some concepts:

Substance is an object/entity characterized through the homogeneity of the composition and of their constituent structure.

Material is anobject/entity of different nature and structure (example, a layered material, consisting of paper and plastic sheets

Material property is a common characteristic for the specific class of materials that characterize the response of material to the external stresses (examples: electric conductivity, thermal conductivity, thermal expansion, friction intensity between two materials etc.)

Material parameter is a physical quantity (scalar, vector, tensor) associated with a material property and which characterizes the state of the material under the action of the external stresses (electric field, temperature, radiation, etc. (example, σ- electric conductivity, ρ-electric resistivity, λ-thermal conductivity, μ - magnetic permeaability, ε - electric permittivity, etc.).

It can mention that a *Guide to the terms used in naming physical quantities* (Annex A) is proposed in the International Standard IS0 310:1992(E), Quantities and units.

It is useful to mention that many classification criteria of materialsare used.

- According to the material composition, there are:

- Inorganic materials,
- Organic materials.

-According to the character of the periodical properties of the chemical elements, there are:

- Metals,

- Metalloids,
- Non-metals.

-According to their state of aggregation and structure:

- Gaseous materials,
- Liquid materials,
- Solid materials, which can be:
    - Crystalline materials,
    - Amorphous materials,
    - Mezomorphous materials.

-According to electric conductivity σ or resistivity ρ:

- Electrical conductive materials, in which electrical currents are of order of (A – kA),
- Semiconductive materials, with currents of order of (μA – mA),
- Electroinsulating materials, with currents of order of (pA – nA).

-According to their applications:

- Materials for the electrical and electronic industry;
- Materials for civil engineering
- Materials for automotive industry
- Materials for food industry.

Short Test for Lesson 1

1. Make a classification of chemical bonding forces, in correlation with the aggregation states.
2. What is the difference between crystalline and amorphous structure?
3. What do you know about smart materials?
4. Define and give examples for:
    - a substance,
    - a material,
    - a material property,
    - a material parameter.
5. Give the general expression for number of atoms/ unit cell calculus.
6. Calculate the number of atoms per unit cell for the body/volume centered cubic (BCC) and for the face centered cubic (FCC) crystal lattice structures.
7. Make a classification of materials aaccording to electric conductivity.

2. Electromagnetic Theory and Material Laws

Contents

2.1. Macroscopic and Microscopic Theories of Electromagnetism

Section Summary: *In this section, a connection between microscopic and macroscopic theory of electromagnetics is done. The macroscopic, phenomenological and classical character of macroscopic theory is explained. The ways for obtaining the macroscopic quantities through averaging the microscopic quantities are also shown.*

Macroscopic theory of electromagnetism is a classical and phenomenological theory.

- **Phenomenological character** is given by the fact th**at** the means of introducing the physical quantities and the laws relating to these quantities, starting from the analysis and synthesis of the experimental data.
- **Classical character** is given by the fact that in this theory the classical mechanical laws are considered.
- **Macroscopic character** is given by the fact that it is admitted a continuous model of the substance and field. Bodies are continuous media characterized by a series of material parameters, as: electrical resistivity ρ, electrical permittivity ε, magnetic permeability μ.

Connection with Microscopic theory of electromagnetism can be explained using the sketch in Fig. 2.1.

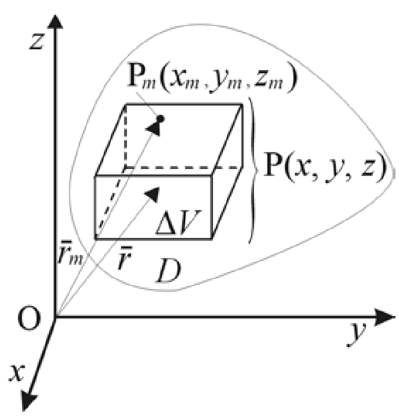


Fig. 2.1. Explanation for the connection between macroscopic and microscopic space

A point P_m with the microscopic coordinates (x_m, y_m, z_m) in the system of coordinates XYZ has the position described by the position vector $\boldsymbol{r}_m$.

A point P with the macroscopic coordinate (x, y, z) in the same system of coordinates XYZ, will include all points P_m from the volume ΔV, considered infinitely small physically.
To define the position vector $\boldsymbol{r}$of point P, it should integrate on the volume ΔV all position vectors $\boldsymbol{r}_m$.
Thus, through averaging the physical quantities from microscopic theory is possible to obtain the physical quantities in macroscopic theory. However, the averaging should be done on the domain of space ΔV, in the interval of time Δt, considered also infinitely small physically.

- **Macroscopic spatial averaging** can be obtained as:

$$\{ f_{micro} \}_{avg.s} = f(r_{macro}, t_{micro}) = \frac{1}{\Delta V} \cdot \int_{\Delta V} f_{micro.}(r_{micro}, t_{micro}) \cdot dV_{micro} \tag{2.1}$$

- **Macroscopic temporal averaging**

$$\{ f_{micro} \}_{avg\,t} = f(r_{micro}, t_{macro}) = \frac{1}{\Delta t} \cdot \int_{\Delta t} f_{micro}(r_{micro}, t_{micro}) \cdot dt_{micro} \tag{2.2}$$

- **Macroscopic spatial and temporal averaging**

$$\{ f_{micro} \}_{avg\,s,t} = f(r_{macro}, t_{macro}) = \frac{1}{\Delta V \cdot \Delta t} \cdot \iint_{\Delta V \Delta t} f_{micro}(r_{micro}, t_{micro}) \cdot \Delta V_{micro} \cdot \Delta t_{micro} \tag{2.3}$$

Some examples regarding the connection between microscopic and macroscopic quantities are given in Table 2.1.

Table 2.1. Connection between microscopic and macroscopic electric quantities

Electrical quantity	**Relation of averaging**	**Unit of measurement**
Electric charge	$q_{macro} = q = \{q_{micro}\}_{avg\ s,t} = N \cdot q_0$	$[q]_{SI} = \mathrm{C}$
Current intensity	$I_{macro} = I = \{I_{micro}\}_{avg\ s,t} = \frac{N \cdot q_0}{\Delta t}$	$[\mathrm{I}]_{SI} = \mathrm{A}$
Current intensity	$J_{macro} = J = \{J_{micro}\}_{avg\ s,t} = \left\{\frac{I_{micro}}{\Delta S_{micro}}\right\}_{avg\ s,t} = = \left\{\frac{1}{\Delta S_{micro}} \cdot \frac{N \cdot q_0}{\Delta t_{micro}}\right\}_{avg\ s,t} = \frac{I}{S}$	$[\mathrm{I}]_{SI} = \mathrm{A/m^2}$
Electric moment	$p_{macro} = p = \{p_{micro}\}_{avg\ s,t} = \{q_{micro} \cdot h_{micro}\}_{avg\ s,t} = q \cdot h$	$[\mathrm{p}]_{SI} = \mathrm{C \cdot m}$
Electric polarization	$P_{macro} = P = \{P_{micro}\}_{avg\ s,t} = \left\{\frac{p_{micro}}{\Delta V_{micro}}\right\}_{avg\ s,t} = \frac{N \cdot q \cdot h}{\Delta V}$	$[\mathrm{P}]_{SI} = \frac{\mathrm{C}}{\mathrm{m^2}}$

2.2. Material Laws in Science

Section Summary: *In this section, a general material law statement is given, and examples of material laws in Mechanics, Thermodynamics and Electrotechnique, with material parameters are enumerated.*

The general statement for a material law is of the form:
„Whenever a material is submitted to a stress of a certain nature (examples: mechanical and electrical forces, thermal stresses, radiations etc.), which represent THE CAUSE of that particular phenomenon, there will appear within the material an EFFECT which depends on the nature and the structure of the material, through a characteristic parameter of material."

In Table 2.2, examples of laws of materials, in Mechanics, Thermodynamics and Electrotechnics, with material parameters, expressed in mathematical forms, are given.

Table 2.2. Laws of materials and the characteristic parameters

Domain	**Material Law**	**Relation**	**Parameters of material**
Mechanics	Law of elasticity	$\frac{F}{S} = E_y \cdot \frac{\Delta l}{l_0}$	Young's modulus E_y [N/m^2]
	Law of friction	$F_f = \mu \cdot N$	Friction coefficient μ [-]
Thermo-dynamics	Law of thermal expansion	$l = l_0(1 + \alpha_l \Delta\theta)$	Linear dilation coefficient $\alpha_l [1/K]$
	Law of specific heat	$Q = m \cdot c \cdot \Delta\theta$	Specific heat c [j/(kg · K)]
	Law of thermal conductivity	$\frac{\Delta Q}{\Delta S \cdot \Delta t} = -\lambda \frac{\Delta T}{\Delta n}$	Thermal conductivity $\lambda\, [W/(m \cdot K)]$
Electro-technics	Law of electrical conduction	$\bar{J} = \sigma \bar{E}$	Electrical conductivity $\sigma\, [1/(\Omega \cdot m)]$ Electrical resistivity $\rho\, [\Omega \cdot m]$
	Law of temporary electric polarization	$\bar{P} = \varepsilon_0 \chi_e \bar{E}$	Electrical susceptibility χ_e Electrical permittivity $\varepsilon_r = 1 + \chi_e$
	Law of temporary magnetization	$\bar{M} = \chi_m \bar{H}$	Magnetic susceptibility χ_m Magneticpermeability $\mu_r = 1 + \chi_m$

	Law of electrolysis	$\frac{\Delta m}{\Delta t} = \frac{1}{F} \cdot \frac{A}{n} \cdot i$	The chemical equivalent *A/n* The electrochemical equivalent *A/Fn*
	Law of printed electric fields	$\bar{J} = \sigma(-\overline{\nabla V} + \bar{E})$, where, $\bar{E} = -S\Delta T$	Contact potential Seebeck coefficient (thermoelectric power) $S = -\frac{\Delta V}{\Delta T}$ [V/K]

2.3. Laws of Materials in Electrical Engineering

Section Summary: *In this section, the statement for the following three laws of materials applied in electrical engineering is given: law of electric conduction, law of electric polarization and law of magnetization. Their particularities are pointed in function of different types of materials – linear or non-linear, homogenous or non-homogenous.*

2.3.1. Electrical conduction law

- General statement in literal form: *Whenever a material is submitted to an electric field of intensity **E**, an electric current of density **J** will be established, with a specific value depending on the nature and structure of the material.*
- General statement in mathematical form:

$$\bar{J} = f(\bar{E}) \tag{2.4}$$

where***J*** is the density of the electric current of conduction and ***E*** is the intensity of the applied electric field.

The electric conduction law **for isotropic materials**, **linear and with no imprinted fields**, in literal form, is: *In linear and isotropic materials, with no imprinted electric fields, in every point and every moment, the current density of conduction **J** is proportional to the intensity of the applied electric field **E***. Thethe mathematical form is:

$$\bar{J} = \sigma\bar{E} \tag{2.5}$$

Or

$$\bar{E} = \rho\bar{J} \tag{2.6}$$

In these relationships, the material parameters involved are:

- electric conductivity σ, and
- electric resistivity ρ.

The relation between these parameters is:

$$\sigma = \frac{1}{\rho} \tag{2.7}$$

In International Unit System of Measurements, the corresponding units of measurements are: $[\sigma]_{SI}$= 1/Ωm, and $[\rho]_{SI}$ = Ωm(Ohm x meter).

For **anisotropic, linear and with no imprinted electric fields,** the law of electric conduction has the expression:

$$\bar{J} = \bar{\bar{\sigma}}\bar{E} \tag{2.8}$$

where the electric conductivity is a tensor parameter:

$$\bar{\bar{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \tag{2.9}$$

*In an anisotropic material, the density of the conduction electric current **J** is non homoparallel with the intensity of the applied electric field **E**.*

For this type of materials, 9 parameters of electric conductivity are needed in designing process of electric or electronic devices.

For **nonlinear materials,** the dependence between ***E*** and ***J*** is graphically described. An example of a nonlinear material is given in Fig. 2.2.

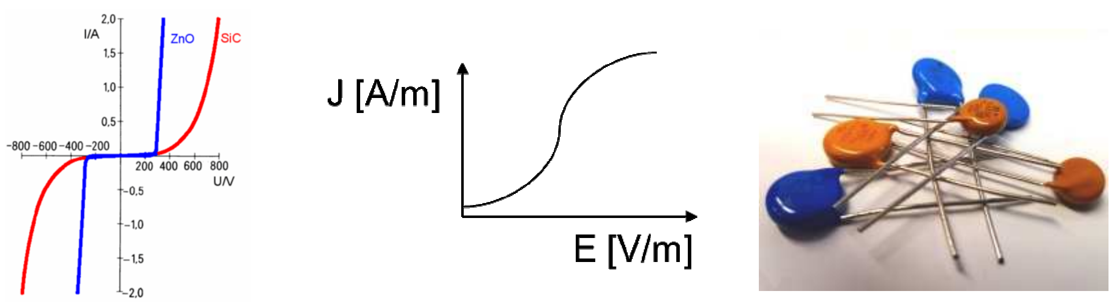


Fig. 2.2.Example of Volt-Ampere characteristics of varistors,as nonlinear materials

All materials could be considered for a specific domain of values of electric field intensity as linear materials. With this consideration, all materials can be arranged on a scale of conductivity, respectively resistivity (Fig. 2.3).

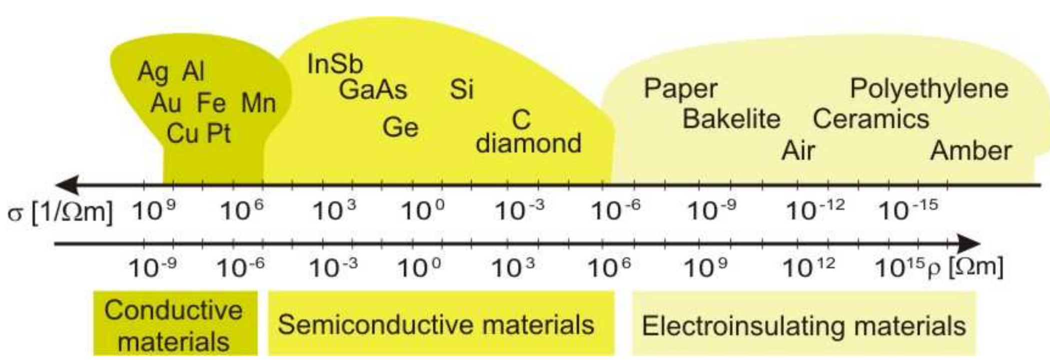


Fig. 2.3. The scale of electric conductivity and resistivity of materials

Taking into account the values of conductivity and resistivity, the materials can be classified in conductive, semiconductive and electroinsulating materials, with the values of resistivity specified in Table 2.3.

Table 2.3. Classification of materials with criterion of electric resistivity

Types of materials	**Values of resistivity**	**Electrical conduction mechanisms**
Conductive materials	$(10^{-8} - 10^{-6})$ Ω·m	Electronic conduction Ionic conduction
Semiconductive materials	$(10^{-5} – 10^{7})$ Ω·m	Electronic conduction
Electroinsulating materials	$(10^{8} - 10^{16})$ Ω·m	Electronic conduction Ionic conduction Molionic conduction

2.3.2. Electrical polarization law

- General statement in literal form: *Whenever a dielectric is submitted to an electric field of intensity **E**, there is established a state of polarization given by the quantity of the electrical polarization **P**$_t$ which depends on the nature and the structure of the material.*
- General statement in mathematical form:

$$\overline{P}_t = f(\overline{E}) \tag{2.10}$$

where**P**$_t$is the temporary electrical polarization and ***E*** is the intensity of the applied electric field.

The temporary electric polarization law for **isotropic, linear and with no imprinted field materials,** in literal form, is: *In linear and isotropic dielectrics, with no permanent polarization, in every point and in every moment, the quantity of temporary electrical polarization **P**$_t$ is directly proportional to the intensity of the applied electric field **E***. Thethe mathematical forms are:

$$\overline{P}_t = \varepsilon_0 \chi_e \overline{E} \tag{2.11}$$

$$\overline{D} = \varepsilon_0 \overline{E} + \overline{P}_t + \overline{P}_p = \varepsilon_0 (1 + \chi_e) \overline{E} = \varepsilon_0 \varepsilon_r \overline{E} = \varepsilon \overline{E} \tag{2.12}$$

In these relationships, the material parameters involved are:

- electric susceptivity χ_e,
- absolute electric permittivity ε, and
- relative electric permittivity ε_r.

The relations between these parameters are:

$$\varepsilon = \varepsilon_0 \varepsilon_r \tag{2.13}$$

$$\varepsilon_r = 1 + \chi_e \tag{2.14}$$

where the value of vacuum absolute electric permittivity is:

$$\varepsilon_0 = \frac{1}{4\pi \cdot 9 \cdot 10^9} \left(\frac{F}{m}\right) \tag{2.15}$$

It should mention that unit of measurement for absolute permittivity is F/m (Farad/meter), and relative permittivity and electric susceptivity are the dimensionless parameters.

For **anisotropic, linear and with no imprinted field dielectrics,** the law of electric polarization has the expression:

$$\overline{P}_t = \varepsilon_0 \overline{\overline{\chi_e}} \overline{E} \quad (2.16)$$

$$\overline{D} = \varepsilon(\overline{\overline{1}} + \overline{\overline{\chi_e}})\overline{E} = \varepsilon_0 \overline{\overline{\varepsilon_r}} \overline{E} = \overline{\overline{\varepsilon}} \overline{E} \quad (2.17)$$

where the electric susceptivity and electric permittivity are tensor parameters:

$$\overline{\overline{\chi_e}} = \begin{bmatrix} \chi_{exx} & \chi_{exy} & \chi_{exz} \\ \chi_{eyx} & \chi_{eyy} & \chi_{eyz} \\ \chi_{ezx} & \chi_{ezy} & \chi_{ezz} \end{bmatrix} \quad (2.18)$$

$$\overline{\overline{\varepsilon_r}} = 1 + \overline{\overline{\chi_e}} \quad (2.19)$$

In an anisotropic material, the temporary electrical polarization $\boldsymbol{P}_t$ *is non homoparallel to the intensity of the applied electric field* $\boldsymbol{E}$.

For this type of dielectrics, 9 parameters of electric permittivity are needed in designing process of electric or electronic devices.

Having in view the values of electric permittivity, the dielectric materials can be arranged on a scale of electric permittivity (Fig. 2.4).

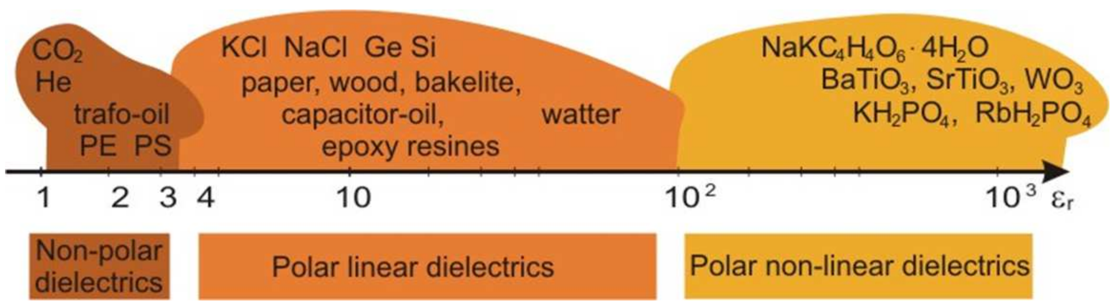


Fig. 2.4. The scale of electric permittivity of dielectrics

Thus, the dielectrics can be classified in non-polar, linear polar and non-linear polar dielectrics, with the values of permittivity specified in Table 2.4.

Table 2.4. Classification of materials with criterion of relative electric permittivity

Types of dielectrics	**Values of relative electric permittivity**	**Electric polarization mechanisms**
Non-polar linear dielectrics	1 to 3	Electronic polarization Ionic polarization
Polar linear dielectric	3 to tens	Electronic polarization Ionic polarization Orientation polarization
Non-linear dielectrics (ferroelectrics)	tens to thousands	Spontaneous polarization

2.3.3. Magnetization law

- General statement in literal form: *Whenever a material is submitted to a magnetic field of intensity **H**, in the material a state of magnetization is established, characterized by the quantity of the magnetization M_t, which depends on the nature, and structure of the material.*
- General statement in mathematical form:

$$\overline{M_t} = f(\overline{H}) \tag{2.20}$$

where M_t is the temporary magnetizationand ***H*** is the intensity of the applied magnetic field.

The temporary magnetization law **for isotropic, linear and with no imprinted field materials**, in literal form, is: *In linear and isotropic materials, with no imprinted electric fields, in every point and every moment, the quantity of temporary magnetization M_t is proportional to the intensity of the applied magnetic field **H***. Thethe mathematical forms are:

$$\overline{M_t} = \overline{\chi}_m \overline{H} \tag{2.21}$$

$$\overline{B} = \mu_0\left(\overline{H} + \overline{M_t} + \overline{M_p}\right) = \mu_0(1 + \chi_m)\overline{H} = \mu_0\mu_r\overline{H} = \mu\overline{H} \tag{2.22}$$

In these relationships, the material parameters involved are:

- magnetic susceptivity χ_m,
- absolute magnetic permeability μ, and
- relative magnetic permeability μ_r.

The relations between these parameters are:

$$\mu = \mu_0\mu_r \tag{2.23}$$

$$\mu_r = 1 + \chi_m \tag{2.24}$$

where the values of vacuum absolute magnetic permeability is:

$$\mu_0 = 4\pi \cdot 10^{-7}\frac{H}{m}$$

It should be mentioned that unit of measurement for absolute permeability is H/m (Henry/meter), and relative magnetic permeability and magnetic susceptivity are the dimensionless parameters.

For **anisotropic, linear and with no imprinted field materials,** the temporary magnetization law has the expression:

$$\overline{M_t} = \overline{\chi}_m \overline{H} \tag{2.25}$$

$$\overline{B} = \mu_0\left(\overline{H} + \overline{M_t} + \overline{M_p}\right) = \mu_0(\overline{H} + \overline{M_t}) = \mu_0(\overline{\overline{1}} + \overline{\overline{\chi_m}})\overline{H} = \mu_0\overline{\overline{\mu_r}}\overline{H} \tag{2.26}$$

where the magnetic susceptivity and the magnetic permeability are tensor parameters:

$$\overline{\overline{\chi_m}} = \begin{bmatrix} \chi_{mxx} & \chi_{mxy} & \chi_{mxz} \\ \chi_{myx} & \chi_{myy} & \chi_{myz} \\ \chi_{mzx} & \chi_{mzy} & \chi_{mzz} \end{bmatrix} \tag{2.27}$$

$$\overline{\overline{\mu_r}} = \overline{\overline{1}} + \overline{\overline{\chi_m}} \tag{2.28}$$

In an anisotropic material, the temporary magnetization $\boldsymbol{M}_t$ *is non homoparallel to the intensity of the applied magnetic field* $\boldsymbol{H}$.

For this type of materials, 9 parameters of magnetic permeability are needed in designing process of electric or electronic devices.

Having in view the values of magnetic permeability, the materials can be arranged on a scale of magnetic permeability (Fig. 2.5).

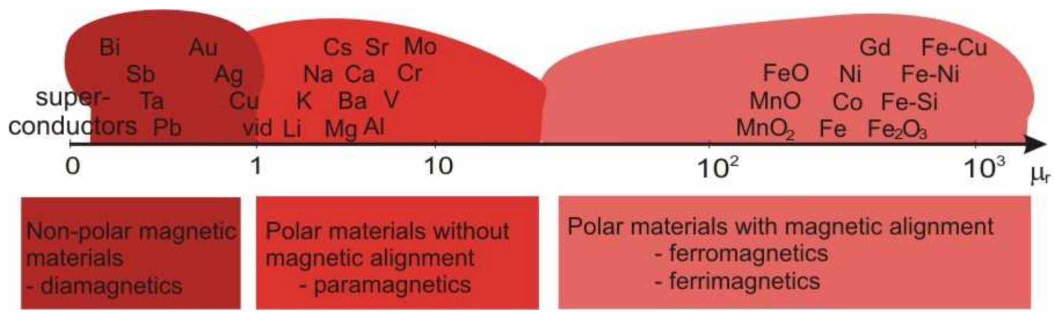


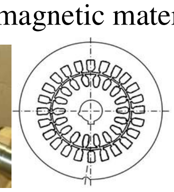
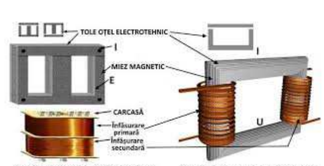
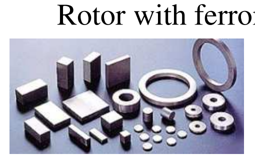
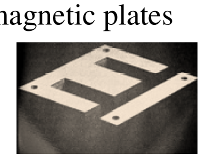
Fig. 2.5. The scale of relative magnetic permeability of materials

Thus, the materials can be classified in non-polar, linear polar and non-linear polar dielectrics, with the values of permittivity specified in Table 2.5while some applications of magnetic materials are shown in Table 2.6.

Table 2.5. Classification of materials with criterion of relative magnetic permeability

Types of materials	**Values of relative magnetic permeability**	**Magnetization mechanisms**
Linear and non-polar magnetic materials	$(1\text{-}10^{-6})$ to $(1\text{-}10^{-3})$	Diamagnetism
Linear polar magnetic materials	$(1 + 10^{-6})$ to $1+10^{-3})$	Paramagnetism
Non-linear magnetic materials (example, ferromagnetics)	tens to thousands	Feromagnetism Ferimagnetism Spontaneous magnetization

Table 2.6.Applications example for magnetic materials in electrical engineering.

Electrical equipment	Image	Component made of magnetic material
Electric motor		  Rotor with ferromagnetic plates
Electric transformer		  Ferromagnetic cores for electrical transformers

Short Test for Lesson 2

1. Express the law of electrical conduction (literary and mathematically) for isotropic and linear materials, specifying the physical quantities and measurement units involved.
2. Draw the material scales of electrical conductivity/ resistivity. Give at least one material example for every material class.
3. Express the law of temporary electric polarization (literary and mathematically) for isotropic and linear materials, specifying the physical quantities and measurement units involved.
4. Draw the material scale of electrical permittivity. Give at least one material example for every material class.
5. Express the law of temporary magnetization (literary and mathematically) for isotropic and linear materials, specifying the physical quantities and measurement units involved.
6. Draw the material scale of magnetic permeability. Give at least one material example for every material class.
7. Give an example of electrical or electronic application for magnetic materials.

3. Electric Conduction in Metals

Contents

3.1. Experimental Determination of Electrical Resistivity

Section Summary: *In this section, the procedure of experimental determination of electric resistivity of materials is described. The principle of resistivity measurement method with four terminals samples is also exposed.*

The experimental determination of electrical resistivity of materials is based on Ohm theorem: "*The voltage U at the terminals of a resistive circuit element is proportional to the intensity of the electric current I passing through that element and to the electrical resistance R of the considered element*". The mathematical expression of Ohm theorem is:

$$U = I \cdot R \tag{3.1}$$

Where, the units of measurement are: for voltage - V, for current intensity - A and, for resistance - Ω.

Using this relation, the expression of electric resistance is obtained:

$$R = \frac{U}{I} \tag{3.2}$$

For a sample of constant section *S*, it is known that the electrical resistance *R* depends on the cross-section *S* of the sample and on the length *L* along which the electric current passes:

$$R = \rho \frac{L}{S} \tag{3.3}$$

where the constant ρ is electrical resistivity of the sample.

For determining of the resistivity of a material, the 4-point probe method is used (Fig. 3.1), in which, two terminals (A and B) are current terminals where current intensity *I* is measured, and other two terminals (C, D) are voltage terminals where voltage *U* is measured.

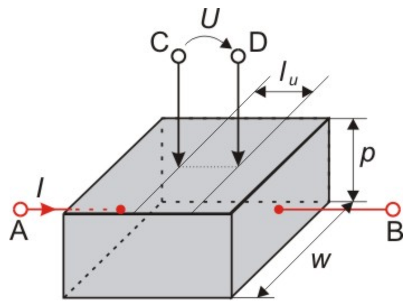


Fig. 3.1. Sample with four terminals for resistivity experimental determination

The electric resistivity is obtained with the relation:

$$\rho = \frac{S}{L}.R = \frac{S}{L}.\frac{U}{I} \ [\Omega m] \quad (3.4)$$

where, with Fig. 3.1, $S=w.p$, and $L=l_u$.

This method has sufficient reproducibility and precision, and it is applied not only for metallic probes and also for semiconductive and electroinsulating materials.

3.2. Mechanism of the Electrical Conduction in Metals

Section Summary: *In this section, the mechanism of electrical conduction in metals is described, taking in account specific hypotheses. It is shown that when an electric field is applied to a metal, a flow of electrons appears in the direction inverse relative to direction of electric field in conductor.*

3.2.1. Metals – as materials with high conductivity

Many of natural substances and materials have high electric conductivity, which correspond to very low resistivity. In conformity with classification of materials, the conductive materials are those with resistivity in the domain of 10^{-9} to 10^{-6} Ωm.

Representative for the class of electrically conductive materials are metals, such as:

- Alkaline metals, which are placed in the first group of the periodic system (Li, Na, K, Rb, Cs, Fr);
- Earth metals, placed in the second group of the periodic system (Be, Mg, Ca, Sr, Ba, and, also, Cu, Ag, Au, Zn, Cd, Hg)
- Transitional metals**,** placed from the group 3 to the group 13.

In Fig. 3.2, a partial view of periodical table in which high conductive metals are shown.

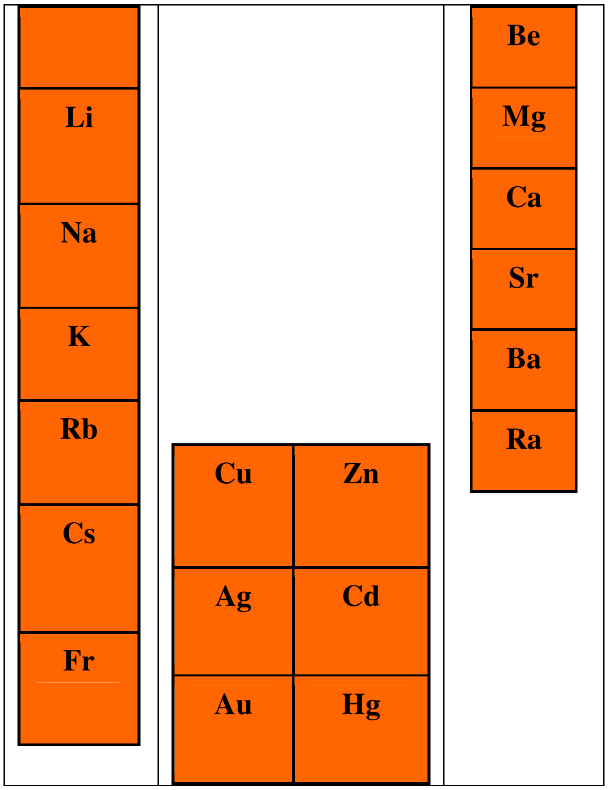


Fig. 3.2. Metals of high conductivity in first and second group of periodic table of elements

Before proceeding to the description of the mechanism of electrical conduction in metals, two concepts must be recapitulated:

- **Electrical conduction** is the process of passing the electric current through a material when an electric field is applied.
- **Electric conduction current** is defined as the ordered movement of free electric charges (electrons or/and ions) under the action of the electric field.

3.2.2. Classical theory of electric conduction

Classical theory of electric conduction was elaborated more than a hundred years ago. P. Drude formulates the classical microscopic theory of electric conduction in 1900, and in 1905, H. A. Lorentz completes this theory with “electronic theory of electric conduction”.

The Drude–Lorentz model of electric conduction is based onclassical theory of physics which treats both electrons and ions as small solid spheres.
Other hypotheses are considered, as:

- It is considered that the crystal lattice of metals consists of the metallic cations through which the electrons move freely;
- Model neglects any long-range interaction between electrons and ions;
- It assumes that the electrons do not interfere between them;
- The only possible interaction is the instantaneous collision between a free electron and an ion.

An image of the metal in bi-dimensional crystalline lattice, formed by electrons and positive ions, is shown in Fig. 3.3. The chaotic movement of electrons is due to the temperature.

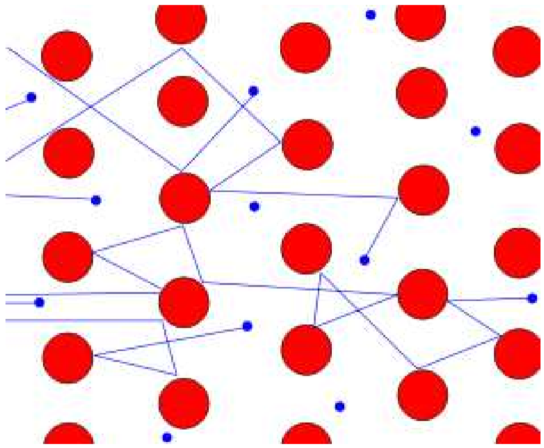


Fig. 3.3. Image of a metal in a bi-dimensional crystalline lattice

3.2.3. The Drude - Lorentz physical model of electric conduction

The Drude - Lorentz physical model of electric conduction consider the following:

- Free electrons have chaotic movement in all directions, under the action of the thermal agitation (T ≠ 0 K);
- Applying an electric field of intensity E (i.e. E≠ 0; T ≠ 0)**, o**ver the **movement of** the thermal agitation, an oriented motion of the electrons will be established. named the drift movement.

In Fig. 3.4, an explanatory sketch for physical model of electric conduction is shown.

- When the switch k is open (Fig. 3.4.a), the movement of electrons is only due to thermal excitation (Fig. 3.4.b);
- When the switch k is closed (Fig. 3.3.a), the movement of electrons is due to both the thermal excitation and the electric field (Fig. 3.4.c). Having in view the sense and direction of electric field intensity ***E***, the direction and sense of drift movement of electron is in opposite side.

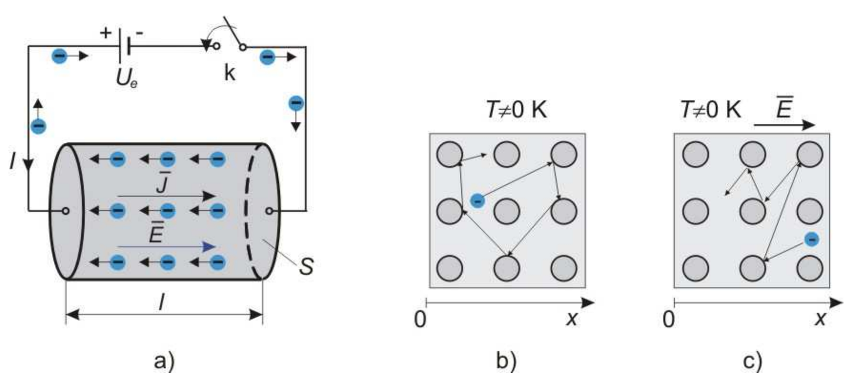


Fig. 3.4. Explanatory sketch for the physical model of electric conduction

The total velocity $\boldsymbol{v}$ of the electron will be:

$$\bar{v} = \overline{v_T} + \bar{v}_e \tag{3.5}$$

where

$\boldsymbol{v}_\mathbf{T}$ is the velocity due to the temperature T ,

$\boldsymbol{v}_\mathbf{e}$ is the velocity due to the electric field of intensity $\boldsymbol{E}$.

Drift velocity of the electrons is the average velocity of the electron movement in the direction of the axis Ox of the applied electric field with intensity $\boldsymbol{E}$.

$$\langle \bar{v} \rangle_{Ox} = \langle \overline{v_T} \rangle_{Ox} + \langle \bar{v}_e \rangle_{Ox} = \langle \bar{v}_e \rangle_{Ox} = v_D \tag{3.6}$$

Thus, when an electric field is applied, a flow of electrons appears in the direction inverse relative to direction of electric field in conductor.

In Fig. 3.3.a, direction of movement of electrons is to the terminal (+), and the conventional sense of the current is in direction to terminal (-) (this is because the conventional sense of current intensity I is defined considering the movement of positive charges).

3.3. Establishing the Expression of Electric Conductivity

Section Summary: *In this section, the method for establishing the expression of electric conductivity in metals, with the classical theory of electric conduction, is detailed. It is demonstrated that the electric conductivity depends on the volume concentration of the electrons in metals, the collision time and the electron mobility as intrinsic factors of influence.*

With the Drude-Lorentz model, the expression of the density of electrical current can be established. Starting from the definition of current density J , de expression is detailed to include the definition of current intensity I and the electric charge of the electrons N.q_0 :

$$J = \frac{I}{S} = \frac{\frac{\Delta q}{\Delta t}}{S} = \frac{N \cdot q_0}{\Delta t} \cdot \frac{1}{S} = \frac{N \cdot q_0}{\Delta t} \cdot \frac{1}{S} \cdot \frac{\Delta l}{\Delta l} \tag{3.7}$$

Having in view the volume of the conductor:

$$V = S \cdot \Delta l \tag{3.8}$$

and the volume concentration n_0 of the conduction electrons:

$$n_0 = \frac{N}{V} \tag{3.9}$$

the final expression of the density of electrical current will be:

$$\bar{J} = n_0 q_0 \bar{v}_d \tag{3.10}$$

The value of density of electric current in metal depends on the volume concentration n_0 of the electrons, electric charge of electron q_0 and on the drift velocity of electrons.

Thus, to know the current density in the metal, the drift velocity should be estimated!

For this, in the Fig. 3.5.a, the description of the interaction force $\boldsymbol{F}_e$ acting of an electron, when an electric field of intensity $\boldsymbol{E}$ is applied is done.
In conformity with the law of dynamics, the electric force will induce an acceleration to the electron:

$$\overline{F_e} = -q_0.\bar{E} \quad (3.11)$$

In conformity with law of dynamics, the electric force $\boldsymbol{F}_e$ induces an acceleration$a$ to the electron of mass m_0:

$$\overline{F_e} = m_0.\bar{a} \quad (3.12)$$

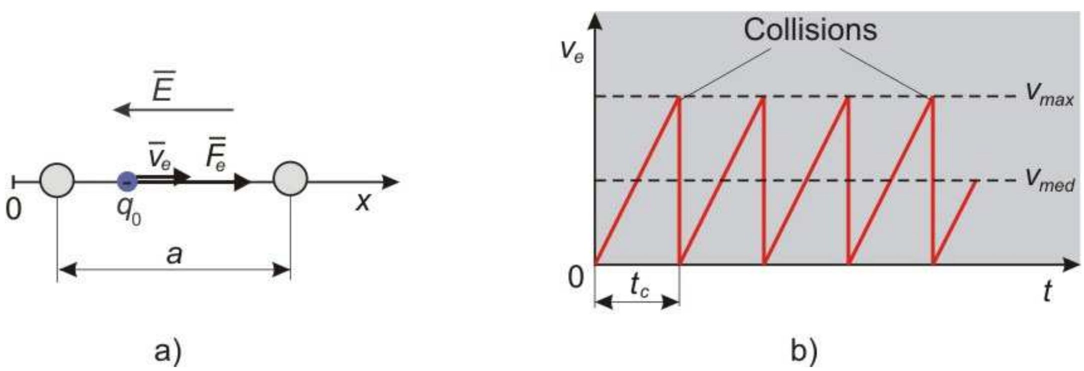


Fig. 3.5. Sketch for drift velocity obtaining

In conformity with the laws of classical mechanics, the acceleration, the velocity and the space of the conduction electron movement are:

$$\bar{a} = -\frac{q_0}{m_0} \cdot \bar{E} \quad (3.13)$$

$$v = at \quad (3.14)$$

$$s = \frac{at^2}{2} \quad (3.15)$$

This model considers that the electron movement is free (without breaks) in a period of time t_c, after which, a collision with the cristaline lattice is produce, and electron loses the accumulated kinetic energy, and drift velocity will be zero (Fig. 3.5.b).

The maximum velocity of the electron can be calculated in function of the average period between two collisionst_c:

$$v_{max} = a.t_c \quad (3.16)$$

The average velocity has the significance as drift velocity of the conduction electrons

$$v_{average} = v_d = \frac{0+v_{max}}{2} = \frac{a \cdot t_c}{2} \quad (3.17)$$

Thus, the final expression of drift velocity will be:

$$\overline{v_d} = -\frac{q_0 \cdot t_c}{2m_0}.\bar{E} \quad (3.18)$$

It can be added the expression of the mobility of the conduction electron μ_e as a parameter which describes the easiness of movement of the electron in the electric field:

$$\mu_e = \frac{v_d}{E} = \frac{q_0 \cdot t_c}{2m_0} \tag{3.19}$$

Electron mobility is a parameter which describe the average time between two collisions of conduction electron.

With the relationships obtained for drift speed and mobility, the electric current density becomes:

$$\bar{J} = \frac{q_0^2 n_0 \cdot t_c}{2m_0} . \bar{E} \tag{3.20}$$

and

$$\bar{J} = n_0 q_0 \mu_e . \bar{E} \tag{3.21}$$

These relations are compared with the law of electric conduction $(\boldsymbol{J} = \sigma . \boldsymbol{E})$ andthus, the expressions for electric conductivity result:

$$\sigma = \frac{J}{E} = \frac{q_0^2 n_0 \cdot t_c}{2m_0} \tag{3.22}$$

$$\sigma = q_0 n_0 \mu_0 \tag{3.23}$$

Electric conductivity in Drude-Lorentz model depends on the volume concentration of the electrons in metals, on the collision time and on the electron mobility.

In Table 3.1, the values of electron mobility μ_0 in different materials are presented.

Table 3.1. Electron mobility in different materials

Category of material	**Material**	**Electron mobility [$m^2/V \cdot s$]**
Metals	Al	0,00435
	Cu	0,00136
	Zn	0,00081
Semiconductors	Si	0,145
	Ga As	0,850
	CdS	0,034
Electrical insulators	SiO_2	≈ 0
	Si_3N_4	≈ 0

In Table 3.2, the values of electric conductivity and resistivity for some metals, obtained with microscopic theory of electric conduction, are presented.

Table 3.2.Electric conductivity and resistivity for some metals

Metal	**n_0 [m^{-3}]**	**t_c [s]**	**σ [1/Ωm]**	**ρ [Ωm]**
Li	$4.6 \cdot 10^{28}$	$0.9 \cdot 10^{-14}$	$0.12 \cdot 10^{8}$	$8.33 \cdot 10^{-8}$
Na	$2.5 \cdot 10^{28}$	$3.1 \cdot 10^{-14}$	$0.23 \cdot 10^{8}$	$4.34 \cdot 10^{-8}$

K	$1.3 \cdot 10^{28}$	$4.4 \cdot 10^{-14}$	$0.19 \cdot 10^{8}$	$5.26 \cdot 10^{-8}$
Cu	$8.5 \cdot 10^{28}$	$2.7 \cdot 10^{-14}$	$0.64 \cdot 10^{8}$	$1.56 \cdot 10^{-8}$
Ag	$5.8 \cdot 10^{28}$	$4.1 \cdot 10^{-14}$	$0.68 \cdot 10^{8}$	$1.47 \cdot 10^{-8}$

The obtained values are in good accordance with experimental values.

However, there are many factors that influence the real values of the conductivity, respectively, of the resistivity of metals.

3.4. Mathiessen Law and Factors which Influence the Electric Conductivity

Section Summary: *In this section, it is demonstrated that the electric conductivity depends, besides the intrinsic factors - the volume concentration of the electrons in metals, the collision time and the electron mobility, also by the extrinsic factors, as temperature, impurities, and environment.*

In a perfect crystal lattice, with all the atoms in position, the electrons would accelerate away until the field was cancelled in some way. In a real situation, however, the atoms are never exactly in the correct crystalline position. This can be caused by thermal vibrations, defects in the lattice (dislocations, vacancies and grain boundaries), impurity atoms, whether substitutional or interstitial. Thus, the conduction electrons are scattered. Each time they are scattered, their velocities are randomized. These scattering events constitute the electrical resistivity. The Mathiessen Law states that:

$$\rho_{total} = \rho_{defects} + \rho_{impurities} + \rho_{thermal} \tag{3.24}$$

The electric resistivity according to the electric scattering events can be expressed as:

$$\rho = \frac{2m_0}{nq_0^2 t_c} \tag{3.25}$$

wheret_c is the average rate of a scattering event, the drift velocity v_d(average velocity) of the conduction electrons is the electric field is $\frac{1}{2}\frac{q_0 E}{m_0} t_c$; for n conduction electrons per unit volume, the current density is: $J = nq_0 v_D = \frac{n}{2}\frac{q_0^2 E}{m_0} = \left(\frac{nq_0^2 E}{m_0}\right)$.

The Mathiessen Law describes the influence of total collision time on the electric resistivity of a metal. Thus, the total collision time for conduction electrons can be written as:

$$\frac{1}{t_{tot}} = \frac{1}{t_{tot}} + \frac{1}{t_{def}} + \frac{1}{t_T} \tag{3.26}$$

where the time components correspond with times of collision of the electrons: with crystalline lattice - t_T, with ionized or neutral impurities - t_{imp}, and with lattice defects - t_{def}.

In Fig. 3.6, the sketches of different types of collisions of the electrons are described.

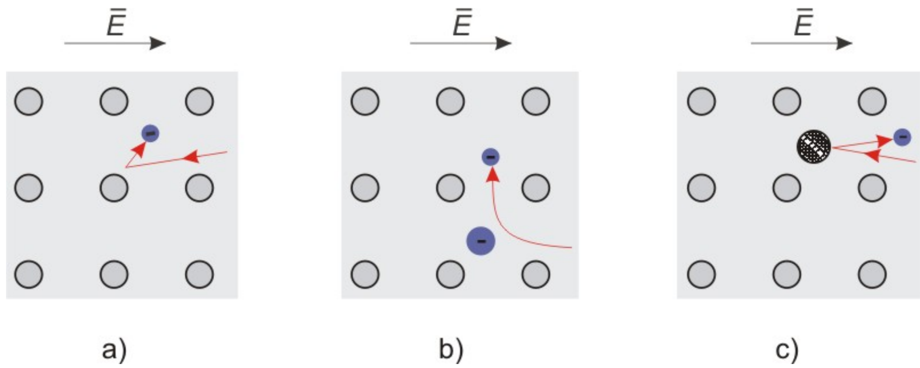


Fig. 3.6. Types of collisions in metallic lattice: a) with crystalline lattice; b) with ionized impurities; c) with neutral impurities.

The expression of the resistivity will be given in function of all type of collisions:

$$\rho = \frac{1}{\sigma} = \frac{m_0}{q_0^2 n_0} \cdot \frac{1}{t_{tot}} = \frac{m_0}{q_0^2 n_0} \cdot \left[\frac{1}{t_{def}} + \frac{1}{t_{def}} + \frac{1}{t_T}\right] \quad (3.27)$$

The resistivity of a metal has:

- a component due to impurities, ρ_{imp},
- a component due to lattice defects (vacancies, dislocations, grain limits, etc) which is the most significant from the three, in terms of contributionρ_{def},
- a component due to thermal vibrations of the atoms of the crystalline lattice, ρ_T.

Equation 3.24 relation is **Mathiessen law.**

Augustus Mathiessenpointed out the following: *when the temperature decreases, the metal conductivity usually improves or, in other words, the electrical resistivity usually decreases with a decrease of temperature:*

Next, the extrinsic factors that influence the resistivity of metals will be analyzed.

3.4.1. Dependence of resistivity on the concentration of components and their nature

For chemical compounds, in general, the resistivity is higher than the one corresponding to the component elements.

In Fig. 3.7, the dependence of the resistivity of metallic compounds on the concentration of components is described.

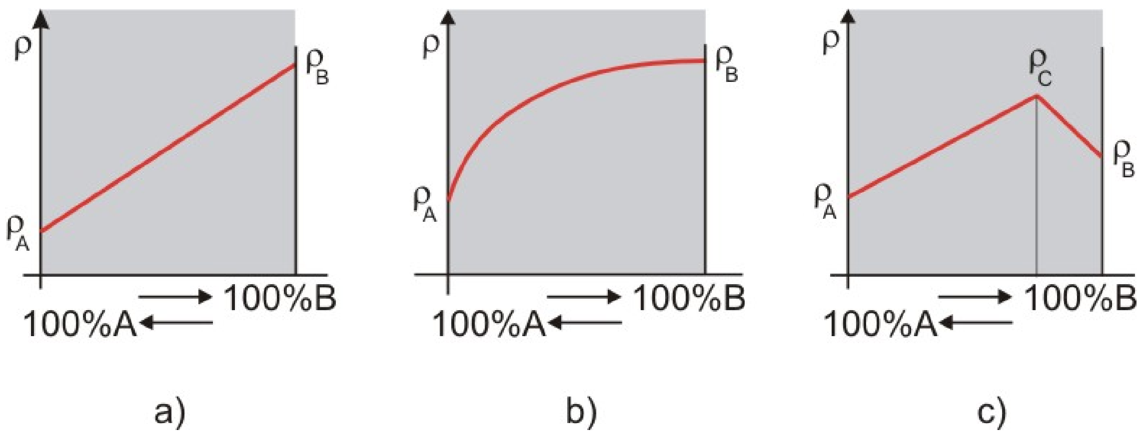


Fig. 3.7. Dependence of the metallic compound resistivity on the concentration of components: a) mechanical mixture with total insolubility in solid state; b) compound with total solubility; c) compound with alloy formation.

3.4.2. Dependence of resisticity on temperature

The resistivity of metals depends on temperature(Fig. 3.8).

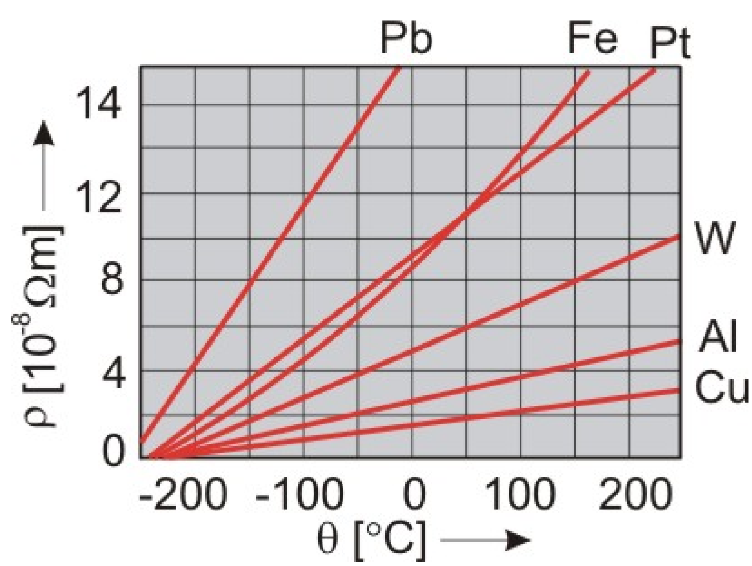


Fig. 3.8. The dependence of the resistivity of some metals with the temperature

The data in Fig. 3.8 shows that in metals, with the increase of the temperature, the resistivity increases. The advantage of using cooper and aluminum as conductive wires in electrical circuit is that the variation of their resistivity with increasing temperature is relative smaller, then other metals.

For metals, there is a domain of temperature for which the temperatute dependence of resistiviti can be considered linearly, and is expressed by relation:

$$\rho = \rho_{T_0} \cdot \left[1 + \alpha_\rho \cdot (T - T_0)\right] \tag{3.28}$$

Where α_ρ is temperature coefficient of resistivity, defined as (for a linear temperature dependency)

$$\alpha_\rho = \frac{1}{\rho_0} \cdot \frac{\rho_\theta - \rho_0}{T - T_0} \ [\mathrm{K}^{-1}] \tag{3.29}$$

$$\alpha_{\rho\_eff} = \frac{1}{\rho_\theta} \cdot \lim_{\Delta\theta \to 0} \frac{\Delta\rho}{\Delta\theta} = \frac{1}{\rho_\theta} \cdot \frac{d\rho}{d\theta} \tag{3.30}$$

which is measured in K^{-1}.

In Table 3.3, the temperature coefficients of resistivity for high conductive metals are presented.

Table 3.3. Temperature coefficients of resistivity for high conductive metals

Metal	**Electrical resistivity at 0°C** **ρ [10^{-8}Ω·m]**	**Temperature coefficient of resistivity, α_T [1/°C]**
Aluminium	2.7	0.0039
Copper	1.6	0.0039
Gold	2.3	0.0034
Iron	9	0.0045
Silver	1.47	0.0038

Note: for metals, resistivity variation in the large spectrum of the temperature is specific (Fig. 3.9).

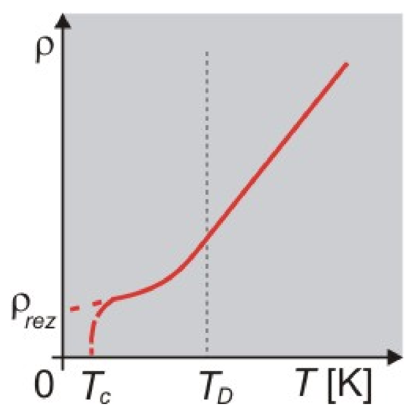


Fig. 3.9. Resistivity variation in the large spectrum of the temperature

In Fig. 3.9, two characteristic temperatures are pointed:

- T_D , called the Debye temperature, at which thelinear dependence begins,
- T_c ,called critical temperature, at which some metals pass in superconductive state. Other metals and alloys do not present this state, and they tend to have a residual resistivity for 0 K of temperature.

Debye temperature is different for different metals: T_{DCu} = 320 K; T_{DAl} = 428 K; T_{DFe} = 470 K.

Below the Debye temperature, the variation of resistivity with temperature is much higher, of the form:

$$\rho = const \cdot T^5 \tag{3.31}$$

This property is exploited in cryogenics, for the transport of electricity at low temperature, with conductors having much lower electrical resistance, which causes lower losses.

Regarding the residual resistivity at 0 K, for the majority of metals, the value of ρ_{rez} can be justified with the relation of Mathiessen: resistivity at 0 K depends on defects and impurities:

$$\rho_{rez} = \frac{m_0}{q_0^2 n} \cdot \left[\frac{1}{t_{imp}} + \frac{1}{t_{def}}\right] = \rho_{def} + \rho_{imp} \tag{3.32}$$

3.4.3. Dependence of resistivity on plastic deformation and pressure

Dependence of resistivity plastic on the pressure is expressed by the relation:

$$\rho_b = \rho_0 \cdot \left[1 + \alpha_p \cdot p\right] \tag{3.33}$$

where α_p is the variation coefficient of resistivity with pressure, with negative values:

$\alpha_{p\ metals} = -(10^{-11} - 10^{-10})\ m^2/N$

With increase of pressure, the resistivity of metals increases. This could be justified by increase of number of defects in metal when pressure applied on a conductor increases.
In the care of a mechanic stresses (traction, compression, torsion), the variation of resistivity is expressed by:

$$\rho_{\sigma m} = \rho_0 \cdot [1 + \alpha_{\sigma m} \cdot \sigma_m] \tag{3.34}$$

where:
$\rho_{\sigma m}$- is the resistivity in the presence of a mechanical tension σ_m,
ρ_o- is the resistivity in the absence of tension
$\alpha_{\sigma m}$- is the variation coefficient of resistivity with mechanical stress

3.4.4. Electric current frequency influence on metal resistivity

At high frequency of alternative currents, the resistivity of all conductive materials depends on the value of frequency *f*.
The electric field penetrates in the materiale only up to the so-called penetration depth δ, thus, the area S of conductive part of the device will be smaller, and, in conformity with resistance relation: R = ρL/S, teh total resistance increses.

The penetration depth is determined with relation:

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \tag{3.35}$$

Where f is frequency of the alternative current, μ is magnetic permeability, and σ is electric conductivity of material.

With this relation, the penetration depth for cooper is:

$$\delta = \frac{0.066}{\sqrt{f}} \qquad (3.36)$$

With increase of frequency f, the penetration depth δ decreases.

For a cooper wire with diameter D, two cases could be stated:

- **the case δ >>*D***, when the resistance in the alternative current has the same value as in the continuous current,
- **the case δ <<*D***, when the resistance in the alternative current has a higher value than the one corresponding to the continuous current.

3.4.5. Dimension effect of the resistivity

The dimension effect is observed to the conductors having the transverse dimension of order of microns: with the decreasing the dimension of conductor, the resistivity grows exponentially (Fig. 3.10).

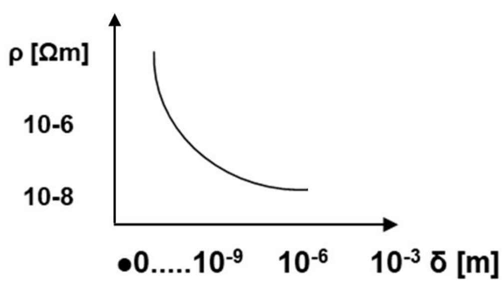


Fig. 3.10. Dimensional effect on the conductor resistivity

Dimension effect could be justified through the fact that with decreasing the dimension of conductor, the time t_{tot}between collision of electrons decreases, and resistivity increases, in conformity with the relation:

$$\rho = \frac{1}{\sigma} = \frac{m_0}{q_0^2 n} \cdot \frac{1}{t_{tot}} \qquad (3.37)$$

Dimension effect is applied in manufacturing thin film resistors.

Short Test for Lesson 3

1. Express the Ohm's law (literary and mathematically) and deduce the measurement unit for electric resistivity. What are the electric conduction and electric conduction current?
2. What are the hypotheses used in the classical model Drude-Lorentz of electrical conduction?
3. Establish the expression of density of conduction electrical current using the classical model of electrical conduction.
4. Establish the expression of drift velocity using the classical model of electrical conduction.
5. Establish the expression of electric conductivity using the classical model of electrical conduction.

4. Energy-Band Model in Crystals and Superconducting State

Contents

4.1. The Energy-Bands Theory in Crystals

Section Summary: *In this section, the energy-bands theory in crystals is detailed and aplied for electrical conduction, and for classification the crystals in conductor, semiconductor and insulating crystals. Based on energy-band theory, the characteristics and justification of superconducting state are done, and some applications of the superconductors are also described.*

The model of energy bands considers that, because of the order at a distance of the constituent atoms, in crystals the energy of the electrons groups in allowed bands and forbidden bands, their order depending on the electronic structure of the constituent elements.

As example, the electronic structure and energy levels for a single atom of Na (sodium)is considered (Fig. 4.1).

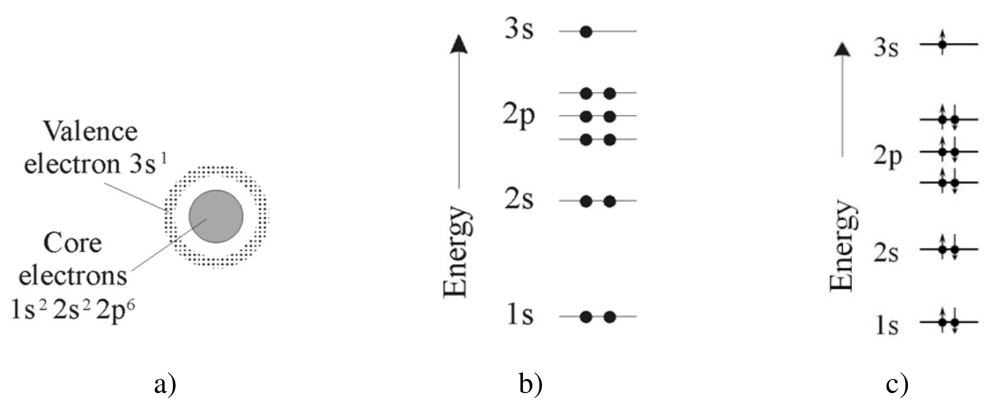


Fig. 4.1.Electronic structure (a) and energy levels (b) with spin electrons view (c) fora single atom of Na

For atomic number for sodium, Z_{Na}=11, the electronic structure is: $1s^2$ $2s^2$ $2p^6 3s^1$, which in energy unidimensional representation is shown in Fig. 4.1.b.

The cases when 2 atoms, 3 atoms and N atoms are at close distances, at distances similar to those in the crystal (of the order of angstroms), are shown in Fig. 4.2.

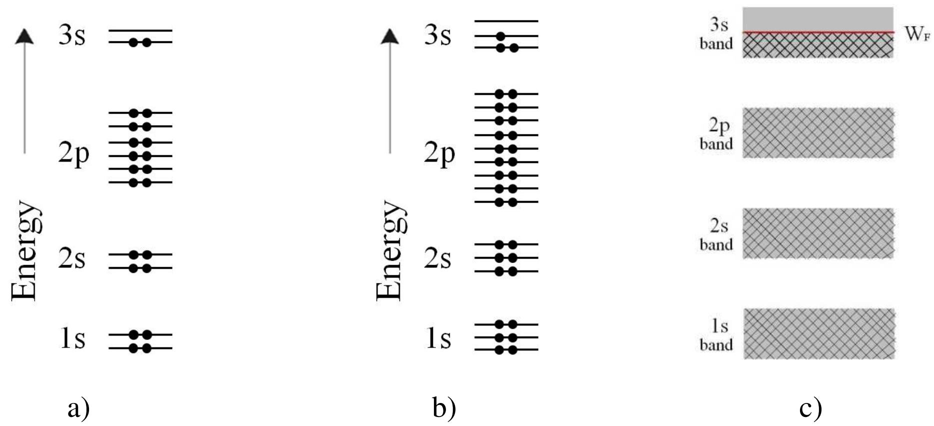


Fig. 4.2. Electronic structure in energy unidimensional representation: a) two close atoms; b) three close atoms; c) N close atoms

Fig. 4.2 shows that in Na crystal, the energy levels are grouped into wider regions, called energy bands, in which some are full occupied with electrons, other are partial occupied with electrons, and the rest are unoccupied energy-bands. The intervals between the allowed energy bands constitute the forbidden bands (impossible to be occupied by electrons).

In the case of atoms with many electrons, it is possible to overlap the immediately higher energy band unoccupied with electrons with the incomplete band occupied with electrons, thus forming a hybrid band incompletely occupied with electrons.

An important quantity in energy-band theory is Femi energy, which specified the last level possible to be occupied with electrons. In Fig. 4.2.c, for sodium crystal, the Fermi energy level is situated in the incomplete occupied energy band 3s, which is half occupied with electrons.

In Fig. 4.3, the formation of hybrid energy band in sodium, magnesium and aluminum crystals is described.

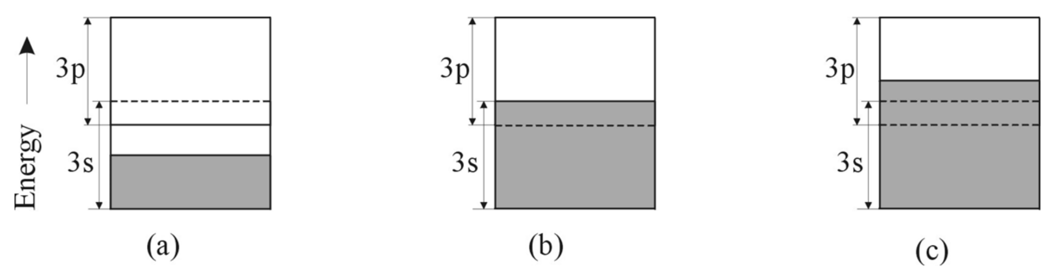


Fig. 4.3. Formation of incompletely occupied hybrid energy-bands in crystals: a) For sodium (Z_{Na} =11), where 3s band is half-filled since there is only one $3s^1$ electron; b) For magnesium (Z_{Mg} =12), where the 3s band is filled and overlaps the empty 3p band; c) For aluminium (N_{Al}=13), where the 3s band is filled and overlaps partially filled 3p band.

The advantage of the energy band model is that it can be used to justify electrical conduction processes in all types of crystals, not only in metallic conductors. A great success was also obtained in justifying the state of superconductivity.

4.2. Electrical Conduction in the Energy-Bands Model

Section Summary: *In this section, the energy-bands model is applied for justification the electric conduction in crystals. The classification of crystals in conductor, semiconductor and electroinsulating crystals are also presented.*

From the macroscopic point of view, the electrical conduction is a process of flow of free electrons in a crystal under action of electric forces established when an electric field is applied to crystal.

The big challenge is to justify why some crystals are highly conductive and others are not. What are the crystals in which some electrons can be considered as quasi-free electrons, which macroscopically can establish an electric current?

For giving the answer to these questions, it is important to take in account the results of quantum theory of energy-bands:

- The electrons from an energy band fully occupied with electrons do not contribute to macroscopic effect of production an electric current when an electric field is applied to crystal;
- Only the electrons in the incompletely occupied bands can contribute to the production of the electric current, since they can be "energized" on the free levels in the incompletely occupied band with electrons.

The distinction between different types of crystals is given by the difference in width of the Fermi forbidden band (ΔW_i), defined by the interval between the last fully occupied energy band - called valence energy band (VB) and the first incompletely occupied energy band - called conduction energy band (CB).

$$\Delta W_i = W_{CV} - W_{BV} \tag{4.1}$$

Thus, in the case of metallic crystals, the Fermi band has zero value (ΔW_i=0). In the case of semiconductors, the width of Fermi forbidden band ($\Delta W_i \neq 0$) is much more reduced than in the case of the insulating crystals, a fact that makes possible the thermal excitation of the electrons from BV to BC (Fig. 4.4).

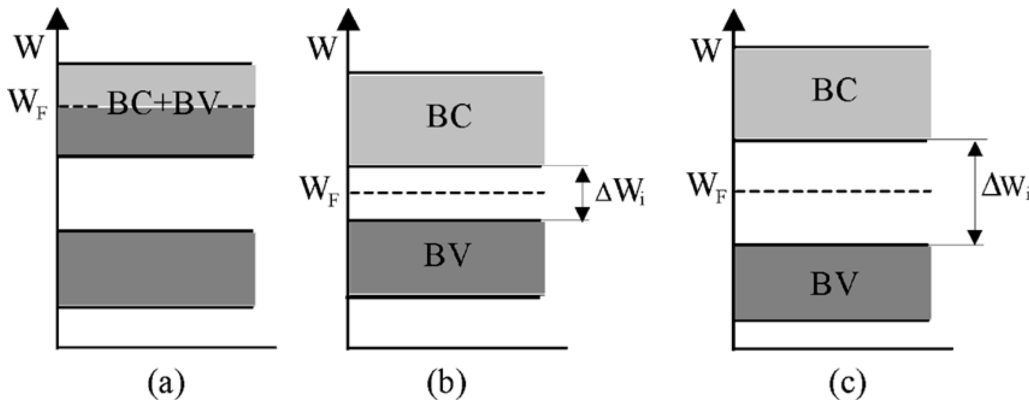


Fig. 4.4. Position of Fermi energy level W_F and width of Fermi forbidden band ΔW_i in crystals: a) case of metallic crystals; b) case of semiconductive crystals; c) case of electroinsulating crystals

In the theory of energy bands, the large differences between the values of electrical conductivity:

- in the case of conductive crystals: $10^8 \div 10^6$ [1/Ωm],
- in semiconductors: $10^5 \div 10^{-7}$ [1/Ωm],
- and in dielectrics: $10^{-8} \div 10^{-18}$ [1/Ωm])

as well as, the influence of some external factors (temperature, electric and magnetic fields, radiations, etc.) are explained on the basis of the different energy spectrum of the electrons in the crystal and the different ways of occupancy with electrons of the energy states in crystals.

A new classification of crystals with criterion of value of resistivity, takes in account the width of Fermi interval ΔW_i:

- for ΔW_i, the crystals are high conductive materials,
- for 0 eV < ΔWi < 3 eV, the crystals are semiconductive materials,
- for ,ΔWi > 3 eV, the crystals are electroinsulating materials.

4.3. Superconducting State – Characteristics, Justification and Applications

Section Summary: *In this section, based on energy-band theory, the characteristics and justification of superconducting state are done, and some applications of the superconductors are also described.*

Superconducting state is the state of a materials characterized by almost the total absence of electrical resistance when an electric field is applied and the current flows in the material.
Superconductivity is the property of some materials to present a very low electrical resistance to the passage of an electrical current.

<u>A short history</u>

- At 10th of July 1908, Kamerlingh Onnes was the first to liquefy helium, by utilizing a number of stages and the Hampson-Linke pre-cooling cycle based on the Joule-Thomson effect. In this way, he lowered the temperature under the boiling point of helium (-269°C = 4,2 K). By reducing the pressure on the liquid helium, he reached a temperature close to 1.5 K.

- In 1911, Kamerlingh Onnes discovered that the resistance of a mercury sample disappears abruptly at temperatures under 4.2 K (Fig. 4.5).

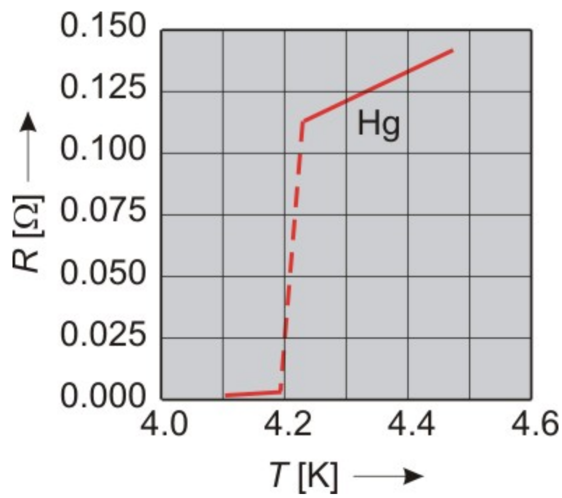


Fig. 4.5. Mercury resistance dependence on temperature

Kamerlingh Onnes reported that the mercury passed into a new state of matter, which, due to its extraordinary electric properties, can be called, superconductive state.

4.3.1. Characteristics of superconduction state

New research has highlighted specific characteristics of the superconducting state (SC).

- **There is a critical superconduction temperature *Tc*,** defined as temperature value in which transition to the superconductive state occurs.
  - A series of metals reach the superconductive state only at temperatures close to 0 K (for example, metals of high conductivity: Au, Ag, Cu, etc.).
  - Some elements reach the SC state at temperatures between 4-10 K, for example: MercuryHg (4.15 K); Vanadium V (5.2 K); LeadPb (7.2K); NiobiumNb (9.3 K).
  - Compounds of transitional metals (like V and Nb) – the superconductive state appears at high temperatures, e.g. TC: Nb_3 Sn (18 K), V_3Ga (15 K), $Nb_3Al_{0,3}Ge_{0,2}$(20.7 K).
  - Ferromagnetic metals (Fe, Ni, Co) do not have superconducting properties.
  - In semiconductive materials (Ge, Si, etc.), the SC state is established only in surface layers at very high pressures.
- **Some crystalline structures are favorable for reaching the superconducting state**.

  Examples: for Hg – rhombohedral structure, V, Pb, Nb – a body-centered cubic (BCC) structure, Nb_3Sn – A_3B type structure.

  Metallic compounds which exhibit the superconductive state at high temperature have a crystalline structure similar to β wolfram (type A_3B), as in Fig. 4.6.

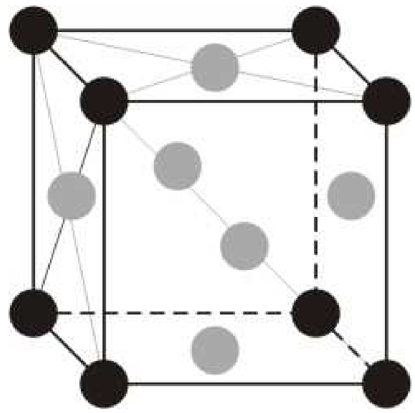


Fig. 4.6. The A3B crystalline structure: atoms of element A are located on the face of the cube and atoms of element B occupy the edges

- **Superconductors have specific thermo-magnetic properties**

  In the superconduction state, with the sudden decrease in electrical resistivity, the thermal conductivity and magnetic properties of superconductors also change.

  The critical magnetic field intensity H_c defines the limit of magnetic field beyond which the superconducting state is destroyed.

  There is a strong connection between critical magnetic field H_c and critical temperature T_c:

$$H_c = H_{c0} = \left[1 - \left(\frac{T_c}{T_{c0}}\right)^2\right] \quad (4.2)$$

  where H_{c0} is critical magnetic field intensity at zero temperature, and T_{c0} is critical temperature at zero critical magnetic field.

  In Fig. 4.7 the dependence $H_c = f(T_c)$ is shown. Only in the domain under the H_c- T_c curve, the material is in superconduction state.

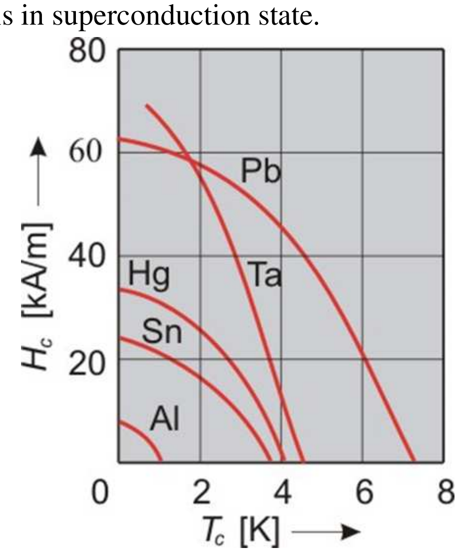


Fig. 4.7. Defining the superconducting state considering critical magnetic field H_c and critical temperature T_c

In Table 4.1, the values of critical temperatures and critical magnetic field are presented.

Table 4.1.a. Values of critical temperatures and critical magnetic fields for some superconductors

Superconductor	**V_3Ga**	**$Nb_3 Sn$**	**$Nb_3(Al_{0,3}Ge_{0,2})$**
T_{c0} [K]	15	18	20.7
H_{c0}, [kA/m]	$20 \cdot 10^3$	$18 \cdot 10^3$	$32 \cdot 10^3$
B_{c0} [T]	25	22	40

Table 4.1.b. Critical temperatures of some superconducting materials and year of applications

Materials	**Tc [K]**	**Remarks**	**Year**
Mercury	4.15	H.K. Onnes	1911
Sulfur-based organic superconductor	8	S.S.P. Parkin et al.	1983
Nb_3 Sn and Nb-Ti	9	Bell Labs	1961
V_3 Si	17.1	J.K. Hulm	1953
Nb_3 Ge	23.2	_	1973
La-Ba-Cu-O	40	Bednorz and Muller	1986
$YBa_2 Cu_3 O_7 - x^a$	92	Wu, Chu, and others	1987
$RBa_2 Cu_3 O_7 - x^a$	~92	(R = Gd, Dy, Ho, Er, Tm, Yb, Lu)	1988
$Bi_2 Sr_2 Ca_2 Cu_3 O_{10} + \delta$	113	Maeda et al.	1988
Tl2 Ca $Ba_2 Cu_2 O_{10} + \delta$	125	Hermann et al.	1988
Hg $Ba_2 Ca_3 Cu_3 O_8 + \delta$	134	R. Ott et al.	1995
Hg0.8 Tl0.2 Ba2 Ca2 Cu3 O8.33	138	Titova and Bryntse et al.	1998
La O Fe As + F	26	H. Hosono et al.	2008
MgB2	39	Nagamatsu et al	2001

Superconductors having a Tc above 77 K (boiling point of liquid nitrogen) are technologically interesting because they do not require liquid helium (boiling point 4 K) or liquid hydrogen (boiling point 20 K) for cooling.

Recently, a new class of superconductors, which is based on layers of iron and arsenic (among others) has been discovered. Examples are parent compounds consisting of LaOFeAs, BaFe2As2, FeSe, and iron phosphide. LaOFeAs is not superconducting, but becomes superconducting when some of the oxygen is replaced by up to 11% fluorine (Tc = 26 K). Replacing the lanthanum with cerium, samarium, neodymium and/or praseodymium leads to a Tc of about 52 K. Doped FeSe has a Tc of 8 K at normal pressure and a Tc of 27 K under high pressure.

- **Superconduction state can be destroyed by the electric currents**

  There is a critical value of the current density ***J***c over which the superconduction state is ruined.

  In Fig. 4.8, only area included in interior of the curves J_c=f(B) corresponds to the superconduction state of a superconductor material.

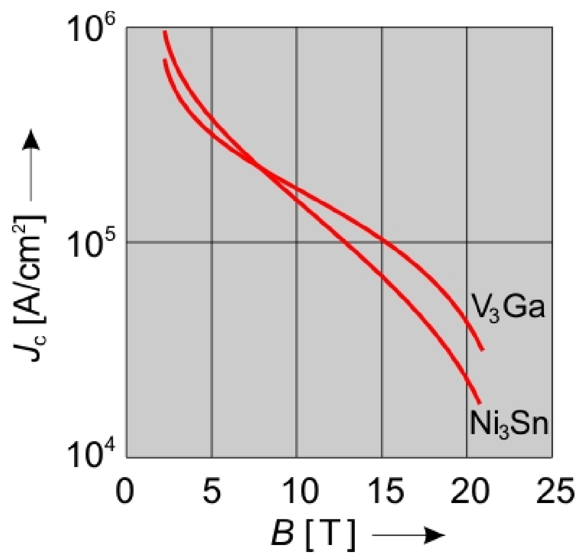


Fig. 4.8.Dependency of the critical density current J_c on the magnetic induction *B* of the magnetic field produced by electric current in superconductors of V_3Gs and Ni_3Sn

- **Meissner effect**

  The Meissner effect is the absence of any magnetic field inside massive superconductors.

When a superconductor is placed in a weak external magnetic field *H*, the field penetrates the superconductor for only a short distance *λ*, called theLondon penetration depth, after which it decays rapidly to zero. This is called theMeissner effect, and is a defining characteristic of superconductivity. For most superconductors, the London penetration depth is on the order of 100 nm.

The Meissner effect is sometimes confused with the kind ofdiamagnetismone would expect in a perfect electrical conductor: according to Lenz's law, when a changing magnetic field is applied to a conductor, it will induce an electrical current in the conductor that creates an opposing magnetic field. In a perfect conductor, an arbitrarily large current can be induced, and the resulting magnetic field exactly cancels the applied field.

The Meissner effect is distinct from this because a superconductor expels all magnetic fields, not just those that are changing. When the material is cooled below the critical temperature, we would observe the abrupt expulsion of the internal magnetic field, which we would not expect based on Lenz's law.

Thus, in superconductive state (when $H < Hc$and $T < Tc$), the magnetic field lines are projected in the exterior from the volume of the superconductor (Fig. 4.9).

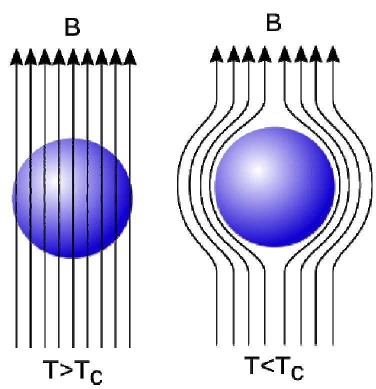


Fig. 4.9. Meissner effect in superconductors

Thus, for temperatures under the critical value T_c, superconductor is behaving as an **ideal diamagnetic material,** i.e. the material is characterized by a diamagnetic susceptibility (χ_m = - 1).

The Meissner effect breaks down when the applied magnetic field is too large. Superconductors can be divided into two classes according to how this breakdown occurs.

- In Type I superconductors, superconductivity is abruptly destroyed when the strength of the applied field rises above a critical value H_c.
- In Type II superconductors, raising the applied field past a critical value H_{c1} leads to a mixed state in which an increasing amount of magnetic flux penetrates the material, but there remains no resistance to the flow of electrical current as long as

the current is not too large. At a second critical field strength H_{c2}, superconductivity is destroyed.

The mixed state is actually caused by vortices in the electronic superfluid, sometimes calledfluxionsbecause the flux carried by these vortices is quantized. Most pureelementalsuperconductors, exceptniobium,technetium,vanadiumandcarbon nanotubes, are Type I, while almost all impure and compound superconductors are Type II.

Conclusion:All these characteristics show that **the superconductive state is a state different from the conductive state**, being characterized by a high degree of ordering and low entropy.

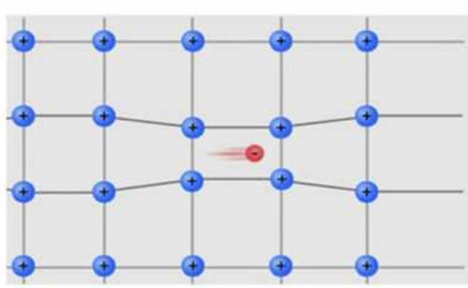
4.3.2. Justification of the superconductive state

Justification of the superconductive state is done with microscopic theory of superconductivity elaborated by Bardeen, Cooper, and Schrieffer in 1957. This theory proves that responsible for modifying the properties of the material in the superconductive state is the ensemble of the electrons in the crystal which pass into a different quantum state.

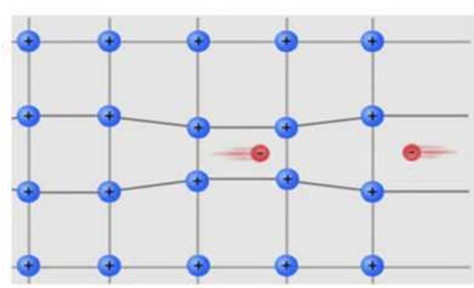
- Through the crystal lattice, an attractive type interaction between two electrons which form a specific connection, called **Cooper pair**. The Cooper pair appears only when the attraction energy between two electrons is greater than the repulsive Coulomb forces between them (Fig. 4.10).

  About Cooper Pair formation:
  http://hyperphysics.phy-astr.gsu.edu/hbase/Solids/coop.html

- The coherence length is defined as the distance at which the attraction force between the electrons of the Cooper pair interacts.



a)



b)

Fig. 4.10. Formation of Cooper pair of electrons in crystalline lattice: a) A moving electron slightly attracts the surrounding positive ions, causing a slight ripple towards its path; b) Another electron passing in the opposite direction is attracted to that displacement

- According to Quantum theory, the Cooper pair has the spin equal to zero**,** thus, is no subjected to the Fermi-Dirac statistical distribution (the maximum number of particles which are in a quantum state is two)but the statistical Bose-Einstein distribution (the number of particles in a given quantum state is unlimited) (Fig. 4.11).

- The energetic spectrum of normal conduction electrons and electrons grouped into cooper pairs is different: a condensation of the energy levels is obtained in the SC state.
- The Cooper pair gathers energy when an electric field is applied and the collective displacement of the Cooper pairs is responsible for producing the macroscopic electric current.

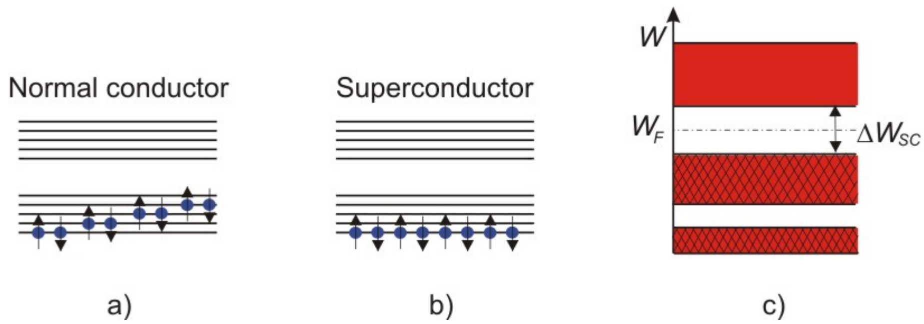


Fig. 4.11. Distribution of electrons in (a) normal conduction, (b) superconduction, and (c) structure of the energy bands for a superconduction crystal at a temperature $T<T_c$

Electron pairs behave differently from unpaired electrons (fermions) (fermioni), which behave according to Pauli's exclusion principle. Electron pairs act as **bosons (elementary particles which have an integer spin quantum number),** and can condense on the same energy levels (they act according to Bose-Einstein statistic).

At sufficiently low temperatures, electrons near the Fermi surface become unstable against the formation of Cooper pairs. Cooper showed such binding will occur in the presence of an attractive potential, no matter how weak.

4.3.3. Applications of the superconductive state

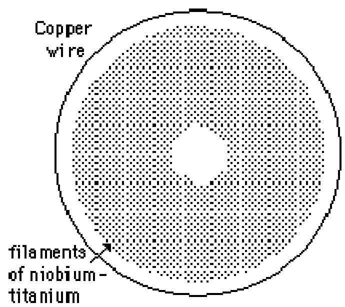
Many are applications of superconduction state, as:

- High intensity superconducting electromagnets,
- High performance electrical machines and transformers,
- Electric energy transmission cables,
- Energy storage devices (e.g. superconductive magnetic energy storage- SMES)
- Josephson junctions for SQUID (superconducting quantum interference devices)
- Large scale application in constructing MRI (magnetic resonance imaging equipment or other medical apparatus),
- Maglev trains based on the magnetic levitation phenomena.

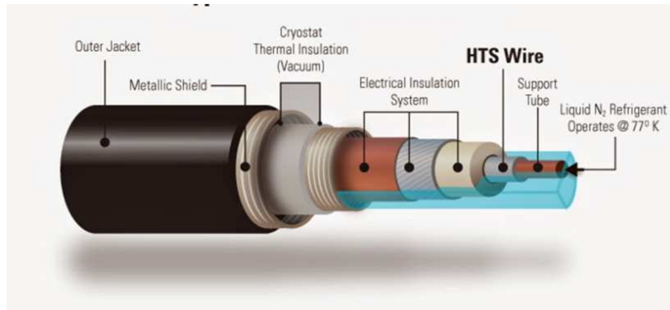
Some examples are given here.

A. Superconducting Niobium-Titanium Cables

As superconductive material is used Nb-Ti, as a type II superconductor, with critical temperature of 10 K, and a critical magnetic field of 5 Tesla. The filaments of Ni-Tiare disposed in a matrix medium of cooper, as in Fig. 4.12.



a)



b)

Fig. 4.12. Superconductive cable for power transmission: a) detail of superconductive wire structure; b) cable structure

Superconducting wire is almost always of a multi-filamentary design in which individual continuous filaments (< 30 μm in diameter) are embedded in a high conductivity matrix (Fig. 4.13).

Fig. 4.13. Samples of superconductors produced by Luvata

When conditions in a localized region of a superconductor exceed the critical surface, that area will start to conduct resistively producing local heating. The resulting heat and current transfer will start a cascade effect and result in a rapid transition of the whole superconductor to the normal state (quenching).

B. Supermagnets

Superconducting electromagnets for high magnetic fields are still manufactured from "old fashioned" Nb-Ti or Nb3Sn alloys. The wires for the electromagnets are composed of fine filaments of aNb-Ti alloy, each of which is only micrometers in diameter. They are imbedded in a matrix of nearly pure copper (for flexibility). Despite their considerably higher transition temperatures, ceramic superconductors have not yet revolutionized new technologies, mainly because of their inherent brittleness, their incapability of carrying high current densities, and their environmental instability.

Table 4.2. Most used superconductor materials and their physical parameters.

Properties	**NbTi (45% Nb; 55% Ti)**	**Nb_3Sn**	**MgB2**
Molecular Weight	140.73	397.43	45.93
Melting Point	1900 °C	2100°C	830°C
Density	5.7g/cm^3	>5.7g/cm^3(20°C)	2.57 g/cm^3
Current density, 4.2K	4x10^{5}A/cm^2	10^6 A/cm^2	10^6 A/cm^2
Upper critical field at 4.2 K	11.5 T	25T	6.5 T
Appearance	Metallic Solid in various form	Silver-gray solid	Dark grey solid

Because of the low cost of its constituent elements, MgB_2 has promise for use in superconducting low to medium field magnets, electric motors and generators, fault current limiters and current leads.

In 2006 a 0.5 Tesla openMRI (Magnetic Resonance Imaging) superconducting magnet system was built .A project at CERN to make MgB_2 cables has resulted in superconducting test cables able to carry 20,000 amperes for extremely high current distribution applications. The IGNITORtokamakdesign was based on MgB_2 for its poloidal coils. Thin coatings can be used in superconducting radio frequency cavities to minimize energy loss and reduce the inefficiency of liquid helium cooled niobium cavities.

In southern France, 35 nations are collaborating to build the world's largest tokamak which should be started in December 2025 as a mega experiment of containing plasma and squeeze energy. To create the optimal conditions for fusion in a tokamak, the hot gas (plasma) must be confined in the center of the vessel. This is the job of powerful magnetic fields, which are created by large electromagnets, witch again are made with superconductors, which must be cold it down up to 4 K (Fig. 4.14).

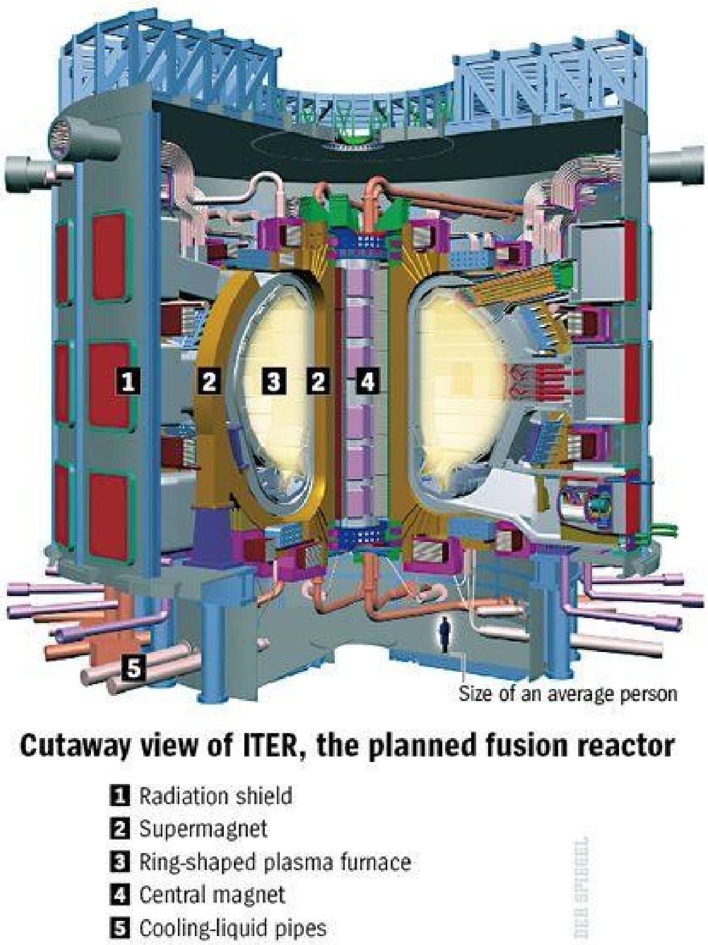


Fig. 4.14.Overview of ITER tokamak

C. Superconductive Electric Machines

The main advantages for use superconductors in electrical machines are:

- Reduced resistive losses but only in the rotor electromagnet.
- Reduced size and weight per power capacity without considering the refrigeration equipment.

The disadvantages are also great:

- The cost, size, weight, and complications of the cooling system.
- A sudden decrease or elimination of motor/generator action if the superconductors leave the SC state.
- A greater tendency for rotor speed instability.
- Motor bearings need to be able to withstand cold or need to be insulated from the cold rotor.
- As a synchronous motor, electronic control is essential for practical operation. This introduces expensive harmonic loss in the super-cooled rotor electromagnet (Fig. 4.15).

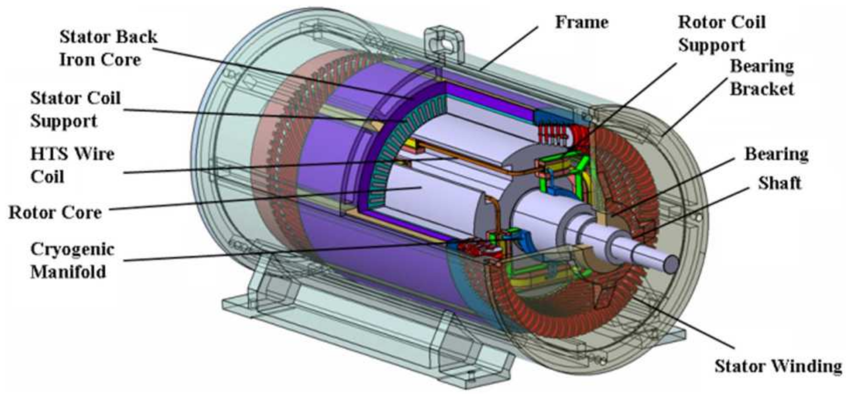


Fig. 4.15. A high temperature superconducting motor conceptual design

D. Superconducting transformers

The main advantages for use superconductors in electrical machines are:

- Energy savings (~50% Reduction), resource savings, not flammable (no oil),
- Compact and lightweight,
- Higher stability, less voltage drops, less reactive power,
- Active current limitation.

The disadvantages are also great:

- Increased design complexity,
- Increased costs and cooling requirements for maintaining the SC state in the HTS materials (Fig. 4.16).

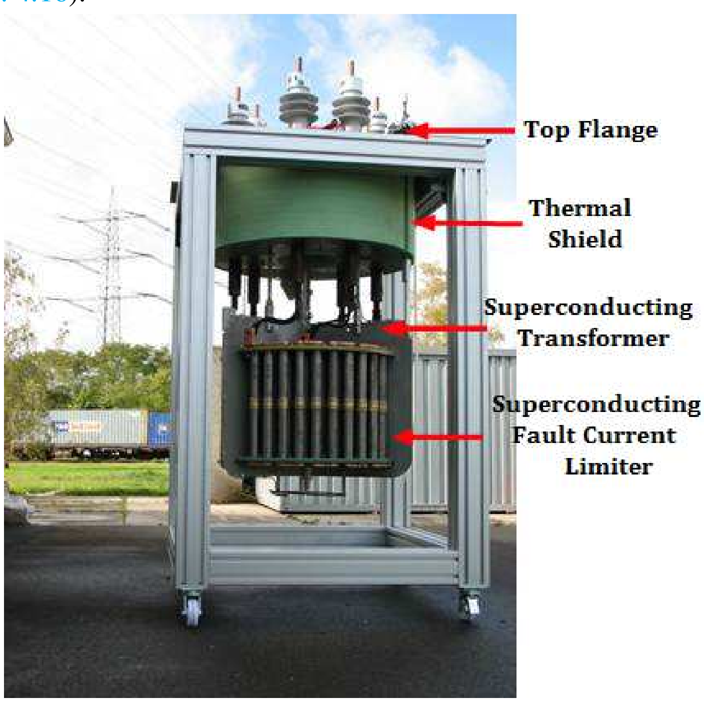


Fig. 4.16.A superconducting transformers

Magnetic levitation trains – MAGLEV

Magnetic levitationis used in some transport trains, by essentially eliminating any friction between the train and other parts such as the tracks.
In Fig. 4.17, it is shown how a permanent magnet is repelled, when this piece is placed above a superconductor at a temperature lower than the critical temperature. The superconductor pushes away its field by acting like a magnet with the same pole causing the magnet to repel (i.e. Meissner effect).

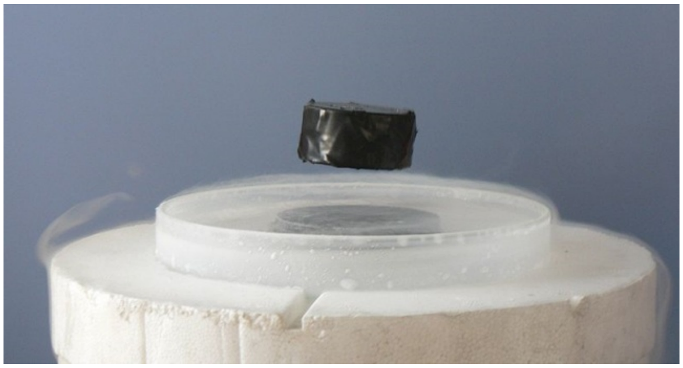
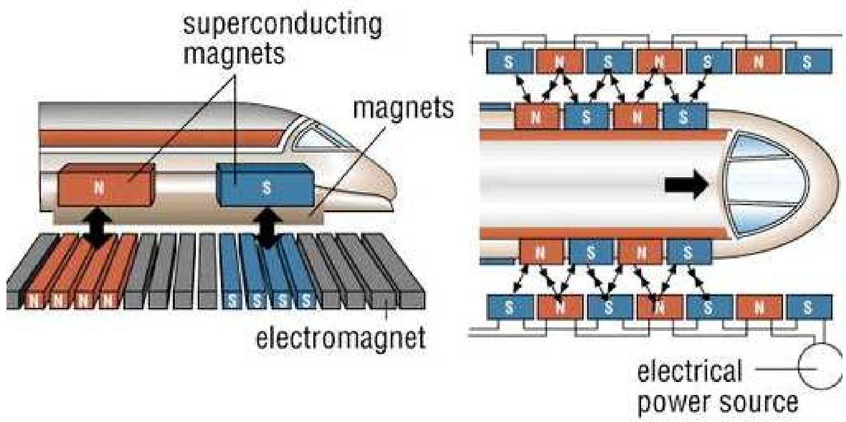


Fig. 4.17. Demonstration of magnetic levitation

A focal point for commercial implementation of magnetic levitation (MAGLEV) technology took place in 1990 when a project was funded in Japan. The Ministry of Transport authorized the construction of the Yamanashi Maglev Test Line, which opened on April 3rd, 1997. In Decembre 2003, the test vehicle reached a top speed of 581 km/h.

In Fig. 4.18, design elements for MAGLEV trains are detailed.



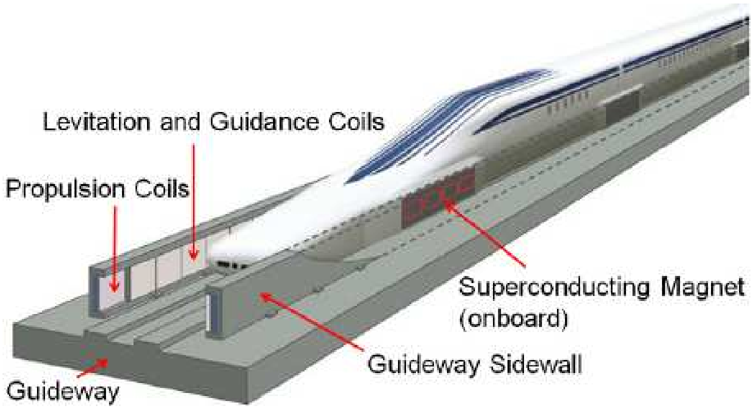


Fig.4.18. Design elements for MAGVEV trains

Superconducting microchips which Josephson junctions

Superconducting microchips are formed by thousands of electronic devices with Josephson junctions (Fig. 4.19).

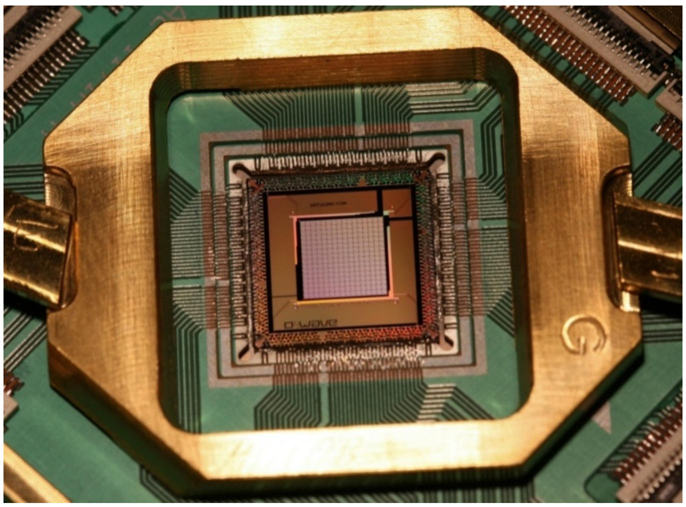


Fig. 4.19. Image of superconducting microchip which formed by thousands of Josephson junctions

A petaflop represents a thousand trillion floating point operations per second. The fastest supercomputers today are of the petaflops (quadrillions) of operations per second order.
Currently (beginning of 2022), the fastest supercomputer in the world, Fugaku, is in Japan and was completed in March 2021. It is rated for 537212 Tflops.
It is presumed that nanometer or sub-nanometer scale devices, along with unconventional switching mechanisms (like Josephson junctions, associated with superconduction materials) are necessary for the next level in computing power

Short Test for Lesson 4

1. Build the energy bands structure for neon crystal (Ne), knowing the atomic number Z_{Ne} = 10;
2. Build the energy bands structure for sodium crystal (Na), knowing the atomic number Z_{Na} = 11.

   Take into account the following study cases:

   a) *a single atom*

   b) *2 atoms situated at very long distances*

   c) *2 atoms situated closely each other*

   d) *5 atoms situated closely each other*

   e) *N atoms situated closely each other*

   Note: Energy structure of the electrons in an atom is described by:

   $1s_2^2\,2s_4^2\,2p_{10}^6\,3s_{12}^2\,3p_{18}^6\,4s_{20}^2\,3d_{30}^{10}\,4p_{36}^6\,5s_{38}^2\,4d_{48}^{10}\,5p_{54}^6\,6s_{56}^2\,4f_{70}^{14}\,5d_{80}^{10}\,6p_{86}^6\,7s_{88}^2\,5f_{102}^{14}\,6d_{112}^{10}\,7p_{118}^6$
3. Are the neon and sodium crystals electrically conductive??
4. Explain the Meissner effect.
5. What is the Josephson Effect, and what applications are developed based on this effect?
6. What characteristics are specific for superconductive materials?
7. What is a Cooper pair?
8. Describe an application of the superconductive state.

5. Electrical Conduction in Semiconductive Materials

Contents

5.1. General Characteristics of Semiconductive Materials

Section Summary: *In this section, general views on the materials which have semiconductive properties are given. For the crystals based on elements of periodical table, the values of energy gap and resistivity at room temperature are also given.*

Many are materials which are used as semiconductors, and many definitions are applied in this field. Semiconductors are:

a. Materials having the electrical conductivity values between conducting materials and insulating material, corresponding to the resistivity values of $\rho = 10^{-6} \div 10^{6}\Omega$m,
b. Materials for which the conductivity/resistivity could be modified by varying the impurity concentration *n*, or by the actions of external factors, like temperature *T*, electric field ***E***, magnetic field ***H***, radiations etc.
   This dependence has general form: $\rho = f(n, T, E, H,$ etc.$)$
c. Crystals with small energy gap: $\Delta W_i < 3$ eV.

Materials with semiconductive properties are shows in Table 5.1.

Table 5.1. Materials with semiconductive properties

- **Chemical elements:**
  B, C, Si, P, S, Ge, As, Se, Sn, Sb, Te, I
- **Two-chemical combinations**:
  AIV BIV (SiC)
  AIII BV (GaP, GaAs, GaSb, InP, InAs, InSb etc.)
  AII BVI (HgS, HgSe, HgTe, CdS, CdTe, ZnS, ZnSe, ZnTe etc.)
  AIV BVI (PbS with ΔW_i =0,39 eV, PbSe with W_i =0,26 eV, PbTe with ΔW_i=0,32 eV)

- **Three-chemical combinations** ($CuSi_2P_3$, In_4SbTe_3, etc.)
- **Semiconductor oxides**:
  Cu_2O (ΔW_i=1,9 eV), ZnO (ΔW_i=3,2 eV), TiO_2 (ΔW_i=3 eV)
- **Lantanidecombinations**(LaTe, EuS, etc.)
- **Organic combinations** (antacid, organic compounds, etc.).

In Table 5.2, the elements which form crystals with semiconductive properties (marked with *) are shown. The Fermi energy gap ΔW_i (noted as E_g) and values of resistivity are also specified.

Table 5.2. Elements in periodic table with semiconductive properties

II B	**III**	**IV**	**V**	**VI**	**VII**
Be	B* Eg=1.1 eV	C* $\rho=10^9\ \Omega m$ Eg=5.2 eV	N	O	F
Mg	Al	Si* $\rho=2,3.10^3\ \Omega m$ Eg=1.12 eV	P* Eg=1.5 eV	S*	Cl
Zn	Ga	Ge* $\rho=4,7.10^{-1}\ \Omega m$ Eg=0.67 eV	As* Eg=1.2 eV	Se* Eg=1.7 eV	Br
Cd	In	Sn* Eg=0.10 eV	Sb* Eg=0.12 eV	Te* Eg=0.36 eV	I* Eg=1.25 eV
Hg	Tl	Pb	Bi	Po	At

5.2. Electrical Conduction in Intrinsic Semiconductors

Section Summary: *In this section, physical model and energy-band model of electrical conduction in intrinsic semiconductors is detailed. The electrical conductivity relationship is obtained and the effect of temperature on intrinsic semiconductors is also pointed.*

5.2.1. Mechanism of electric conduction

Electrical conduction in semiconductors is based on different mechanisms compared with electrical conduction in metals.

First, the electrical conduction processes are presented for intrinsic semiconductors, which are semiconductive materials without impurities.

A silicon crystal is different from an insulator because at any temperature above absolute zero temperature, there is a finite probability that an electron in the lattice will be knocked loose from its position, leaving behind an electron deficiency called a "hole". The conductivity of a semiconductor can be modeled in terms of the band theory of solids. The band model of a semiconductor suggests that at ordinary temperatures there is a finite possibility that electrons can reach the conduction band and contribute to electrical conduction.For intrinsic semiconductors, the physical model and energy-band model of electrical conduction, sinthetically presented in Fig. 5.1.

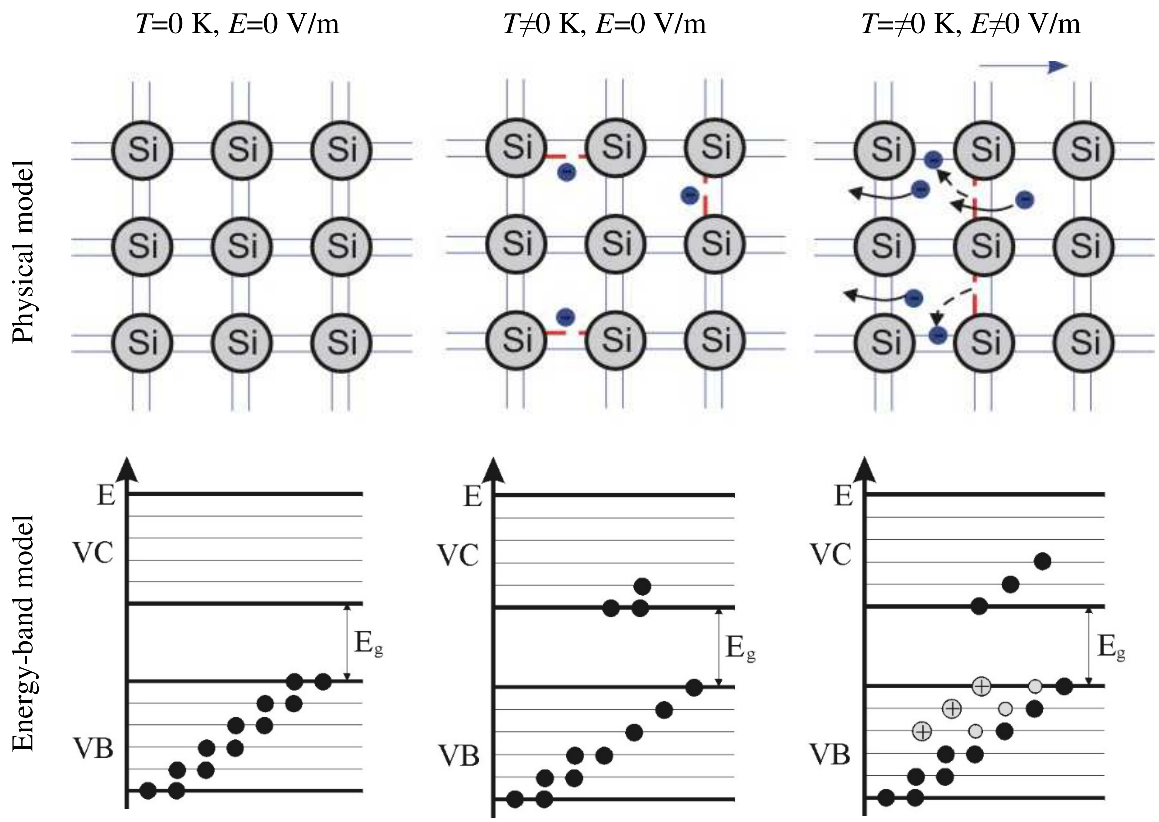


Fig. 5.1. Physical and energy-band models of electrical conduction in intrinsic semiconductors

The physical model of electric conduction considers that the electrical conduction is achieved by the ordered displacement of the quasi-free electrons (produced by thermal excitation) in the opposite direction of the applied electric field ***E***and by the ordered displacement of the uncompensated (positive) bonds in the direction of the applied electric field.

The energy-band model of electric conduction considers that the electrical conduction achieved by the electrons from conduction band (CB) and by holes from valence band (VB).

Note: Holes are fictive particles introduced in energy-band theory for easier calculation of the electrical conductivity.

5.2.2. Electrical conductivity relationship

The expression of the electrical conductivity for intrinsic semiconductors is obtained with the following relations:

- Law of electrical conduction:

$$\bar{J} = \sigma \bar{E} \tag{5.1}$$

- Density of electric current:

$$\bar{J} = nq_0 v_n^* + nq_0 v_p^* \tag{5.2}$$

- Electric conductivity:

$$\sigma = \frac{J}{E} = \frac{nq_0 v_n}{E} + \frac{pq_0 v_p}{E} \tag{5.3}$$

In these relations, n is number of electrons in conduction band (CV), and p is the number of holes in valence band (VB). For intrinsic semiconductors, $n=p=n_i$, unde$n_i$represents concentration of intrinsic charges (intrinsic carrier density). Having in view the definition of electron and hole mobilities,

$$\mu_n = \frac{v_n}{E}, \tag{5.4}$$

$$\mu_p = \frac{v_p}{E}, \tag{5.5}$$

The expression of electric conductivity is obtained:

$$\sigma_i = n_i q(\mu_n + \mu_p) \tag{5.6}$$

Electric conductivity of the intrinsic semiconductors depends on the concentration of intrinsic charges n_i and the mobility of electrons from CV and holes in VB.

In Table 5.3, the values of electric conductivity for crystals of silicon (Si) and germanium (Ge) are given.

Table 5.3. Mobility and electric conductivity for crystals of silicon (Si) and germanium (Ge)

Physical properties	**Silicon**	**Germanium**
Energy gap E_g [eV]	1.12	0.67
Electron mobility μ_n[m^2/ (V·s)]	0.135	0.39
Hole mobility μ_p[m^2 /(V·s)]	0.048	0.19
Intrinsic carrier density n_{i0} [carriers/m^3]	1.5 x 10^{16}	2.4 x 10^{12}

Density d_m [kg/m^3]	2330	5320
Resistivity ρ_i [Ωm]	2300	0.47

5.2.3. Temperature dependency of resistivity

With the following relations, the dependence of intrinsic resistivity with temperature is obtained:

- Volume concentration of electrons in CB:

$$n = N_C e^{\left[-\frac{W_C - W_F}{kT}\right]} \quad (5.7)$$

- Volume concentration of holes in CV:

$$p = N_V e^{\left[\frac{W_V - W_F}{kT}\right]} \quad (5.8)$$

- Volume concentration of intrinsic electric charge:

$$n \cdot p = n_i^2 = N_V N_C e^{\left[-\frac{W_C - W_F}{kT}\right]} \quad (5.9)$$

It results:

$$n_i = \sqrt{N_V N_C} \cdot e^{\left[-\frac{\Delta W_i}{2kT}\right]} \quad (5.10)$$

where N_C is the number of electrons in the conduction band, N_V – number of electrons in the valence band, W_V – energy level in the valence band, W_C – energy level in the conduction band, W_F –Fermi energy level

Equation 5.10 can also be written as (electrical conductivity dependence with temperature):

$$\sigma = n_0 q_0 (\mu_n + \mu_p) \cdot e^{\left(-\frac{E_g}{2kT}\right)} \quad (5.11)$$

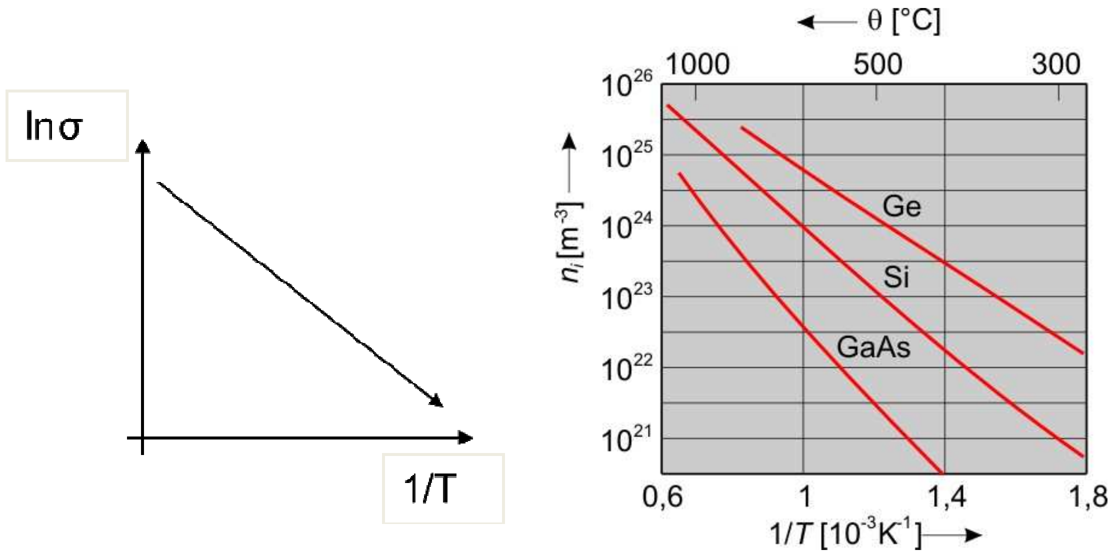
Since there is a logarithmic dependency between the electrical conductivity and temperature (Figure 5.2), we get:

$$\ln \sigma = \ln \sigma_0 - \frac{E_g}{2kT} \quad (5.12)$$

The temperature effect on intrinsic semiconductors can be expressed as:

$$n_i \cong n_0 \cdot e^{\left(-\frac{E_g}{2kT}\right)} \quad (5.13)$$

The graphic of electric conductivity dependence on temperature is given in Fig. 5.2.a: conductivity increases exponentially with increasing the temperature.



a) b)

Fig. 5.2.Temperature dependence of the electric conductivity for intrinsic semiconductors:a) general dependence; b) intrinsic electric charge dependence on temperature.

5.3. Electrical Conduction in Extrinsic Semiconductors

Section Summary: *In this section, physical model and energy-band model of electrical conduction in extrinsic semiconductors is detailed. The electrical conductivity relationship is obtained and the effect of temperature on extrinsic semiconductors is also pointed.*

The extrinsic semiconductors are semiconductors in which specific impurities are added.

The extrinsic conduction of n-type is obtained if a pure semiconductor crystal doped with donor impurities. Thus, in the crystals lattice of IVA group elements (silicon, germanium), the impurity atoms of a V group elements are introduced, as **P, As or Sb,** named **donor atoms.**

The extrinsic conduction of p-type is obtained if a pure semiconductor crystal is doped with acceptor impurities. Example, the impurity atom of a three valence group element III, such as **B, Al or Ga**, as **acceptor atoms**, is introduced in the silicon or germanium crystal lattice:

5.3.1. Electrical conduction in n-type semiconductors

For extrinsic semiconductors of n-type, the physical model and the energy-band model of electrical conduction, synthetically presented in Fig. 5.3, are usefull in justification of the particularity of electric conduction.

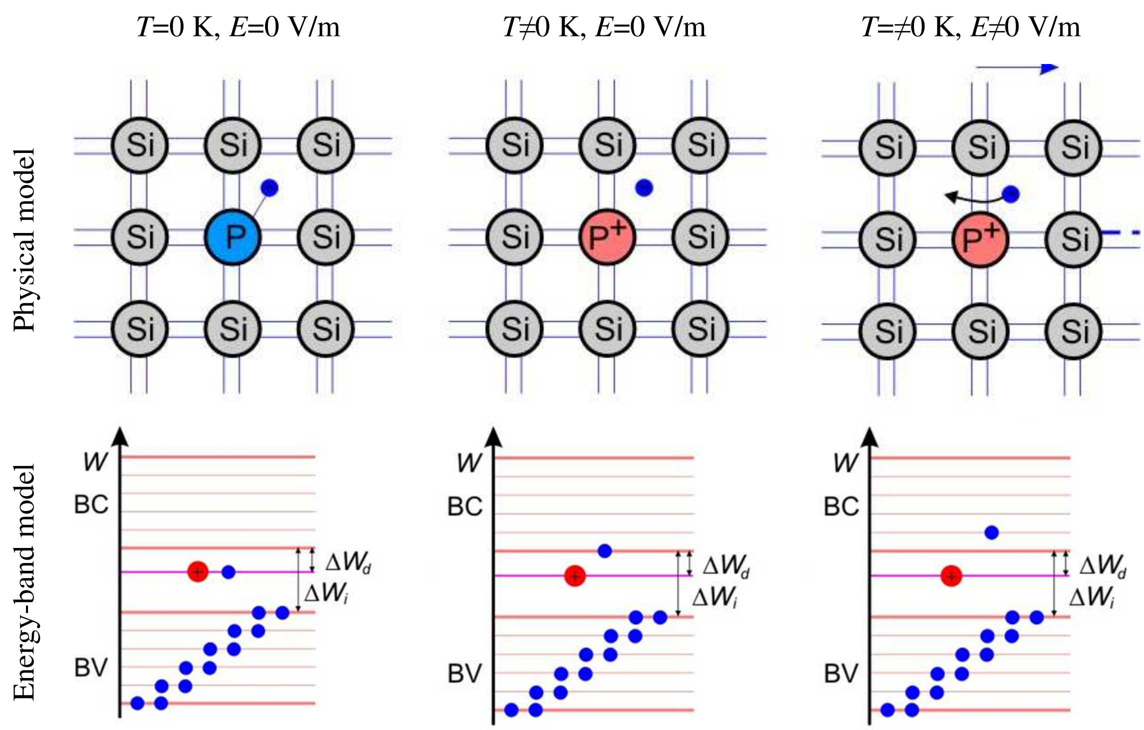


Fig. 5.3.Physical and energy-band models of electrical conduction in n-type extrinsic semiconductors

The physical model considers that electrical conduction in n-type semiconductor is achieved by electrons coming from the donor impurity atoms, which have an ordered displacement in the direction of the applied electric field *E*.

The energy-band model considers that electrical conduction in n-type semiconductor is achieved by achieved by electrons from conduction band (CB) coming by thermal excitation (or other stimulus) from the donor levels of donor impurity atoms. The activation energy is $\Delta Wd << \Delta Wi$.

5.3.2. Electrical conduction in p-type semiconductors

For extrinsic semiconductors of p-type, the physical model and the energy-band model of electrical conduction, sinthetically presented in Fig. 5.4, are usefull in justification of the particularity of electric conduction.

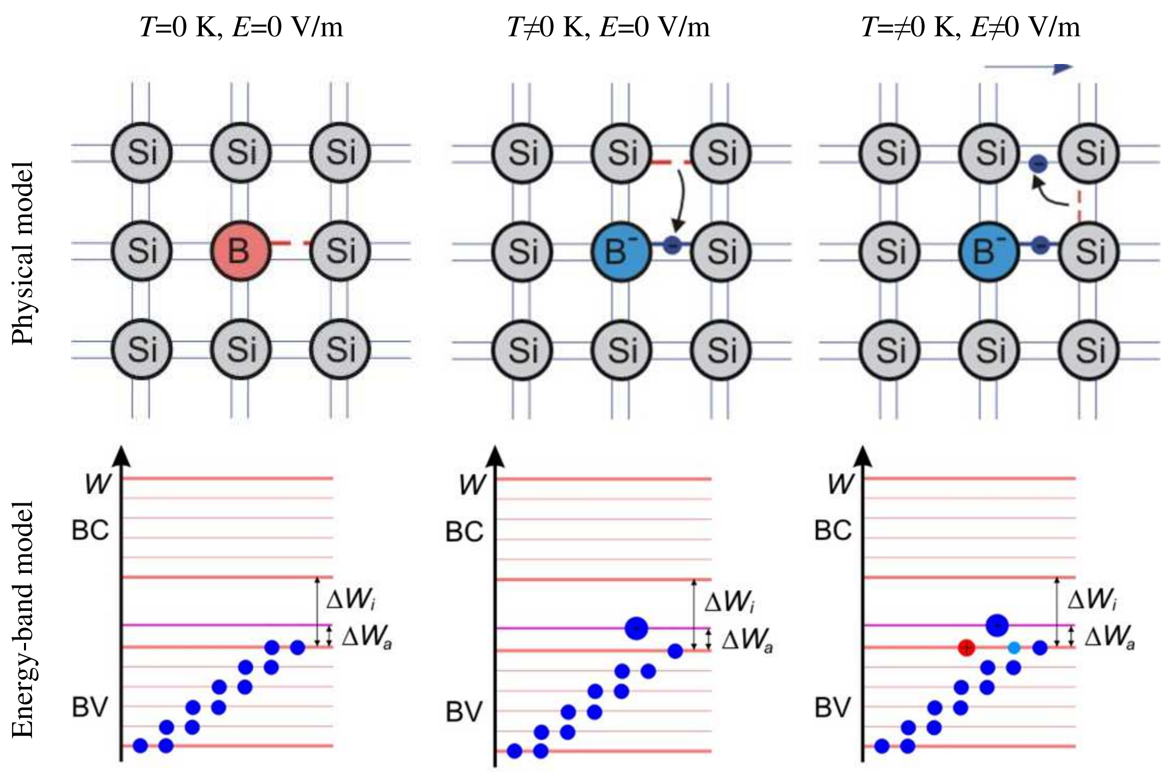


Fig. 5.4. Physical and energy-band models of electrical conduction in p-type extrinsic semiconductors

The physical model considers that electrical conduction in p-type semiconductor is achieved by the uncompensated bonds (holes) coming from the acceptor impurity atoms, and which have an ordered displacement in the direction of the applied electric field *E*.

The energy-band model considers that electrical conduction in p-type semiconductor is achieved by the holes from valence band (VB) which are generated by electrons transition from VB to the acceptor levels of acceptor impurity atoms. The activation energy is $\Delta W_a << \Delta W_i$.

5.3.3. Electrical conductivity relationship for extrinsic semiconductors

The number of charge carriers in n-type semiconductors and corresponding conductivity are (where n_e are "donor" electrons and n_i are intrinsic conduction electrons):

$$n_{"n"} = n_e + n_i \tag{5.14}$$

$$n_{"n"}(T) = N_{0e} \cdot e^{\left(-\frac{W_d}{kT}\right)} + N_{0i} \cdot e^{\left(-\frac{W_{iF}}{2kT}\right)} \backslash \tag{5.15}$$

$$\sigma_{"n"} = N_{0e} q_0 \mu_n \cdot e^{\left(-\frac{W_d}{kT}\right)} + N_{0i} q_0 (\mu_n + \mu_p) \cdot e^{\left(-\frac{W_{iF}}{2kT}\right)} \tag{5.16}$$

The number of charge carriers in p-type semiconductors and corresponding conductivity are (where p_e are "acceptor" electrons and p_i are intrinsic conduction electrons):

$$n_{"p"} = p_e + n_i \qquad (5.17)$$

$$n_{"p"}(T) = N_{0e} \cdot e^{\left(-\frac{W_a}{kT}\right)} + N_{0i} \cdot e^{\left(-\frac{W_{i\mathrm{F}}}{2kT}\right)} \qquad (5.18)$$

$$\sigma_{"p"} = N_{0e} q_0 \mu_p \cdot e^{\left(-\frac{W_a}{kT}\right)} + N_{0i} q_0 (\mu_n + \mu_p) \cdot e^{\left(-\frac{W_{i\mathrm{F}}}{2kT}\right)} \qquad (5.19)$$

The general expression of electrical conductivity in extrinsic semiconductor is:

$$\sigma = n_a q_0 \mu_p + n_0 q_0 (\mu_n + \mu_p) \cdot e^{\left(-\frac{E_g}{2kT}\right)} \qquad (5.20)$$

Table 5.4. Acceptor and donor energy with different impurities in silicon and germanium crystals

Doping	**In silicon (ΔW_i = 1.12 eV)**		**In germanium (ΔW_i = 0.67 eV)**	
	ΔW_d [eV]	**ΔW_a [eV]**	**ΔW_d [eV]**	**ΔW_a [eV]**
P	0.015		0.0120	
As	0.049		0.0127	
Sb	0.039		0.0096	
B		0.045		0.0104
Al		0.057		0.0102
Ga		0.065		0.0108
In		0.160		0.0112

5.3.4. Effect of Temperature on Extrinsic Semiconductors

The effect of temperature is highlighted for a semiconductor of n-type in which the intrinsic electric conductionis also taken into account.

The expression of electric conductivity for this case is:

$$\sigma_{"n"} = N_{0e} q_0 \mu_n \cdot e^{\left(-\frac{W_d}{kT}\right)} + N_{0i} q_0 (\mu_n + \mu_p) \cdot e^{\left(-\frac{W_{i\mathrm{F}}}{2kT}\right)} \qquad (5.21)$$

The dependence of electric conductivity on temperature, taking in account and dependence of charge carrier mobility on temperature (Fig. 5.5.a) is given in Fig. 5.5.b.

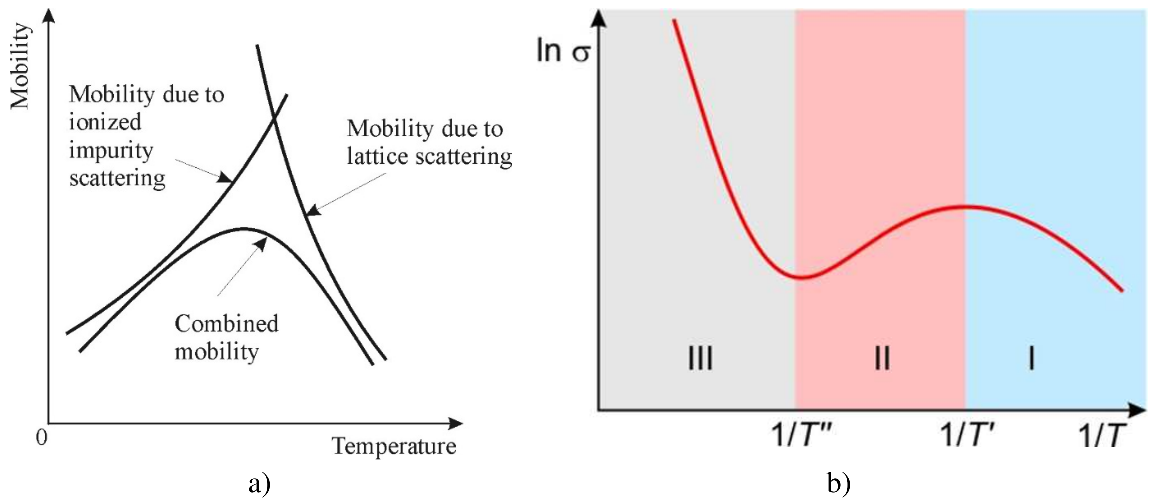


Fig. 5.5. Temperature influence on the n-type semiconductor:
a) influence on charge carrier mobility; b) influence on electric conductivity

Fig. 5.5.b shows that there are three specific domains of electric conduction:

- Domain I – conduction is of extrinsic type, for temperature below T'. Conductivity values are relatively lower. When the temperature increases (when 1/T decreases) the electrical conductivity increases exponentially.
- Domain II – conduction is of metallic type, for temperatures between T' and T". Conductivity values are relatively higher. When the temperature increases (when 1/T decreases) the electrical conductivity decreases.
- Domain III – conduction is of intrinsic type, when temperatures are up to T". Electrical conductivity increases greatly. When the temperature increases (when 1/T decreases) the electrical conductivity increases exponentially.

Usually, the electronic devices are designed to operate in the domain of extrinsic conduction (domain I).

Short Test for Lesson 5

1. Describe literally and graphically, the physical model and the energy bands model of intrinsic electrical conduction in semiconductors.
2. Describe literally and graphically, the physical model and the energy bands model of *n*-type extrinsic electrical conduction in semiconductors.
3. Describe literally and graphically, the physical model and the energy bands model of *p*-type extrinsic electrical conduction in semiconductors.
4. What observations can be done, related to the manner of controling the electrical conduction process in semiconductors?

6. Applications of Semiconductive Materials

Contents

6.1. Controlled Conduction and p-n Junction. Applications

Section Summary: *In this section, the elements regarding the controlled conduction in semiconductor devices which are based on the properties of p-n junction are detailed. A general views on the applications of materials with semiconductive properties are also systemic presented.*

The controlled conductionis conditioned by the realization of the crystalline structure of the intrinsic semiconductive materials and subsequently, by the introduction in the semiconductors of a certain quantity of donor and acceptor impurities.

Two types of currents are established in non-uniform spatial distribution of charges in extrinsic semiconductors:

- **Electric conduction current,** with the current densitydescribed by the relations:

$$\bar{J} = \sigma\bar{E} = (\sigma_n + \sigma_p) \cdot \bar{E} \quad (6.1)$$

where the value of electric conductivity of extrinsic semiconductor is function of the *n* concentration of electrons and p concentration of hole, and their motilities μ_n,and μ_p:

$$\sigma_{extrinsic} = \left(\sigma_n + \sigma_p\right) = q_0(n\mu_n + p\mu_p) \quad (6.2)$$

- **Electric diffusion current,** with the current densitydescribed by the relations) note that $\nabla$is the gradient operator):

$$\bar{J}_{dif} = q_0(D_n\nabla n - D_p\nabla p) \quad (6.3)$$

whereD_n and D_p are the diffusion coefficients of charge carriers.

A way of controlling the electrical conduction through which the dynamic balance between conduction and diffusion currents is obtained is the p-n junction.

6.1.1. Processes in the p-n junction

The property of unidirectional conductionis realized with junctions of different semiconductive materials, by putting in common two semiconductors with different charge carrier concentration.

The physical model of the *n- p* junction formation is shown in Fig. 6.1.

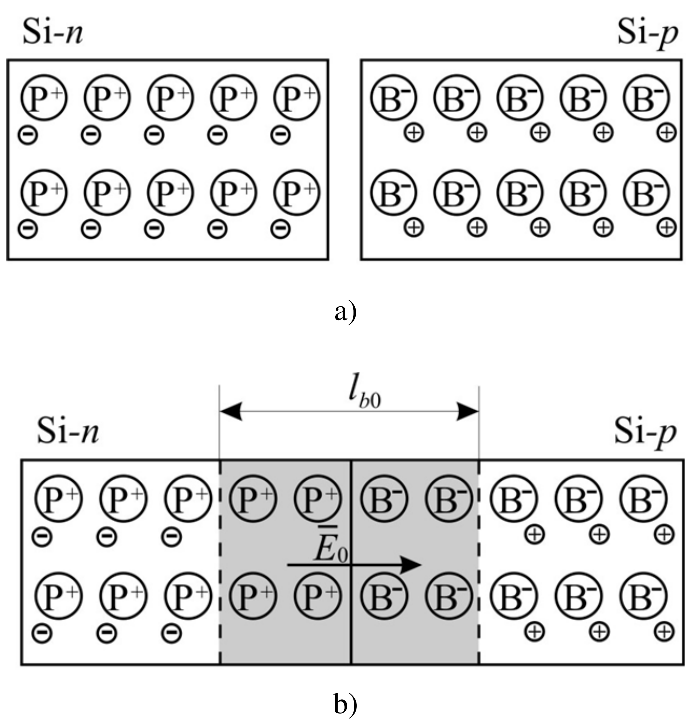


Fig. 6.1. Physical model of n-p junction: a) two n-Si and p-Si crystals in no contact; b) two n-Si and p-Si crystals in contact, with formation of n-p junction

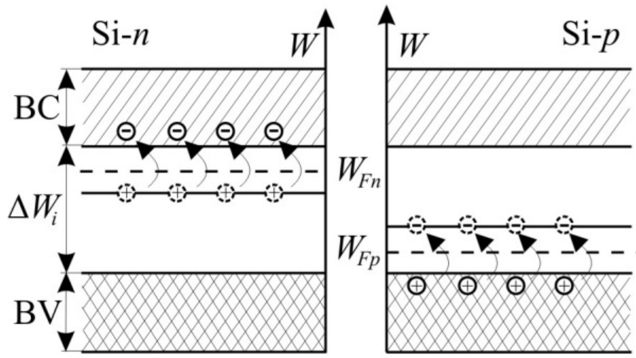
In Fig. 6.1.a, the state of two semiconductors, not in contact, is described.

- In n-Si crystal, the atoms of the donor impurities, in this case - phosphorus atoms (P), lose the valence electrons and become positive ions (noted P^+), fixed in the crystalline lattice. Overall, in the crystal, there is also an equal number of *quasi-free electrons*, uniformly distributed in the volume of the semiconductor.
- In p-Si crystal, the atoms of the acceptor impurities, in this case - boron atoms (B), receive electrons from the atoms of the silicon crystalline lattice, and they become negative ions (noted B^-), fixed in the crystalline lattice. Overall, in the crystal, there is an equal number of *uncompensated bonds*, equivalent to the number of positive particles, called *holes*.

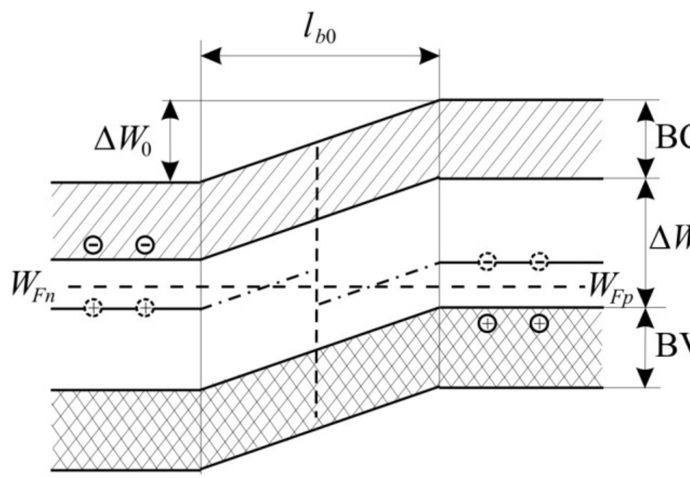
In Fig. 6.1.b, the state of the two semiconductors in contact is described.

- Because of the unbalance between the excess of the mobile electrons in the n-semiconductive material and the excess of holes in p-semiconductive material, in the junction region there is a recombination of electrons with holes.
- In junction area, a space-charge region is formed with the contribution of the impurity ions P^+ and B^-, which generate an electric imprinted field of intensity $\boldsymbol{E}_0$.
- The electric field of intensity $\boldsymbol{E}_0$ acts with a force ($\boldsymbol{F}_e$=q.$\boldsymbol{E}_0$) on the electrons in p-Si and the holes in n-Si crystal, which prevents new recombination in the junction area.

The properties of n-p junction can be explained also in energy-band model (Fig. 6.2).



a)



b)

Fig. 6.2. Energy-band model of n-p junction

Fig. 6.2.a shows the structures of energy bands for the case of n-Si and p-Si single-crystals, which are not in contact: the ionization processes of the impurity atoms correspond to the electrons presence in BC (for n-Si) and to the holes in BV (for p-Si). The Fermi levels, noted with W_{Fn} and W_{Fp} have specific positions.

Fig. 6.2.b shows the energy band structures for the case when n-Si and p-Si crystals are put in contact: the equalizing of the Fermi energies (W_{Fn}=W_{Fp}) determines a variation of the bands

structures, in function of the width of the l_{b0} junction. This variation of levels position corresponds to the energy variation ΔW_0 which is the cause of the appearance of the electric imprinted field at the contact.

The n-p junction is characterized by unidirectional electric conduction (Fig. 6.3).

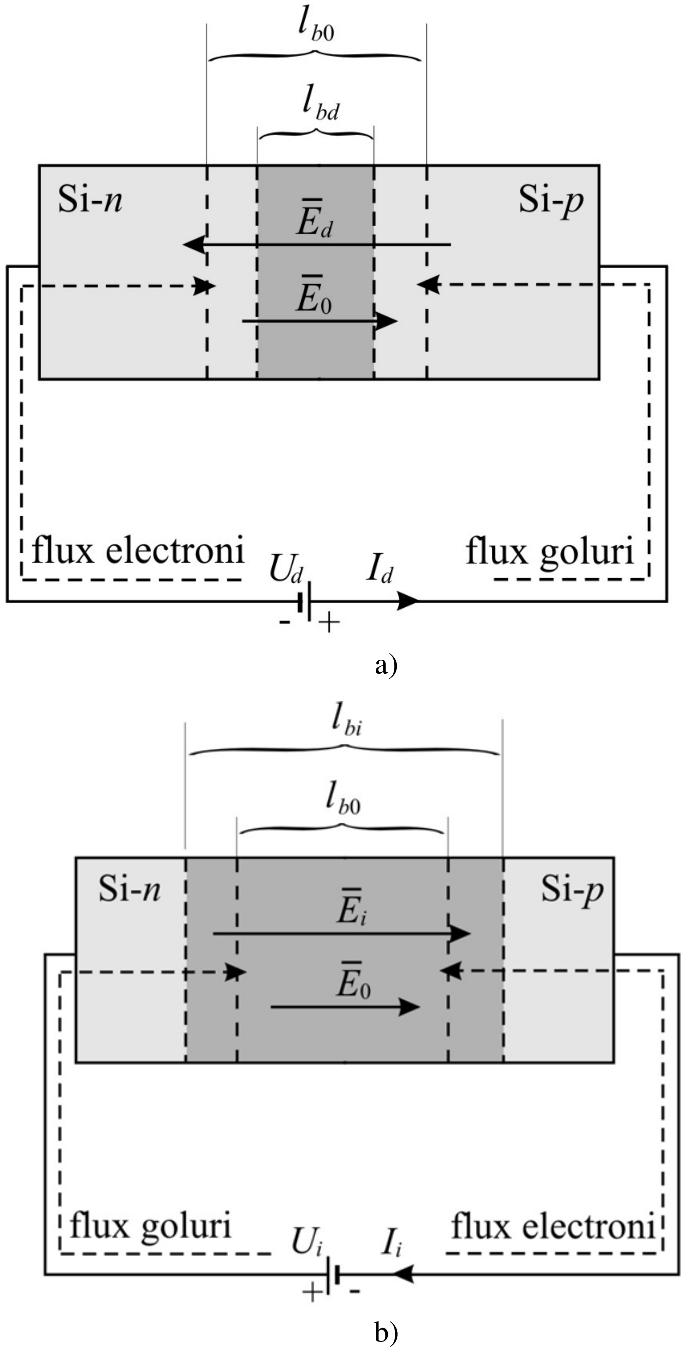


Fig. 6.3. Unidirectional conduction in n-p junction: a) forward supply; b) reverse supply

In forward supplying (Fig. 6.3.b), the external electric field is of opposite sense of the internal field, the value of total electric field having overall a lower intensity, which determines the decrease of the junction width from l_{b0} to l_{bd}. The width of depletion region decreases. This is favorable to passing the electric current.

Requirements for materials used in controlled electrical conduction are specifically:

- To allow an easy obtaining, controllable and highly reproducible electrical conductivity
- To allow obtaining of a greater electric conductivity
- To have small variation with disturbances as electric field frequency, temperature etc.
- To have small values of electric permittivity, to be stable with frequency.

Nowadays, the technologies of Ge, Si an GaAs is mainly applied for obtaining specific n-p junctions (Table 6.1).

Table 6.1. Semiconductive materials for controlled conduction by n-p junctions

Material	μ_n [m²/Vs]	μ_p [m²/Vs	D_n [m²/s]	D_p [m²/s]	n_i [m⁻³]	σ_i [1/(Ωm)	ε_r	$T_{melting}$ [K]	E_{str} [MV/m]
Si	0.135	0.048	$3.5\cdot10^{-3}$	$1.2\cdot10^{-3}$	$1.45\cdot10^{16}$	$4.08\cdot10^{-4}$	11.7	1693	30
Ge	0.390	0.190	$10\cdot10^{-3}$	$4.9\cdot10^{-3}$	$2.40\cdot10^{19}$	$2.13\cdot10^{0}$	16.0	1210	8
Ga As	0.860	0.025	$22\cdot10^{-3}$	$0.6\cdot10^{-3}$	$9.00\cdot10^{12}$	$1.27\cdot10^{-6}$	11.1	1713	35

In Table 6.2, for main semiconductive materials, the specific properties and industrial applications are given.

Table 6.2. Specific properties and industrial applications of main classes of semiconductive materials

Semi-conductive material	ΔW_i at 0 K, [eV]	μ_n at 300 K, [cm²/Vs]	μ_p at 300 K, [cm²/Vs]	Industrial applications
Si	1.12	1900	425	Rectifiers, solar cells, radiation detectors, transistors, tiristors

Ge	0.78	3900	1700	Rectifiers, radiation detectors, transistors
Se	1.6÷1.9			Rectifiers
GaP	2.33	100	150	Luminescent diodes
GaAs	1.52	7000	450	Tunnel diodes, laser
InP	1.42	4500	150	Filters, infrared cells
InAs	0.43	27000	450	Filters infrared, Hall cells
InSb	0.24	76000	760	Filters infrared, Hall supplied, Hall cells, magnetoresistive cells
Cu_2O	2.06		100	Rectifiers
Zn O	3.2	200		Fluorescent cells
Zn S	3.5-3.8	-	-	
Zn Se	2.8	-	-	
Cd S	2.5	-	-	Fotorezistors, X ray dozimeter
Cd Se	1.84	240	-	Fotorezistors
Pb S	0.37	640	350	
Pb Se	0.45	1400	640	Fotoelements
PbTe	0.45	2100	840	
Bi_2 Te_3	0. 5	310	400	Thermoelectric cooling systems
Bi_2 Se_3	0.28	600	-	

6.2. Semiconductive Materials for Sensors - Varistors and Thermistors

Section Summary: *In this section, the main characteristics of the materials for varistors and thermistors, applied for voltage-current sensors and temperature sensors. The properties of these materials are also systemic presented.*

Based on the variation of the electric conductivity with electrical or thermal stresses, the semiconductive materials are used for build many kinds of sensors, as varistors and thermistors.

<u>6.2.1. Materials for varistors</u>

Varistors are the passive circuit components whose electrical resistivity modifies with the variation of the electric field.

Materials for varistors should have high sensitivity to electric fields. This is shows by the slope of current-volt characteristic, I=f(U).

The current-volt characteristic is given under the forms:

$$I = AU^{\alpha} ; U = BI^{\beta} \quad (6.4)$$

The parameters α and β define the shape of the I=f(U) and U=f(I) characteristics.

As materials for varistors are used SiC and metallic oxides as: ZnO, TiO2, CaO, MnO2, CuO, etc.

In Fig. 6.4, the current-volt characteristic for ZnO and SiCvaristors are shown.

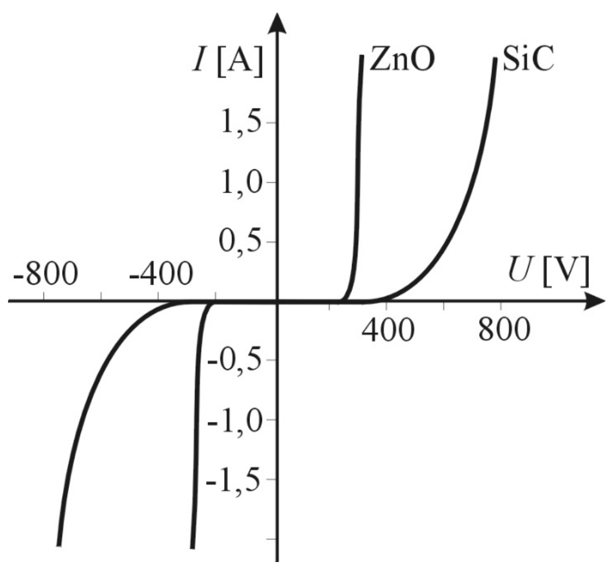


Fig. 6.4. Current-volt characteristic for ZnO and SiCvaristors

It is noted that the SiCvaristor can be used at a higher voltage than the ZnOvaristor. The α parameters are: $\alpha_{SiC} \approx 5$ and $\alpha_{ZnO} \approx 25$.

6.2.2.Materials for thermistors

Thermistors are the passive circuit components whose electrical resistivity modifies with the variation of the temperature.

Materials for thermistors should have high sensitivity to the temperature. This is shows by the slope of resistance-temperature characteristic, *R*=f(*T*).

Usually, the thermistors are of negative type (NTR in Fig. 6.5), with the resistance - temperature characteristic given under the form:

$$R_T = R_{T0} \cdot e^{\left[B\cdot\left(\frac{1}{T}-\frac{1}{T_0}\right)\right]} \tag{6.5}$$

The *B* parameter characterizes the sensitivity of the thermistor, named index thermal sensitivity, measured in degree Kelvin [K].

The coefficient of resistance variation with temperature is defined as:

$$\alpha_{RT} = \frac{1}{R_T} \cdot \frac{dR_T}{dT} \tag{6.6}$$

In Fig. 6.5, the resistance- temperature characteristic is shown for NTR aand PTR thermistors, comparative with the characteristic of a metallic wire.

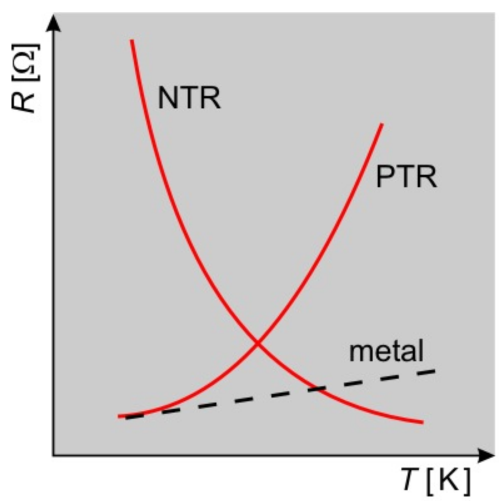


Fig. 6.5.Resistance-temperature characteristics for negative-thermistors (NTR), positive-thermistor (PTR) and metals.

From Fig. 6.5we can denote that:

- For NTR, thermistor resistance exponentially decreases with increasing the temperature.

- For PTR, thermistor resistance exponentially increases with increasing the temperature.
- For metallic wires, resistance linearly increases with increasing the temperature.

For NTR thermistors, metallic oxides, p- or n-doped, are used. Examples:

- Fe_2O_3 oxide, doped with titanium (Ti) ions, will has a behaviour of n-type semiconductor;
- NiO oxide, doped with litium (Li) ions, will has a behaviour of as p-type semiconductor.

For PTR thermistors, barium titanate ($BaTiO_3$) combined with strontium titanate ($SrTiO_3$ are used. A conduction of p-type results by By the substitution of the Ba^{2+} bivalent ions with La^{3+} trivalent ions or the Ti^{4+} tetravalent ions with Sb^{5+} or Nb^{5+} five-valence ions.

The resistance - temperature characteristic for PTR thermistors is of form:

$$R_T \cong A + C \cdot e^{BT} \tag{6.7}$$

Some specific parameters of NTR and PTR thermistors are given in Table 6.3.

Table 6.3. Specific parameters of NTR and PTR thermistors

Type	*B*, [K]	*R*$_{25}$, [Ω]	Temperature domain [ºC]	Critical temperature [ºC]	Maximum voltage [V]	Temperature coefficient [10^{-2} K^{-1}]
NTR	2650÷4200 3350÷4650 2600÷3680	1,1 - 32 15 ÷ 85 3,3÷330 ×10^3	0 ÷ 120 - 25 ÷ 155 - 25 ÷ 100			
PTR		50÷60 100 250 36 ÷ 50		30÷105 70 6 115	25 265 25 180	7 ÷ 40 35 5 35

In Table 6.3, R_{25} is the thermistor resistance at reference temperature of 25ºC.

6.3. Materials for Opto-Electronic and Electro-Optic Conversion

Section Summary: *In this section, the main characteristics of the materials used in opto-electric and electro-optic Conversion. The properties of these materials are also presented.*

6.3.1. Materials for Opto-Electric Conversion

Opto-electronic conversion is the process of transforming the optical energy in electrical energybased on the increasing the free charge carrier volume concentration of a semiconductor under the action of optical signal, resulting the modification of electrical conductivity of the semiconductive material.

Internal photoelectric effectappears if the energy received from the photons by the electrons is smaller than the extraction work, but sufficient for the production of mobile charge carriers in the semiconductor, by breaking of some connections in the crystalline lattice.

This process is produced with following condition:

$$W_r = \hbar\omega \geq \Delta W_i \qquad (6.8)$$

Where the optical wave pulsation is:

$$\omega = 2\pi f = 2\pi \cdot \frac{c}{\lambda} \qquad (6.9)$$

With these relations, the limit of wavelength for which the absorbtion processes are produced is:

$$\lambda_i = \frac{2\pi\hbar c}{W_i} \qquad (6.10)$$

Radiations with $\lambda > \lambda_i$ (limit of wavelength) are no longer absorbed by semiconductor.

In Fig. 6.6.a, energy-band model of opto-electric conversion through the absorption mechanisms by the optical radiation is shown.

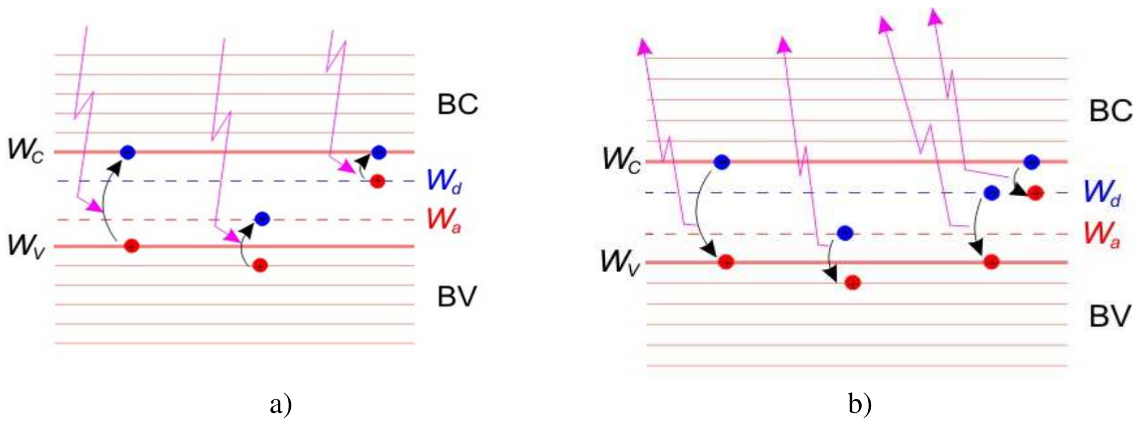


Fig. 6.6. Energy-band models for opto-electric (a) and electro-optic (b) conversions

Specific requirements are for semiconductive materials used in opto-electric conversion:

- to have high sensitivity to electromagnetic radiation within a wide wavelength range
- to have a reduced optical threshold;
- the recombination processes of the excessive carriers to be less intense
- the charge carrier mobility to be higher so that the response velocity to be higher

The most often used materials with opto-electric function are: Ge doped with Au or Hg, InSb, $Hg_{0.8}Gd_{0.2}$.

In Table 6.4, some characteristics for semiconductive materials used for opto-electronical conversion are shown.

Table6.4.Characteristics for semiconductive materials used for opto-electronical conversion

Semi-conductive	CdS	GaP	CdTe	GaAs	Si	Ge	PbS	InAs	InSb	PbSe
ΔW_i [eV]	2.4	2.24	1.5	1.35	1.12	0.67	0.42	0.39	0.23	0.23
λ_i [μm]	0.52	0.56	0.83	0.92	1.1	1.8	2.9	3.2	5.4	5.4

	Silicon doped with						Germanium doped with					
	In	**Ga**	**Bi**	**Al**	**As**	**Sb**	**Au**	**Hg**	**Cd**	**Cu**	**Zn**	**B**
ΔW_e **[eV]**	0.155	0.072	0.070	0.0685	0.05	0.042	0.15	0.09	0.06	0.04	0.03	0.01
λ_e **[μm]**	8	17	18	18	23	29	8,3	14	21	30	38	120

6.3.2. Materials for Electro-Optical Conversion

Electro-optical conversion represents the process of transformation of the electrical signal into optical signal, on the basis of the radiative recombination effect of the mobile charge carriers from a semiconductor in the case when an electric field of a certain value is being applied.
In Fig. 6.6.b, energy-band model of electro-optic conversion through the recombination mechanisms with optical radiation is shown.

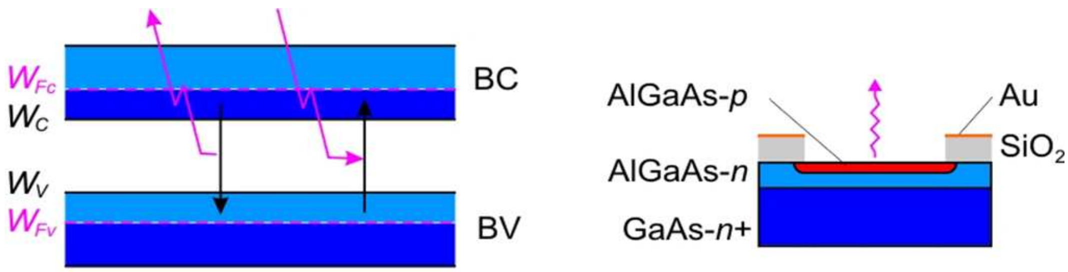
Spontaneous emission can appear.The common type of light emitting consists of spontaneous recombination between an excited electron from BC and a hole from BV. The energy difference is emitted in the form of a light quantum (photon).

- Fluorescence, when $\tau = (10^{-5} \div 10^{-8})$ s ;
- Phosphorescence, when $\tau = (1 \div 10^{-4})$ s.

Stimulated emission appears by **t**he stimulated recombination, which generates a coherent optical radiation, of a constant and unique frequency, phase and direction.

Stimulated emission is applied for:
- Laser diodes (**L**ight **A**mplification through **S**timulated **E**mission of **R**adiation)
- LED-devices (**L**ight **E**mitting **D**iodes.

In Fig. 6.7, the photo-emissive diode (LED) principle: a) energy band configuration in the conditions of the population inversion; b) the structure of a LED diode.



a) b)

Fig. 6.7. Photo-emissive diode (LED) principle: a) energy band configuration in the conditions of the population inversion; b) the structure of a LED diode

LED-s emit radiations of different colors, depending on the wavelength where the emission efficiency is maximum. Examples:

- infrared – GaAs (700nm ÷ 1mm / 430THz÷300GHz, 1.24meV ÷ 1.7eV)
- red-yellow – GaAsP,
- red – GaPZn,
- green-yellow – GaPN,
- blue - ZnSe.

In Table 6.5, the corresponding optical wave lengths and, respectively, frequencies for different colors are given.

Table 6.5. Optical wave lengths and characteristic frequencies for different colors

Red	~10÷780 nm	~ 480÷405 THz
Orange	~ 590-650 nm	~ 510-480 THz
Yellow	~ 575-590 nm	~ 530-510 THz
Green	~ 510-560 nm	~ 600-530 THz
Azure	~ 485-500 nm	~ 620-600 THz
Blue	~ 452-470 nm	~ 680-620 THz
Indigo	~ 380-424 nm	~ 790-680 THz

In Table 6.6, the characteristics of some semiconductive materials used for electro-optic conversion are shown. In this table, the significance of parameters is:

n_i– volume concentration of charge carriers;

r_i– recombination velocity of mobile charge carriers at thermal equilibrium,
τ_i – life time of mobile charge carriers at thermal equilibrium,
r_r – radiative recombination velocity,
τ_r – life time of the excessive carriers at un-balance.

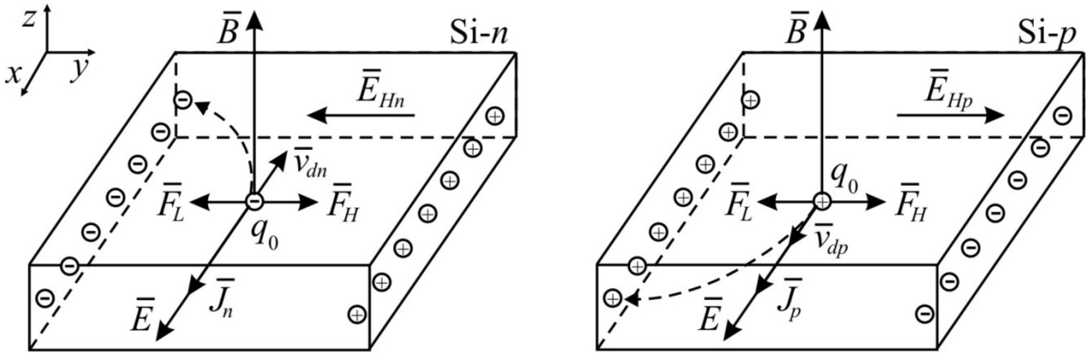
Table 6.6. Characteristics of some semiconductive materials used for electro-optic conversion

Semiconductor	**n_i, [10^{18} m^{-3}]**	**r_i, [m^{-3} s^{-1}]**	**τ_i, [s]**	**r_r, [m^{-3} s^{-1}]**	**τ_r, [s]**	**$\lambda_{emission}$, [μm]**
Si	0.015	$4.5 \cdot 10^{-1}$	4.6 h	$4 \cdot 10^{15}$	2500	-
GaSb	4.3	$2.4 \cdot 10^{8}$	5000	$2.7 \cdot 10^{19}$	0.37	1.55
InAs	1600	$5.38 \cdot 10^{13}$	15	$4.2 \cdot 10^{19}$	0.24	3.1
PbS	710	$2.42 \cdot 10^{13}$	15	$4.8 \cdot 10^{19}$	0.21	4.3
PbTe	4000	$1.33 \cdot 10^{13}$	2.4	$5.3 \cdot 10^{19}$	0.19	6.5
PbSe	6200	$1.54 \cdot 10^{15}$	2	$4 \cdot 10^{19}$	0.25	8.5
GaAs	$9 \cdot 10^{-6}$	$8.1 \cdot 10^{-2}$	-	$1 \cdot 10^{22}$	0.001	0.83-0.91
InSb	2000	$1.6 \cdot 10^{14}$	0.62	$8.3 \cdot 10^{19}$	0.12	5.2

6.4. Semiconductive Materials for Magneto-Electric Conversion

Section Summary: *In this section, the main characteristics of the materials used in magneto-electric conversion are detailed. The Hall effect is described and the properties of materials for Hall sensors also presented.*

Many magneto-electric effects are used for magneto-electric conversion. Typical is Hall effect, described in Fig. 6.8, for two cases: when the material is of n-type semiconductor (Fig. 6.8.a), and when the material is of p-type semiconductor (Fig. 6.8.b).



a) b)

Fig. 6.8. Hall effect model: a) in Si-n semiconductor; b) in Si-p semiconductor

The expression of intensity of induced Hall electric field is obtained with following relations:

A. For Si-n semiconductor

$$\bar{v}_{dn} = -\frac{q_0 <\tau_n>}{m_n^*} \cdot \bar{E} \tag{6.11}$$

$$\bar{J}_n = -nq_0 \bar{v}_{dn} = \frac{q_0^2 n <\tau_n>}{m_n^*} \bar{E} \tag{6.12}$$

$$\bar{F}_{Ln} = -q_0 (\bar{v}_{dn} \times \bar{B}) = \frac{q_0^2 <\tau_n>}{m_n^*} (\bar{E} \times \bar{B}) \tag{6.13}$$

$$-q_0 \bar{E}_{Hn} = \bar{F}_{Ln} \tag{6.14}$$

$$\bar{E}_{Hn} = -\frac{q_0 <\tau_n>}{m_n^*} (\bar{E} \times \bar{B}) \tag{6.15}$$

$$\bar{E}_{Hn} = -\frac{1}{q_0 n} \cdot (\bar{J}_n \times \bar{B}) \tag{6.16}$$

$$R_{Hn} = -\frac{1}{q_0 n} \tag{6.17}$$

B. For Si-n semiconductor

$$\bar{v}_{dp} = +\frac{q_0 <\tau_p>}{m_p^*} \cdot \bar{E} \tag{6.18}$$

$$\bar{J}_p = pq_0 \bar{v}_{dp} = \frac{q_0^2 p <\tau_p>}{m_p^*} \bar{E} \tag{6.19}$$

$$\bar{F}_{Lp} = q_0 (\bar{v}_{dp} \times \bar{B}) = \frac{q_0^2 <\tau_p>}{m_p^*} (\bar{E} \times \bar{B}) \tag{6.20}$$

$$q_0 \bar{E}_{Hp} = \bar{F}_{Lp} \tag{6.21}$$

$$\bar{E}_{Hp} = \frac{q_0 <\tau_p>}{m_p^*} (\bar{E} \times \bar{B}) \tag{6.22}$$

$$\bar{E}_{Hp} = \frac{1}{q_0 n} \cdot \left(\bar{J}_p \times \bar{B}\right) \tag{6.23}$$

$$R_{Hp} = \frac{1}{q_0 p} \quad (6.24)$$

The results of these calculi indicate that Hall constant in n-semiconductor depends on volume concentration n of conduction electrons, and in p-semiconductors, on the volume concentration p of holes.

In a hybrid semiconductor (with n-type and p-type semiconductor), the Hall constant is of form:

$$R_H = \frac{\mu_p - \mu_n}{n_i q_0 (\mu_p + \mu_n)} \quad (6.25)$$

For metals, hall constant can be calculated with the same relation as for n-type semiconductor.

Because volume concentration of conduction electron is higher, the value of R_H is much smaller than in semiconductor materials (Table 6.7).

Table 6.7. Hall constants in metals

Metal	R_H, [m^3/C]	n_o, [m^{-3}]	n_{atoms}, [m^{-3}]
Silver	$-8{,}4 \cdot 10^{-11}$	$0{,}75 \cdot 10^{29}$	$0{,}59 \cdot 10^{29}$
Gold	$-7{,}2 \cdot 10^{-11}$	$0{,}85 \cdot 10^{29}$	$0{,}48 \cdot 10^{29}$
Aluminum	$-3{,}0 \cdot 10^{-11}$	$2{,}1 \cdot 10^{29}$	$0{,}60 \cdot 10^{29}$
Copper	$-5{,}5 \cdot 10^{-11}$	$1{,}1 \cdot 10^{29}$	$0{,}85 \cdot 10^{29}$

In semiconductors, Hall constant is $R_H = 10^{-6} \div \mathbf{10}^{-1}$ m^3/C, for this reason, the semiconductor materials are used in the construction of Hall sensors.

Short Test for Lesson 6

1. Describe literally and graphically, the **physical model of *p-n* junction** formation.
2. Describe literally and graphically, the **energy bands model of *p-n* junction** formation.
3. Describe mathematically, literally and graphically, at least two applications of semiconductive materials.
4. By comparing their properties, what semiconductive materials are a suitable replacement for silicon (Si) in the electronics industry? What is the reasoning for which Si replacement materials are actively being looked for?

7. Dielectrics in Electrical Engineering

Contents

7.1. Dielectrics – Definitions, Characterization, Classification

Section Summary: *In this section, definition, characteristic parameters and classification of dielectrics are presented. Examples of various classes of dielectrics are also given.*

Dielectrics arematerials used either for insulating conductive parts, or for obtaining capacitive elements of the certain components, circuits or systems.
A dielectric is a solid, liquid or gaseous (non-ionized) which is often considered to be a material that does not contain free electric charge.

- An ideal (perfect) insulatorhas no free electric charges ($q_{free}= 0$).
- An ideal dielectric is the dielectric containing only bounded electric charges ($q_{bonded} \neq 0$).
- A real dielectric has $q_{free}\neq 0$ and $q_{bonded} \neq 0$.

The property of obtaining certain capacitive elementsthroughdielectrics is strictly related to the polarization phenomenon.

In Fig. 7.1, the models for electric moments corresponding to the atom with one electron (Fig. 7.1.a) and for the multi-electronic atom (Fig. 7.1.b) are shown.

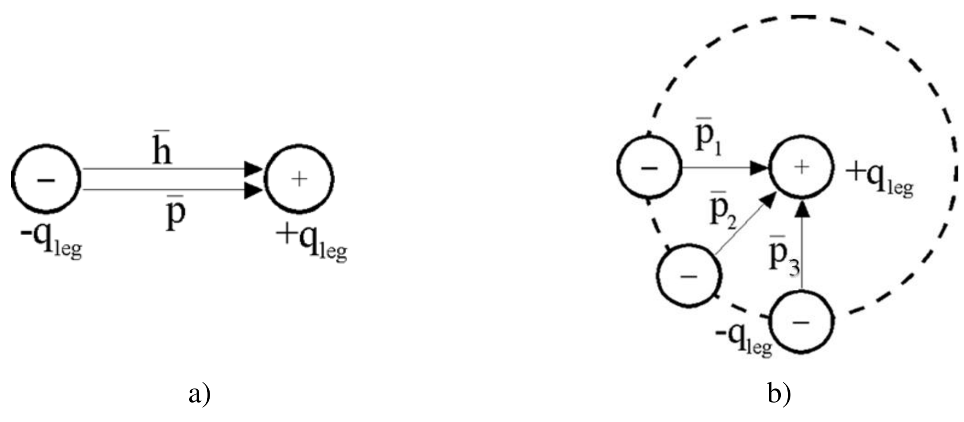


Fig. 7.1, Models for electric moments corresponding to: a) atom with one electron; b) multi-electronic atom

Electric moment of an atom with one electron:

$$\bar{p} = q_b \bar{h} \tag{7.1}$$

Electric moment of a multi-electron atom:

$$\bar{p} = \sum_{i=1}^{N} q_{bi} \bar{h}_i \tag{7.2}$$

Unit of measurmenet for electric moment is C·m (Coulomb x meter).

Electric polarisation, measured in C/m^2:

$$P = \frac{\Delta p}{\Delta V} = n \cdot p \tag{7.3}$$

Connection between electrical induction $\bar{D}$ and electric polarization $\bar{P}$ is given by the relations:

$$\bar{D} = \varepsilon_0 \bar{E} + \bar{P}_t + \bar{P}_p = \varepsilon_0 (1 + \chi_e) \bar{E} = \varepsilon \bar{E} \tag{7.4}$$

The material parameters χ_e and ε have been presented in lecture 2 of this course, where a classification of the dielctrics as also detailed.

7.1.1. Non-polar dielectrics

Non-polar dielectrics are dielectric materialsfor which the structural unit has total electric moment null ($\boldsymbol{p}_{tot}=0$).

In Fig. 7.2 the structures of non-polar molecules of CO_2 molecule (Fig. 7.2.a) and CH_4 methane molecule (Fig. 7.2.b) are shown.

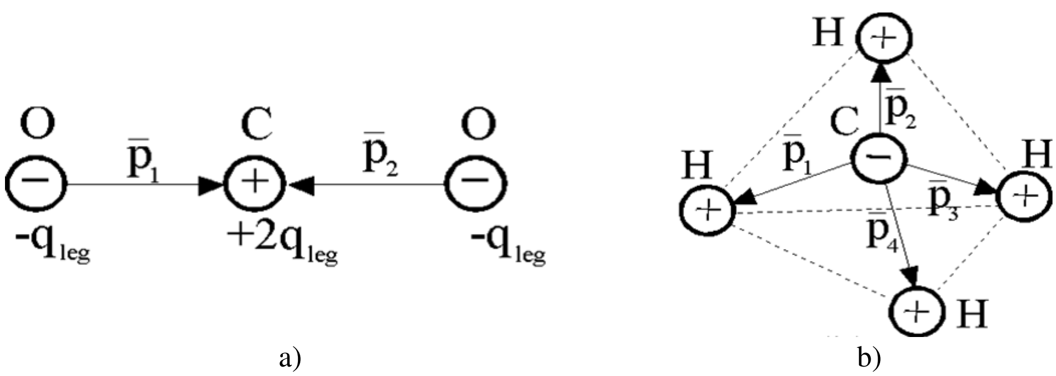


Fig. 7.2. Structures of non-polar molecules: a) CO_2 molecule; b) CH4 methane molecule

Examples of non-polar dielectrics

- Gases: H_2, O_2, N_2, CO_2, CH_4
- Crystals: Si, Ge, C - diamond
- Hydrocarbons; C_6H_6, paraffin, polyethylene, etc.

Characteristic for tthese type of dielectrics is the low values of electric permittivity and losses in alternative electric fields.

7.1.2. Polar dielectrics

Polar dielectrics are dielectric materialsfor which structural unit hastotal electric moment non-null ($\boldsymbol{p}_{tot} \neq 0$).

In Fig. 7.3 the structures of polar molecules of water (H_2O) is shown.

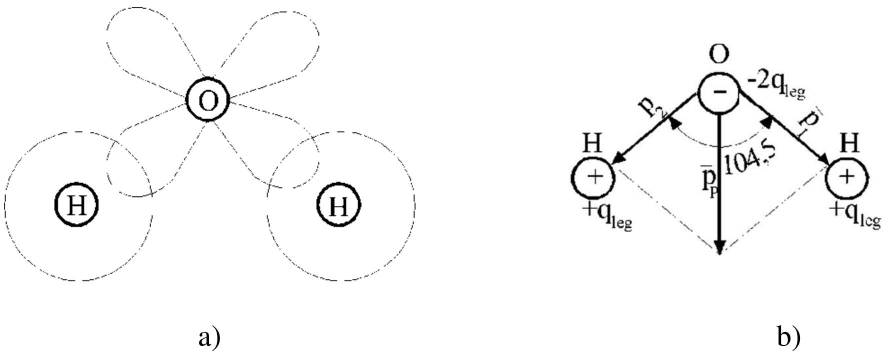


Fig. 7.3. Structures of water polar molecule: a) distribution of electronic clouds between atoms of hydrogen and oxygen; b) permanent electric moment of water molecule

Examples of polar dielectrics are given in Table 7.1.

Table 7.1. Values of permanent electric moment for some polar dielectrics

Molecule	CO	HI	HBr	HCl	NH_3	H_2S	SO_2	H_2O
$\boldsymbol{p_p}$ **[10^{-30}·Cm]**	0,4	1,2	2,6	3,4	5,0	5,3	5,3	6,2

Table 7.2 shows the electrical permittivity values for the classes of dielectrics with applications in electrical engineering.

Table 7.2. Electrical permittivity values for the classes of dielectrics

Types of dielectrics	$\varepsilon_r = 1 + \chi_e$
Non-polar linear dielectrics	1 - 3
Polar linear dielectrics	3 - tens
Non-linear dielectrics (ferroelectrics)	tens - thousands

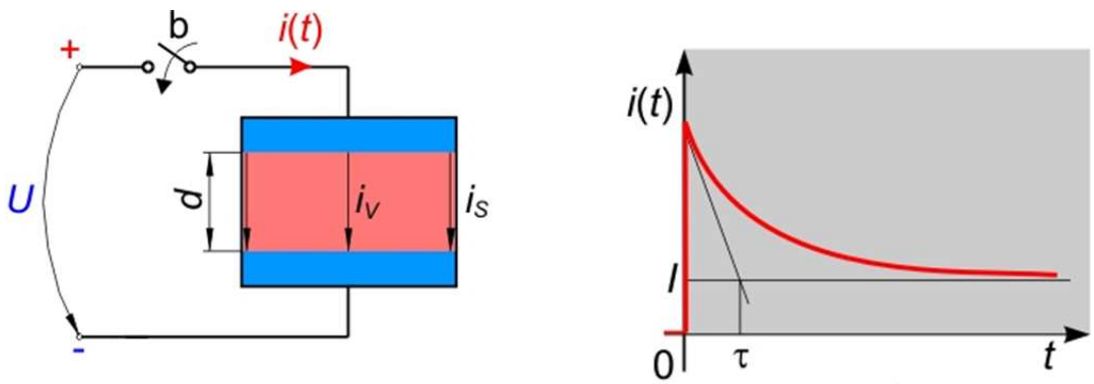
7.2. Particularities of Electrical Conduction in Dielectrics

Section Summary: *In this section, the method of measurement of electric resistivity of dielectrics is detailed, and the particularities of electric conduction in dielectrics are also presented.*

7.2.1. Experimental determination of resistivity in solid dielectrics

Dielectrics are materials with no or with very small quantities of free electric charges. Thus, dielectrics are generally electroinsulating materials, and the methods of determination of electric resistivity in dielectrics are common with those of measurement in electroinsulating materials.

In Fig. 7.4, the scheme and the curve of electric current in the electroinsulating sample supplied in direct current (DC) is shown.



a) b)

Fig. 7.4. DC measurement of electric current in electroinsulating sample: a) Electric schema; b) Electric current curve i=f(t) at supplying the sample.

The curve $i(t)$is obtained by overlapping the following components:

- a charge loading current of capacitor i_{cap}(t), defined by the geometrical capacity C and the resistance R of the circuit, component decreasing in time with the time constant $\tau = RC$;
- a polarization currenti_{pol}(t), decreasing in time,
- an absorption currenti_{abs}(t), decreasing in time, determined by the phenomena of incomplete conduction in dielectric;
- a conduction currenti_{cond}(t), of constant value in time, determined by the displacement of the free electric charge.

$$i(t) = i_{cap}(t) + i_{pol}(t) + i_{abs}(t) + i_{cond}(t) \quad (7.5)$$

After the passing of the transient regime, only conduction current component will pass through the sample, the value of which depends on the value of the sample resistivity.

The higher the resistivity, the lower the conduction current is.

In Fig. 7.3.a, the current components which pass on the sample are indicated:

- Current which pass through the volume of sample (i_V), with permanent value I_V
- Current which pass through the surface of sampel (i_S), with permanet value I_S.

Total current for permanent regime is:

$$I = I_V + I_S \tag{7.6}$$

For dielectric/electroinsulating sample, it is defined the electric conductance G, and, corresponding, the volume conductance G_V and surface conductance G_S, measured in $1/\Omega$.

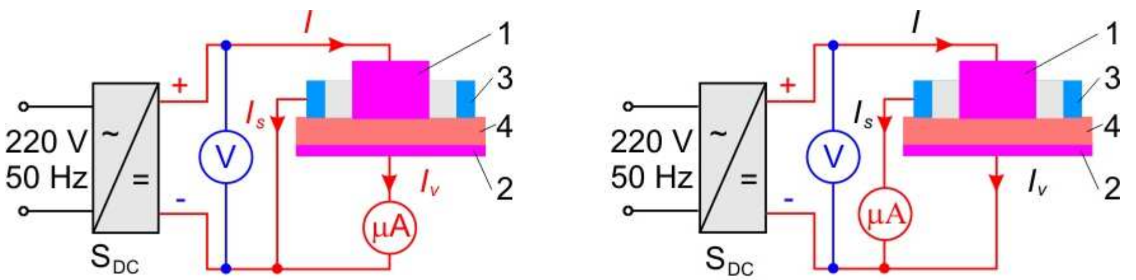
$$G = \frac{I}{U} = G_V + G_S \tag{7.7}$$

It is also defined the electric resistance R, and, corresponding, the volume resistance R_V and surface resistance R_S, measured in Ω.

$$R = \frac{1}{G} = \frac{R_V R_S}{R_V + R_S} \tag{7.8}$$

The sample resistivity has also two components: volume resistivity ρ_V and surface resistivity ρ_S.

In Fig. 7.5, the corresponding measurement schemes are shown.



a) b)

Fig. 7.5.Resistivity measurement schemes with three electrodes: a) for volume resistivity; b) for surface resistivity

The arrangement and dimensions of electrodes and sample for volume resistivity determination are described in Fig. 7.6.

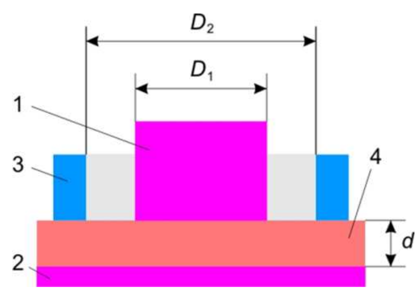


Fig.7.6. Arrangement and dimensions of electrodes and sample for volume resistivity determination

The measurement system with three electrodes includes:

- Cylindrical metallic electrode (1),

- Plate metallic electrode (2),
- Ring metallic electrode (3),
- Insulating /dielectric sample (4).

Volume resistivity ρ_V is determined with the following relationships:

$$R_V = \frac{U}{I_V} \qquad (7.9)$$

$$R_V = \rho_V \cdot \frac{d}{S_{eff}} \qquad (7.10)$$

$$\rho_V = \frac{R_V \cdot S_{eff}}{d} \qquad (7.11)$$

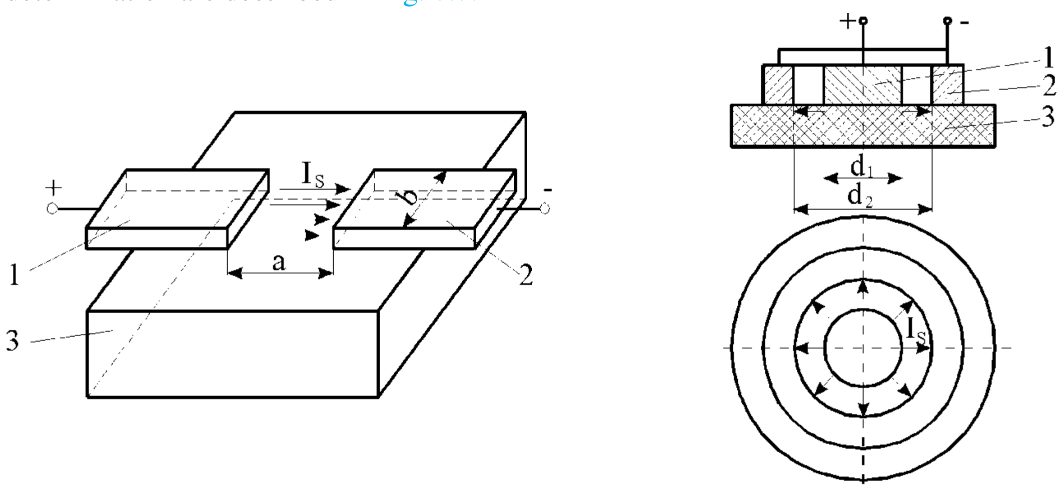
Where:

$$S_{eff} = \frac{\pi \cdot D_{av}^2}{4} \qquad (7.12)$$

$$D_{av} = \frac{D_1 + D_2}{2} \qquad (7.13)$$

Volume resistivity of electroinsulating materials/ real dielectrics / is about $(10^8 \div 10^{18})$ Ωm.

The arrangement and dimensions of electrodes and sample for volume resistivity determination are described in Fig. 7.7.



a) b)

Fig. 7.7. Arrangement and dimensions of electrodes and sample for surface resistivity determination: a) scheme with plan electrodes; b) scheme with cylindrical electrodes.

The scheme in Fig. 7.7.a includes: plan electrodes (1, 2) and sample (3).

The scheme in Fig. 7.7.b includes:

- Cylindrical metallic electrode (1),
- Ring metallic electrode (2),
- Insulating sample (3).
- Inferior plate electrode (is not shown),

Surface resistivity $\rho_S$is determined with the following relationships:

$$R_S = \frac{U}{I_S} \tag{7.14}$$

$$R_S = \rho_S \cdot \frac{a}{b} \tag{7.15}$$

$$\rho_S = \frac{R_S \cdot \pi \cdot D_{av}}{(D_2 - D_1)/2} \tag{7.16}$$

Surface resistivity of dielectrics is about ($10^6 \div 10^{16}$) Ω.

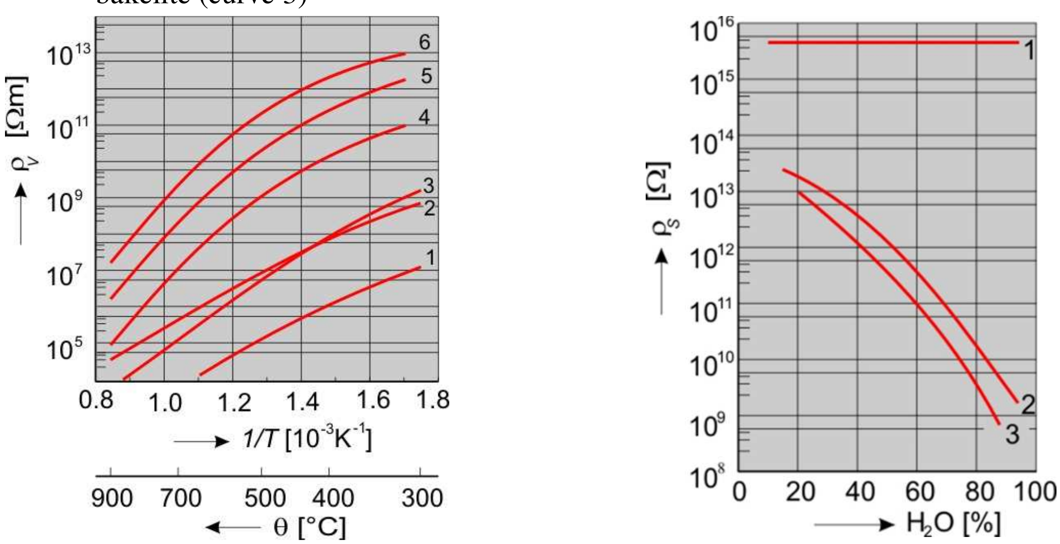
In Fig. 7.8.a, dependence of volume resistivity on temperature is shown, for different dielectric ceramics:

- mulite - SiO2 Al2O3 BaO (curve *1*),
- Al_2O_3 96% (curve *2*),
- steatiteAl_2O_3MgO (*c*urve *3*),
- Al_2O_3 (curve *4*),
- Al_2O_3 99,5% (curve *5*),
- BeO 99,5% (curve *6*).

Note: In insulation materials /real dielectrics/ the volume resistivity decrease with increasing of the temperature.

In Fig. 7.8.b, dependence of surface resistivity on humidity is shown, for different dielectrics:

- ceresine (curve 1)
- alkaline glass (curve 2)
- bakelite (curve 3)



a) b)

Fig. 7.8. Volume resistivity dependence on temperature (a) and variation of surface resistivity on humidity (b) in different insulating materials /real dielectrics

Surface resistivity characterizes the conduction phenomenon through the surface layers of material and depends on the dielectric nature and external factors: humidity, contamination with different substances, oxidation degree, etc.

7.2.2. Electrical conduction mechanisms in dielectrics

In dielectrics, three types of electrc conduction processes are established when an electric field is applied:

- Ionic or electrolytic conduction, characterized by ionic conductivity $\sigma_{ion,}$
- Electronic conduction, determined by free electronic charges and corresponding electronic conductivity σ_e
- Mol-ionic or electrophoresis conduction, caracterised by mol-ionic conductivity σ_{mol}

Total electric conductivity will be:

$$\sigma_{tot} = \sigma_{ion} + \sigma_e + \sigma_{mol} \tag{7.17}$$

In dielectric crystals, electric conduction processes are mixed, the charge carriers being mainly electrons and ions.

Characteristic for dielectrics is the strong dependence of the temperature of total conductivity.

Assuming the analogy insulator-semiconductor, for the electronic conductivity, in an insulating crystal, the relation of conductivity can be described as:

$$\sigma_e = C_i \cdot e^{\left[-\frac{\Delta W_i}{2kT}\right]} + C_d \cdot e^{\left[-\frac{\Delta W_d}{2kT}\right]} + C_a \cdot e^{\left[-\frac{\Delta W_a}{2kT}\right]} \tag{7.18}$$

where:

C_i stands for the constant dependent on the intrinsic conduction mechanism (transition of electrons from BV to BC).

C_d is the constant dependent on the state number of donor atoms, which supply electrons in BC, the activation energy being ΔW_d.

C_a is the constant, which characterizes the transitions of electrons from BV on the acceptor levels of impurity atoms, with the activation energy ΔW_a.

With increasing the temperature, the total conductivity in real dielectrics increases exponentially.

Characteristic for dielectrics is the strong dependence of the total conductivity on intensity of electric field applied on dielectrics.

In Fig. 7.9.a, the general dependence of electric conductivity of dielectrics on the electric field intensity *E* is shown, and in Fig. 7.9.b is given this dependence for epoxy resins (RE) and poly-tetra-flour-ethylene (PTFE) in the electric field (0-4) MV/m.

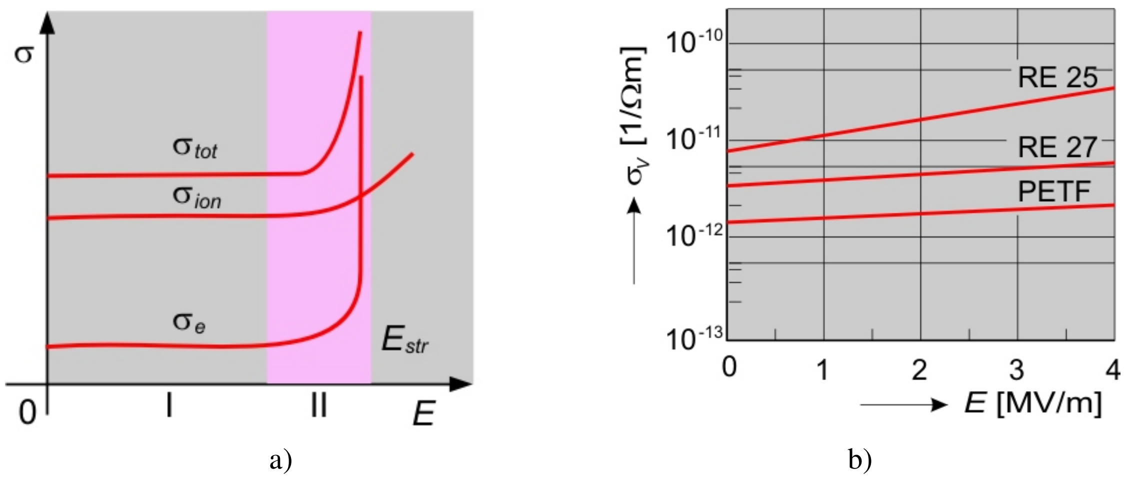


Fig. 7.9. Dielectric conductivity dependence on electric field intensity: a) general dependence; b) dependence for epoxy resins (RE) and poly-tetra-flour-ethylene (PTFE)

General dependence (Fig. 7.9.a) has two regions:

- Ohm region (noted I), in which σ=constant**,** conductivity does not varies with electric field intensity
- Poole region (noted II), in which σ=f(E), where a strong dependency of electric field appears.

At values of electric fields of about 10^4 MV/m all dielectrics loss their electroinsulating properties: electrical breakdown is developing.

7.3. Electrical Breakdown in Dielectrics

Section Summary: *In this section, the processes of electric breakdown in dielectrics are described. Also, the electrical breakdown prevention principles of design are presented for avoiding any possible flashover and breakdown.*

Electrical breakdown corresponds to loss of electroinsulating property of a dielectric material.

Quantities which characterise the electric breakdown process are:

- Electrical breakdown voltageU_{str}, defined as the maximum voltage for which the for which the dielectric still does not break.
- Dielectric strength or breakdown strengthE_{str}, defined as the maximum intensity of electric field for which the dielectric still does not break.

In homogenious electric field, the connection between these quaantities is:

$$E_{str} = \frac{U_{str}}{d} \qquad (7.19)$$

where$U_{str}$is measured in kV, or MV and E_{str}in kV/mm or MV/m.

Electrical breakdown is determined through different forces and interactions, that act on the dielectric and cause the loss of electrical insulating properties. Thus, there are different types of electrical breakdown:

- electro-mechanic breakdown, when the main factor of breakdown is the mecanical forces,
- electro-thermal, when the main factor of breakdown is incresing the temperature
- electrochemical, when chemical interactions could determine the breakdown.

A special case is appearance of partial discharges in some dielectrics, as paper or plastics, under the action of high voltage electric field.

In Fig. 7.10, an inception of partial discharges in the insulation of a high voltage cable is shown.

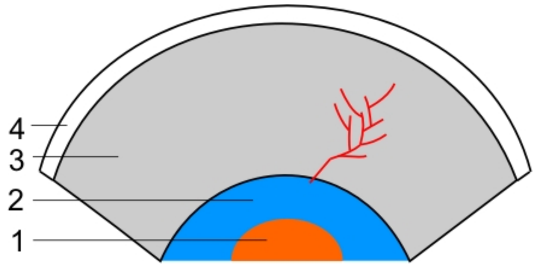


Fig. 7.10. Electrical tree developed in high voltage cable insulation:, in which: 1-conductor core; 2-semiconductive material; 3-electroinsulating material; 4-screen

The electric arborescence begins at the surface of conductor core in dielectric, and in time is developed until it reaches the metallic screen, when the insulation breaks completely.

Electrical breakdown in gases is specific. Voltage breakdown depend on preassure$p$ and distance between electrodes d (Fig. 7.11).

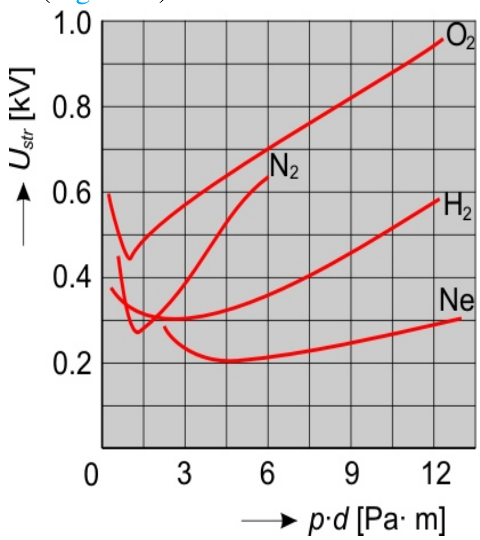


Fig. 7.11.Paschen curves for: neon, hydrogen, nitrogen and oxygen

Gases lose their electrical insulating properties at kV units of order values for a distance between the electrodes of 1 mm.

Example, for air Estr = 3.2 kV/mm, in normal conditions.

7.3.1. Experimental testing to electrical breakdown

Dielectrics are tested to breakdown with different methods and procedures. The methods of breakdown testing are classified with different criteria:

A. According to the type of the applied voltage wave, there are:

- Tests with continuous voltage (tests specific for cables),
- Tests with alternative voltage, at the frequency of 50 Hz,
- Tests with voltage pulse.

B. According to the time criterion, there are:

- Tests with shortor long-term voltage, where the breakdown of thermal type is predominant,
- Tests with sudden voltage application (voltage pulse), when the pure electrical breakdown is predominant.

InFig. 7.12, the scheme of breakdown testing the solid dielectrics with the test of short or long term voltage application is shown.

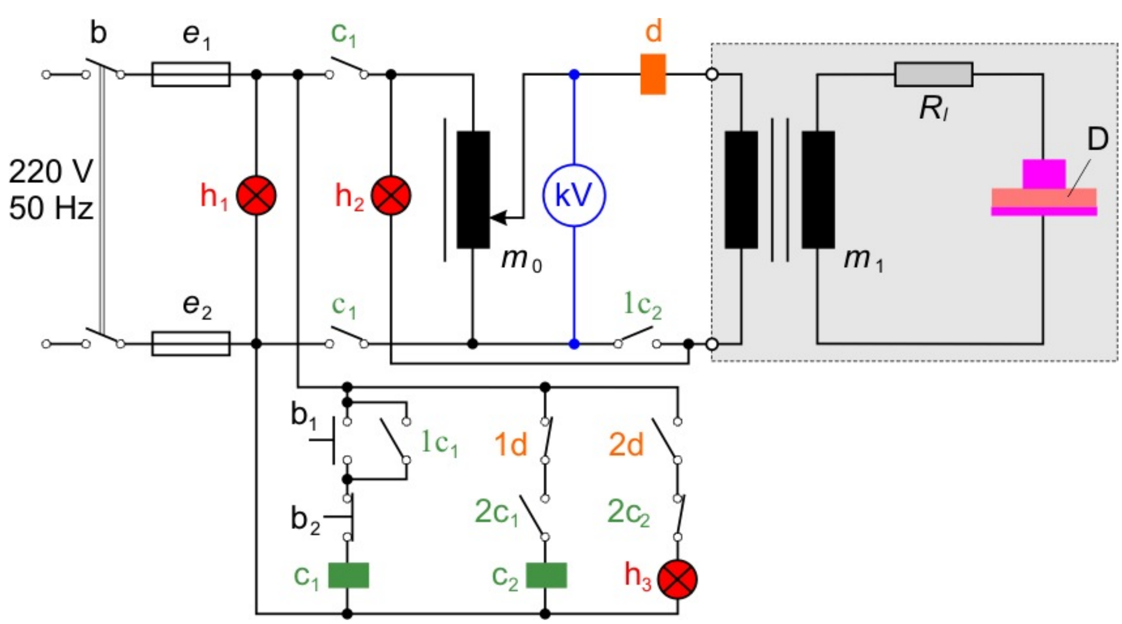


Fig. 7.12. Electrical breakdown testing with AC voltage

The significance of the devices used in Fig. 7.12 is:

Devices	**Symbol in diagram**
Single pole switch	*b*
AC contactors	c_1, c_2
Over-current relay	*d*
High voltage signal lamp	h_1
High voltage signal lamp	h_2

Breakdown signal lamp	h_3
Autotransformer	m_0
High voltage transformer	m_1
Resistor for breakdown current limiting	R_l
AC Voltmeter	V
High voltage cell with metallic enclosure and connected	D

Electrical breakdown prevention principles are used in designing electroinsulating systems:

(a) Avoiding the sharp edges and extremities
(b) The adequate setting of distance among components
(c) Avoiding the accumulation of electric charge
(d) Isolations and enclosing
(e) The proper ventilation
(f) The avoidance of the accidental voltages.

Short Test for Lesson 7

1. What are the differences between the notion of dielectric and electroinsulating material?
2. Define (mathematically and literary) the electric moment and the electric polarization.
3. Give examples and describe the structuree of a non-polar and a polar molecule.
4. What is the electrical breakdown phenomena?
5. Mention three prevention principles against the electrical breakdown of dielectrics.
6. Which factors influence the electrical breakdown strength? Which is the value of electrical breakdown for air?

8. Dielectric Polarization in Constant Electric Fields

Contents

8.1. Polarization Approach using the Microscopic Theory

Section Summary: *In this section, the microscopic theory of polarization is presented, as basis for justification of the polarization processes which are developed when a constant or variable electric field is applied. The general expressions of electric susceptivity and relative permittivity are obtained, and a classification of the polarization states is done.*

Electric polarization *P* is a macroscopic quantity which characterizes the polarization state of a dielectric. This quantity is obtained by temporal and spatial averaging of microscopic polarization $\boldsymbol{P}_{\mathrm{micro}}$:

$$\bar{P} = [\bar{P}_{micro}]_{med\ s,t} \tag{8.1}$$

Experimentally, it was deduced that the application of an electric field of intensity $\boldsymbol{E}$ induces an electric moment $\boldsymbol{p}_{\mathrm{e}}$ in each structural units of the dielectric. The contribution of all electric moments, having a volume concentration *n*, determines the electric polarization characterized by the quantity $\boldsymbol{P}$:

$$\bar{P} = n\bar{p}_e \tag{8.2}$$

Also experimentally, it was deduced that the magnitude of the electric moment $\boldsymbol{p}_{\mathrm{e}}$ depends on the active electric field of intensity $\boldsymbol{E}_0$, applied to the each structural unit.

$$\bar{p}_e = \alpha_e \bar{E}_0 \tag{8.3}$$

where$\boldsymbol{E}_0$ is determined with the relation:

$$\bar{E}_0 = \bar{E} + \frac{\gamma}{\varepsilon_0}\bar{P} \tag{8.4}$$

Combining these relations, an expression of electric polarization is obtained:

$$\bar{P} = \frac{n\alpha_e}{1-\frac{\gamma n\alpha_e}{\varepsilon_0}}\bar{E} \tag{8.5}$$

Taking into account the temporary polarization law:

$$\bar{P} = \varepsilon_0 \chi_e \bar{E} \tag{8.6}$$

The following expressions are obtained:

- for electric susceptivity:

$$\chi_e = \frac{\frac{n\alpha_e}{\varepsilon_0}}{1-\frac{\gamma n\alpha_e}{\varepsilon_0}} \quad (8.7)$$

- for the relative electric permittivity:

$$\varepsilon_r = 1 + \frac{\frac{n\alpha_e}{\varepsilon_0}}{1-\frac{\gamma n\alpha_e}{\varepsilon_0}} \quad (8.8)$$

Significance of each parameter and its units of measure are noted here:
$\boldsymbol{p}_e$ - electrical moment of the structural unit, in C·m,
n=N/V - volume concentration of structural units, in number of units on m^3,
α_e – polarizability
$\boldsymbol{E}$ – applied electric field intensity, in V/m,
E_0 – active electric field intensity, in V/m,
γ – structural coefficient,
χ_e – electric susceptibility,
ε_r– relative permittivity.

Note: material parameters χ_e and ε_r have different values, depending on the polarization mechanism and influencing factors involved in polarization processes.

The temporary polarizationstateis characterized by appearance of the polarization processes when an electric field is applied, and after canceling the electric field, the dielectric returns to its initial state, without polarization phenomena (after passing the transient regime).

The temporary polarization includes: displacement polarization (electronic polarization and ionic polarization) and orientation polarization (in the case of polar dielectrics).
The displacement polarization is determined by the displacement from the rest position of the bound electric charges under the action of the electric field and the establishment of the electronic electric moment (electronic polarization) or the ionic electric moment (ionic polarization).

In the case of polar dielectrics (which have permanent electric moments $\mathbf{p}_p$), when a constant electric field is applied, the permanent electric moments tend to orient themselves according to the direction of the applied electric field (orientation polarization).

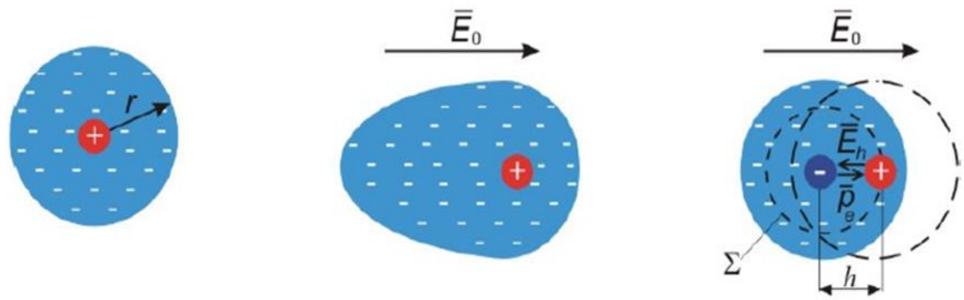
The permanent polarization state is characterized by appearance of the polarization processes when an electric field is applied, and after canceling the electric field, the dielectric maintains its state of polarization.
The permanent polarization is specific for some dielectrics through:

- Spontaneous polarization (of thermal origin),
- Piezoelectric polarization (of mechanical origin).

8.2. Electronic Polarization in Dielectrics

Section Summary: *In this section, the mechanism of electronic polarization developed when a constant electric field is applied to a dielectric is described. The expressions of electric susceptivity and relative permittivity are obtained, and some examples are given.*

The electronic polarization stateoccurs whenever an electric field is applied (Fig. 8.1). A displacementof the negative electric charges (electronic clouds) from the center of the positive charges (nucleus) of each dielectric's atom appears which leads to the establishing of dipolar electric moments, dependent on the applied electric field, as well as on the dielectric's nature.



a) b) c)

Fig. 8.1. Model of electronic polarization: a) a multi-electron atom in absence of electric field; b) deformation of the electronic cloud under the action of active electric field of intensity $\mathbf{E}_o$; c) model for establishing the electric moment $\boldsymbol{p}_e$.

In Fig. 8.1.c, the direction of active electric field applied on the multi-electron atom is shown, and also the direction of electric moment $\boldsymbol{p}_e$, given by the electric charges included in sphere of surface Σ, which is displaced with distance *h* on the center of nucleus (positive charges).

With this model, the electronic polarizability is obtained:

$$\alpha_e = 4\pi\varepsilon_0 r^3 \tag{8.9}$$

Using general expression of relative permittivity, the expression of relative permittivity in the case of electronic polarization is obtained:

$$\varepsilon_e = 1 + \frac{4\pi\varepsilon_0 r^3}{1-4\pi\varepsilon_0 r^3} \tag{8.10}$$

In these relations, the significance of parameters is:

- *n* - concentration of atoms, in number of atoms/m^3
- r - radius of atom, in m
- γ - structural coefficient

The structural coefficient has different values in function of type of dielectric structures:
Examples:
γ ≈ 0 for gases,
γ ≈ 1/3 for BCC (CVC) crystals

Notes:

1. For gases, the expression of relative permittivity corresponding to the electronic polarization, is:

$$\varepsilon_{r\ gasses} = 1 + 4\pi\varepsilon_0 r^3 \quad (8.11)$$

Thus, the values of relative permittivity for gases are of order:

$$\varepsilon_{r\ gasses} \approx 1 + 4\pi \cdot 6.02 \cdot 10^{26} \cdot 10^{-30} \approx 1 + 10^{-3} \quad (8.12)$$

2. The electronic polarization appears in all dielectrics when a constant or variable electric field is applied to the dielectric. However, the relative permittivity value remains low compared to the case of other polarization mechanisms.
3. The calculation of electronic polarizability requires the knowledge and application of laws from the theory of electromagnetism.

   For the calculation of electric moment $\boldsymbol{p}_e$ -, it is considered that the electronic cloud is a rigid sphere with the charge - Zq_0, displaced with the distance h towards the nucleus center.

Electric moment $\boldsymbol{p}_e$ is:

$$\bar{p}_e = Zq_0 \cdot \bar{h} \quad (8.13)$$

According to electric flux law for a sphere with radius h:

$$\int_\Sigma \bar{D} \cdot d\bar{S} = q_\Sigma \quad (8.14)$$

$$\int_\Sigma \varepsilon_0 \bar{E}_h \cdot d\bar{S} = \int_V \rho_V \cdot dV \quad (8.15)$$

The volume density of electric charges ρ_V is expressed by:

$$\rho_v = \frac{-Zq_0}{\frac{4\pi r^3}{3}} \quad (8.16)$$

Where the volume of sphere is:

$$V_{sphere} = \frac{4\pi r^3}{3} \quad (8.17)$$

With these relations it is obtained:

$$\int_\Sigma \varepsilon_0 E_h \cdot dS = -\int_V \frac{-Zq_0}{\frac{4\pi r^3}{3}} \cdot dV \quad (8.18)$$

The surface area Σ of sphere is:

$$S = 4\pi h^2 \quad (8.19)$$

And

$$\varepsilon_0 E_h 4\pi h^2 = \frac{Zq_0}{\frac{4\pi r^3}{3}} \cdot \frac{4\pi h^3}{3} \quad (8.20)$$

With the relations:

$$\varepsilon_0 E_h 4\pi r^3 = Zq_0 \cdot h \quad (8.21)$$

$$p_e = Zq_0 \cdot h \tag{8.22}$$

It results:

$$p_e = 4\pi r^3 \varepsilon_0 E_h \tag{8.23}$$

and electric polarizability:

$$\alpha_e = 4\pi\varepsilon_0 r^3 \tag{8.24}$$

Electric polarizability α_e dependents on atomic radius.
Example, for hidrogen atom ($r = 10^{-10}$ m)

$$\alpha_e = 4\pi \cdot \frac{1}{4\pi \cdot 9 \cdot 10^9} \cdot 10^{-30} \cong 10^{-40} F \cdot m^2 \tag{8.25}$$

In Table 8.1 the electronic polarizability for some gases are given.

Table 8.1. Electronic polarizability for gases

Atom	H	He	Ne	Ar	C	Li	Na	K
α_e [10^{-41}F·m^2]	7,34	2,34	4,45	17,80	16,68	134,4	300,4	378,2

For nonpolar and poorly polar, dielectric permittivity is determined by electronic susceptivity components.

For rare gas atoms, polarizability has lower values compared to alkaline metals. Polarizability is influenced in the case of alkaline metals (Li, Na, K) by valence electrons, which are poorly bounded.

Electronic polarisability is established in a short time, in 10^{-14}-10^{-15} seconds.
Returning to the old equilibrium state is done same quickly.

8.3. Ionic Polarization in Dielectrics

Section Summary: *In this section, the mechanism of ionic polarization developed when a constant electric field is applied to a dielectric is described. The expressions of electric susceptivity and relative permittivity due to ionic type of polarization are obtained, and some examples are given.*

The ionic polarization stateoccurs whenever an electric field is applied in dielectrics where the bonds are ionic in nature. The process consists in the ions' displacement from the initial position in the presence of the applied electric field, which determines the appearance of supplementary electric moments.
In Fig. 8.2, a model of ionic polarization is described, in the case of the distribution of ions in one-dimensional crystal, in the absence of the electric field, and in the presence of the electric field.

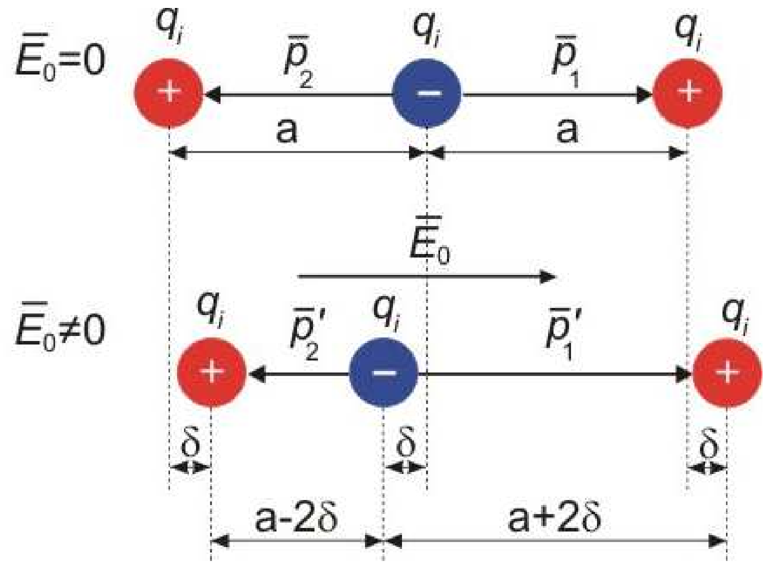


Fig. 8.2. Model of ionic polarization in the case of one-dimensional crystal: in the absence of the electric field ($\boldsymbol{E}_o = 0$), and in the presence of the electric field ($\boldsymbol{E}_o \neq 0$)

With this model, the ionic polarizability is obtained:

$$\alpha_i = 8\pi\varepsilon_0 a^3 \tag{8.26}$$

By utilizing the general expression of susceptibility and relative permittivity, in the case of ionic polarization we obtain:

$$\chi_e = \frac{8\pi n \cdot a^3}{1 - 8\pi n \cdot a^3 \gamma} \tag{8.27}$$

and

$$\varepsilon_r = 1 + \chi_e = 1 + \frac{8\pi n \cdot a^3}{1 - 8\pi n \cdot a^3 \gamma} \tag{8.28}$$

In these relations, the significance of parameters is:

- n - concentration of atoms, in number of atoms/m^3
- a - distance between atoms, numerically equal to the crystal lattice constant, in m
- γ - structural coefficient.

Note: The calculation of ionic polarizability requires the knowledge and application of laws from the theory of electromagnetism.

For evaluated the electric moment induced by electric field applied to the dielectric, the distribution of ions in one-dimensional crystalline lattice of ionic crystal is considered (Fig. 8.3).

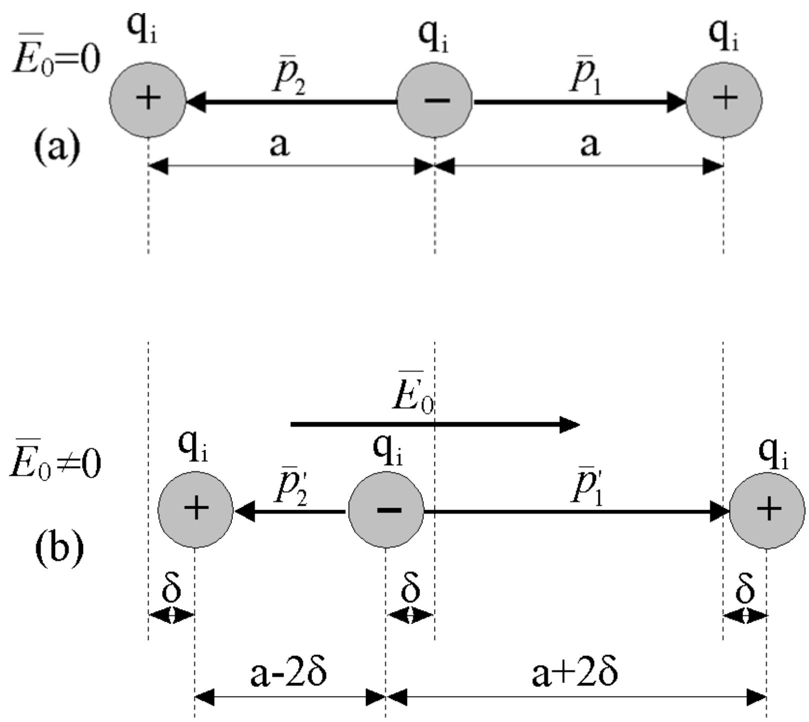


Fig. 8.3. Distribution of ions in one-dimensional crystalline lattice of ionic crystal: a) in absence of electric field; b) in presence of electric field

At $\boldsymbol{E}_o$=0, electric moments of the structural cell (Fig. 8.3.a) are equal and with opposite sens:

$$|\overline{p_1}| = |\overline{p_2}| = q_i \cdot a|\overline{p_1}| \tag{8.29}$$

Thus, the ionic electric moment for this structural unit is null:

$$p_i = 0 \tag{8.30}$$

When an electric field is applied ($\boldsymbol{E}_0$≠0), the ions are displaced (positive ones in the sens of electric field and negative ones in opposite sens) on the distance δ, resulting an electric induced moment on the applied electric field direction (Fig. 9.3.b):

$$p_i = p_1' - p_2' = q_i(a + 2\delta) - q_i(a - 2\delta) = q_i 4\delta \tag{8.31}$$

At equilibrium, the energy required to displace the ions is equalized by the electric energy:

$$W_{depl} = W_{el} \tag{8.32}$$

Where the energy of dipole displacement in electric field is:

$$W_{depl} = \frac{-q_i^2}{4\pi\varepsilon_0 \cdot (a-2\delta)} + \frac{-q_i^2}{4\pi\varepsilon_0 \cdot (a+2\delta)} - \frac{-2q_i^2}{4\pi\varepsilon_0 \cdot a} \tag{8.33}$$

And electric energy of the electric dipole in lectric field is:

$$W_{el} = -p_i E_0 \tag{8.34}$$

With these relations we obtain:

$$-p_i E_0 = \frac{-q_i^2}{4\pi\varepsilon_0 \cdot (a-2\delta)} + \frac{-q_i^2}{4\pi\varepsilon_0 \cdot (a+2\delta)} - \frac{-2q_i^2}{4\pi\varepsilon_0 \cdot a} \tag{8.35}$$

Or

$$-p_i E_0 = \frac{-q_i^2}{4\pi\varepsilon_0}\left(-\frac{1}{a-2\delta} - \frac{1}{a+2\delta} - \frac{-2}{a}\right) \tag{8.36}$$

Or

$$-p_i E_0 = \frac{-q_i^2}{4\pi\varepsilon_0} \cdot \frac{8\delta^2}{a(a^2-4\delta^2)} \tag{8.37}$$

for $a \gg 4\delta$

It is obtained:

$$p_i = 8\pi\varepsilon_0 a^3 \cdot E_0 \tag{8.38}$$

Thus, the electric polarizability for ionic crystals is:

$$\alpha_i = 8\pi\varepsilon_0 a^3 \tag{8.39}$$

Polarizability depends on lattice constant of ionic crystal.
Ionic crystals have permitivity values of order 7÷8 to tens of units.
Ionic polarisability is established in a longer time of $10^{-12} \div 10^{-13}$ s in comparison with electronic polarisability.

8.4. Orientation Polarization in Dielectrics and General Case

Section Summary: *In this section, the mechanism of orientation polarization developed when a constant electric field is applied to a dielectric is described. The expressions of electric susceptivity and relative permittivity due to orientation polarization in polar dielectrics are obtained, and some examples are given. The general case when all types of polarization are produced in also analyzed.*

The dipolar or orientation polarization occurs in the dielectrics that provide permanent electric moments $\boldsymbol{p}_p$. The polar dielectrics hold two factors that determine the dipoles' orientation: the local electric field of intensity $\boldsymbol{E}_o$and thermal movement due to temperature T.In Fig. 8.4, the model of orientation polarization in polar dielectrics are shown, in the absence of the electric field, and in the presence of the electric field.

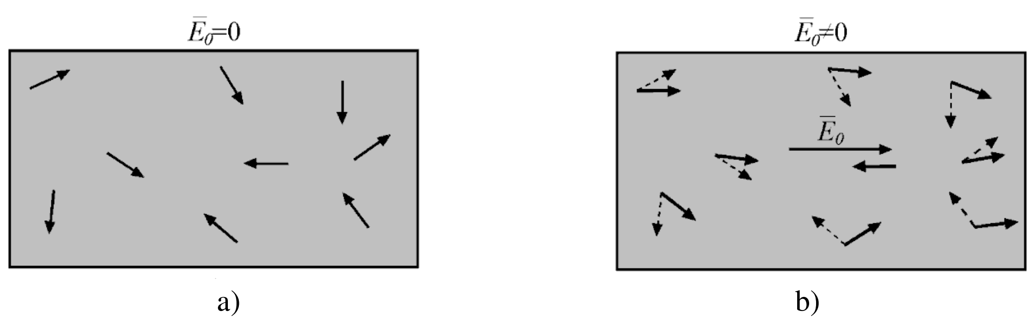


Fig. 8.4. Model of orientation polarization in polar dielectrics: a) in the absence of the electric field; b) in the presence of the electric field

The electric polarizability in this case will be:

$$\alpha_0 = \frac{p_p^2}{3kT} \tag{8.40}$$

The polarizability α_o depends on the magnitude of the permanent electric moment p_p and the temperature *T*.

Considering the general relation for susceptivity and relative permittivity, it results:

$$\chi_e = \frac{\frac{np_p^2}{3k\varepsilon_0 T}}{1-\gamma\frac{np_p^2}{3k\varepsilon_0 T}} \tag{8.41}$$

$$\varepsilon_r = 1 + \chi_e = 1 + \frac{\frac{np_p^2}{3k\varepsilon_0 T}}{1-\gamma\frac{np_p^2}{3k\varepsilon_0 T}} \tag{8.42}$$

Water is a polar dielectric. The permittivity of water on the temperature is shown in Fig. 8.5.

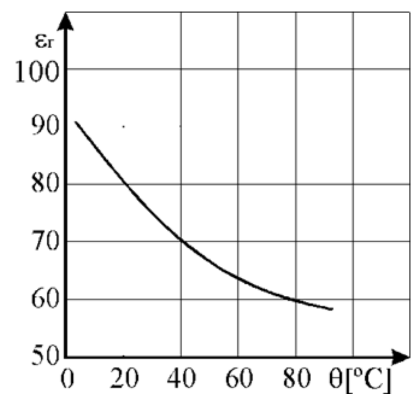


Fig. 8.5. Dependence of water permittivity on temperature

Note: In the case of dielectric with co-existence of electronic, ionic and orientation polarization, the material parameters are sumarized, as:

- Total polarization:

$$\bar{P}_{tot} = \bar{P}_e + \bar{P}_{ion} + \bar{P}_{or} \tag{8.43}$$

Or

$$\bar{P}_{tot} = n_e\alpha_e\bar{E}_0 + n_i\alpha_i\bar{E}_0 + n_0\alpha_0\bar{E}_o \tag{8.44}$$

Or

$$\bar{P}_{tot} = \sum n_k\alpha_k\bar{E}_0 \tag{8.45}$$

Taking into account the expression of active electric field intensity E_0:

$$\bar{E}_0 = \bar{E} + \frac{\gamma}{\varepsilon_0}\bar{P} \tag{8.46}$$

Where E is intensity of applied electricfield, and P is the total polarization, it results:

$$\bar{P}_{tot} \cdot \left(1 - \frac{\gamma}{\varepsilon_0}\sum n_k\alpha_k\right) = \left(\sum n_k\alpha_k\right)\bar{E} \tag{8.47}$$

Or

$$\bar{P}_{tot} = \frac{\sum n_k \alpha_k}{1 - \frac{\gamma}{\varepsilon_0} \sum n_k \alpha_k} \bar{E} \tag{8.48}$$

From this relation it results the expression of total electric susceptivity:

$$\chi_{tot} = \frac{\frac{\sum n_k \alpha_k}{\varepsilon_0}}{1 - \frac{\gamma}{\varepsilon_0} \sum n_k \alpha_k} \tag{8.49}$$

Thus, the expression of total electric permittivity will be:

$$\varepsilon_r = 1 + \chi_{tot} = 1 + \frac{\frac{\sum n_k \alpha_k}{\varepsilon_0}}{1 - \frac{\gamma}{\varepsilon_0} \sum n_k \alpha_k} \tag{8.50}$$

Short Test for Lesson 8

1. Describe the electric polarization model in the general case according to the microscopic theory.
2. Describe the electronic polarization model (literary, mathematically and schematic description).
3. Describe the ionic polarization model (literary, mathematically and schematic description).
4. Describe the orientation polarization model (literary, mathematically and schematic description).
5. Define the mixed polarization model (literary and mathematically) in the case of a dielectric with co-existence of electronic, ionic and orientation polarization mechanisms.

9. Dielectric Polarization in Harmonic Electric Fields

Contents

9.1. Complex Permittivity of Dielectrics

Section Summary: *In this section, the complex permittivity is defined, together with the components. The significance of these material parameters is shown.*

To characterize the polarization phenomena in harmonic (sinusoidal) regime, the term complex permittivity is introduced.

When an harmonic intensity field is applied to a dielectric, the intensity of electric field $E(t)$ and electric induction $D(t)$ vary in time as:

$$E(t) = \sqrt{2}E_{eff}\sin(\omega t) \quad (9.1)$$

$$D(t) = \sqrt{2}D_{eff}\sin(\omega t - \delta_h) \quad (9.2)$$

whereE_{ef} and D_{ef} are the effective values of electric field intensity and, respectively, effective value of electric induction, ω= 2πf is pulsation of electric field, and δ_h is delay angle between $E(t)$ and $D(t)$.

So, the electric induction will be dephased with the angle δ.

The complex permittivityis numerically equal to the ratio between the electric induction phasor $\underline{D}$ and the electric field intensity phasor $\underline{E}$:

$$\underline{\varepsilon}_r = \frac{1}{\varepsilon_0}\cdot\frac{\underline{D}}{\underline{E}} = \frac{1}{\varepsilon_0}\cdot\frac{D_{eff}}{E_{eff}}e^{-j\delta_h} = \frac{1}{\varepsilon_0}\cdot\frac{D_{eff}}{E_{eff}}(\cos\delta_h - j\sin\delta_h) \quad (9.3)$$

The complex permittivity has two terms, correlated with the angle losses tangent, tgδh:

$$\underline{\varepsilon}_r = \varepsilon_r' - j\varepsilon_r'' \quad (9.4)$$

With the components:

$$\varepsilon_r'' = Im\{\underline{\varepsilon}_r\} = \frac{1}{\varepsilon_0}\cdot\frac{D_{eff}}{E_{eff}}\sin\delta_h \quad (9.5)$$

$$\varepsilon_r' = Re\{\underline{\varepsilon}_r\} = \frac{1}{\varepsilon_0}\cdot\frac{D_{eff}}{E_{eff}}\cos\delta_h \quad (9.6)$$

The report of these components is tangent of δ_h

$$\tan\delta_h = \frac{\varepsilon_r''}{\varepsilon_r'} \quad (9.7)$$

Significance of tangent of dielectric losses angle is shows in Fig. 9.1.

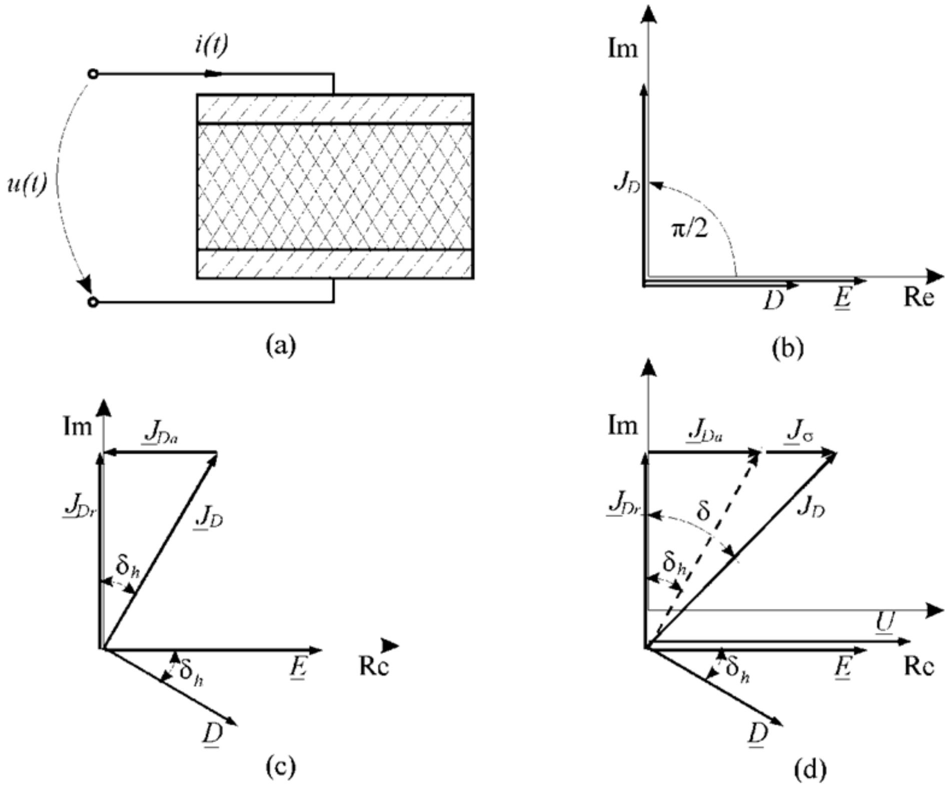


Fig. 9.1. Dielectric in harmonic electric field: a) dielectric supplied in sinusoidal field; b) phasor diagram of an ideal dielectric (without losses); c) phasor diagram of a dielectric with losses (due to polarisation); d) phasor diagram of a real dielectric with losses (through polarisation and conduction processes

In Fig. 9.1.b, the current in dielectric is diphase with π/2 in advance of *E* and *D*.

When a viscosity effect is present, the density of displacement current is diphase with an angle $\delta_h < \pi/2$ (Fig. 9.c).

For a real dielectric, in which viscosity effect exists and also free electric charges exists too, the diphase angle is higher $\delta >> \delta_h$ (Fig. 9.d).

The following relations are deduced:

$$\underline{\varepsilon}_r = j\omega \underline{D} = j\omega\varepsilon_0(\varepsilon_r' - j\varepsilon_r'')\underline{E} \quad (9.8)$$

$$\underline{J}_r = \sigma \underline{E} \quad (9.9)$$

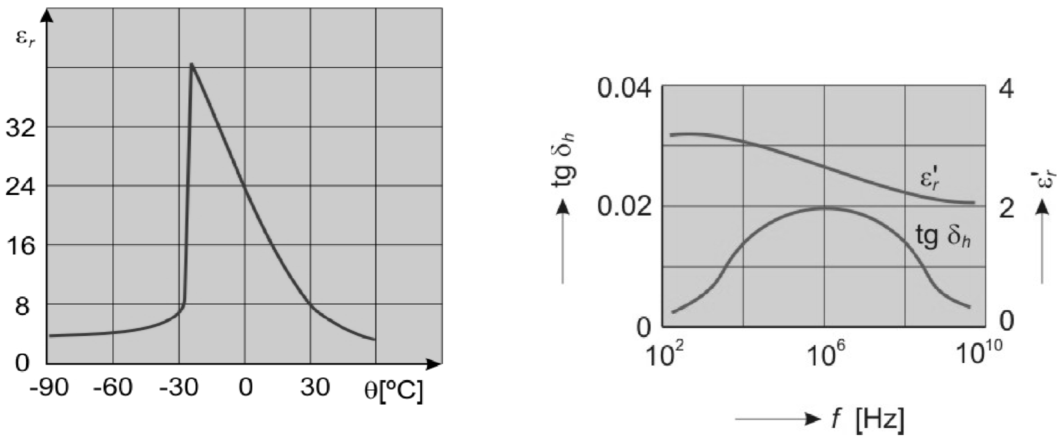
$$\underline{J}_{tot} = \underline{J}_\sigma + \underline{J}_D \quad (9.10)$$

$$\tan\delta_h = \frac{\varepsilon_r''}{\varepsilon_r'} = \frac{|\underline{J}_{Da}|}{|\underline{J}_{dr}|} \quad (9.11)$$

$$\tan\delta = \tan\delta_h + \frac{\sigma}{\omega\varepsilon_0\varepsilon_r'} \quad (9.12)$$

In harmonic fields, there are two material parameters that describe the behavior of dielectrics:relative permittivity and loss factor, or tg δ.These two parameters strongly depend on the structure of the dielectric, the frequency of the applied field and the temperature.

In Fig. 9.2, as an example, the influence of temperature and frequency on the permittivity and loss factor are given for nytro-carbureted hydrogen and for a polyester (Mylar) film.



a) b)

Fig. 9.2.Dependence of the permittivity and the loss factor with: a) temperature - the case fornytro-carbureted hydrogen; b) frequency - the case forpolyester (Mylar) film

9.2. Losses in Dielectrics

Section Summary: *In this section, the specific losses in dielectrics are established. The loss factor is determined with an equivalent circuit in series and an equivalent circuit in parallel. Total losses in dielectrics is also established.*

In each dielectric submitted to the action of the constant or cvasi-stationary electric field, energy losses will appear under the form of heat, because the material conductivity is not zero, and there are dielectric losses too.

- **Losses through conduction**

$$p_\sigma = \sigma_{tot} E_{eff}^2 \qquad (9.13)$$

- **Losses through polarization**

$$p_d = \omega \varepsilon_0 \varepsilon_r' \tan\delta_h \, E_{eff}^2 \qquad (9.14)$$

Total volume density of losses, in W/m^3 are:

$$p_{tot} = p_\sigma + p_d + p_{supl} \qquad (9.15)$$

Or

$$p_{tot} = \omega \varepsilon_0 \varepsilon_r' \tan\delta \, E_{eff}^2 \qquad (9.16)$$

In which$\tan\delta = \tan\delta_h + \frac{\sigma}{\omega\varepsilon_0\varepsilon_r'}$ (equation 9.12).

Establishing the total losses in the entire dielectric volume

- A dielectric with thickness *d*, introduced between the armatures of a plan capacitor with the cross-section S, is stressed with a voltage U, with frequency f.

The effective value of electric field intensity will be

$$E_{eff} = \frac{U}{d} \tag{9.17}$$

Total active power accumulated by capacitor due to the dielectric losses, in W, will be :

$$P = p_{tot} \cdot S \cdot d = \omega\varepsilon_0\varepsilon_r' \tan\delta \cdot \left(\frac{U}{d}\right)^2 \cdot Sd \tag{9.18}$$

Reactive power developed by capacitor, in VAr, will be:

$$Q = q_{tot} \cdot S \cdot d = -\omega\varepsilon_0\varepsilon_r' \cdot \left(\frac{U}{d}\right)^2 \cdot Sd \tag{9.19}$$

With the expression of plane capacitor capacity:

$$C = \frac{\varepsilon_0\varepsilon_r' S}{d} \tag{9.20}$$

It results:

$$P = \omega \tan\delta \cdot CU^2 \tag{9.21}$$

$$Q = -\omega CU^2 \tag{9.22}$$

The angle losses tangent

$$\tan\delta = \frac{P}{|Q|} = \frac{1}{\tan\varphi} \tag{9.23}$$

Where φ is angle between voltage and total current flowing through dielectric.

The loss factor can be determined with an equivalent circuit in series or an equivalent circuit in parallel (Fig. 9.3).

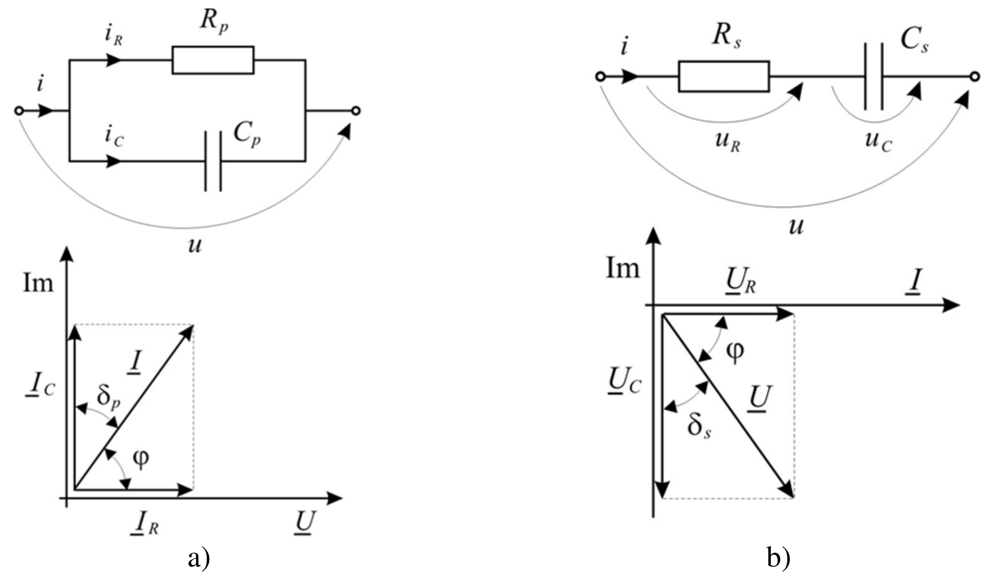


Fig. 9.3. Establishing the expression of factor fosses: a) equivalent RC circuit in series; b) equivalent RC circuit in parallel

For an equivalent RC parallel scheme, illustrated by the elements R_p and C_p, it results:

$$\tan\delta_p = \frac{I_R}{I_C} = \frac{1}{R_p \omega C_p} \tag{9.24}$$

For an equivalent RC series scheme, illustrated by the elements R_s and C_s, it results:

$$\tan\delta_s = \frac{U_R}{U_C} = \omega R_s C_s \tag{9.25}$$

The general parameters of some dielectrics are given in Table 9.1.

Table9.1. Material parameters for dielectrics

Electroinsulating materials	**Volume resistivity at 20^{0}C, ρ_v [Ωm]**	**Relative permittivity at 20^{0}C, 50 Hz ε_r**	**Loss tangent at 20^{0}C, 50 Hz tg δ**	**Dielectric strength at 20^{0}C, 50 Hz E_{str}, [kV/mm]**
Pure paraffin	10^{17}	1.9	$<10^{-5}$	-
Technical paraffin	10^{15}	2.2	$<10^{-4}$	-
Polyethylene	10^{15} - 10^{16}	2.20 – 2.50	$<10^{-5}$	20 – 50
Polytetrafloroethylene	10^{15} - 10^{16}	2.10 – 2.50	$<10^{-4}$	20 – 80
Polystyrene	10^{12}- 10^{15}	2.2 – 2.4	$<10^{-4}$	50 – 70
Polyvinylchloride - PVC	10^{11}- 10^{13}	4.2 – 4.5	$<10^{-2}$	40 – 70
Colophony	10^{15} - 10^{17}	2.0 – 2.5	-	-
Phenolformaldehyderesin	10^{13}	4.0 – 8.5	0.03 – 0.20	10 – 50
Plexyglass	10^{14}	3.4	10^{-2}	10 – 50
Pure paraffin	10^{17}	1.9	$<10^{-5}$	-
Technical paraffin	10^{15}	2.2	$<10^{-4}$	-
Polyethylene	10^{15} - 10^{16}	2.20 – 2.50	$<10^{-5}$	20 – 50

Polytetrafloroethylene	10^{15} - 10^{16}	2.10 – 2.50	$<10^{-4}$	20 – 80
Polystyrene	10^{12}- 10^{15}	2.2 – 2.4	$<10^{-4}$	50 – 70
Polyvinylchloride - PVC	10^{11}- 10^{13}	4.2 – 4.5	$<10^{-2}$	40 – 70
Colophony	10^{15} - 10^{17}	2.0 – 2.5	-	-
Phenolformaldehyde resin	10^{13}	4.0 – 8.5	0.03 – 0.20	10 – 50
Plexyglass	10^{14}	3.4	10^{-2}	10 – 50

Short Test for Lesson 9

1. Define the complex permittivity (mathematically and literary), specifying the physical measurement quantities involved in the relationships.
2. Draw and explain the phasor diagram of a linear and isotropic dielectric placed between the armatures of a capacitor in the following cases:
   a) ideal dielectric, without losses;
   b) real dielectric, with losses through polarization;
   c) real dielectric, with losses through polarization and conduction.
3. Establish the expression of losses angle tangent, using phasor diagrams for serial and parallel equivalent schemes of a capacitor with losses.

10. Magnetic Materials in Electrical Engineering

Contents

10.1. Magnetic State of Materials – Characterization, Classifications

Section Summary: *In this section, a description of magnetic state of materials with macroscopic and microscopic theories of electromagnetism is done. The significance of the physical quantities which characterizes the magnetic state in materials is given. The classifications of materials are done, with criterions value of magnetic permeability and values of atom magnetic momentum.*

In electrical engineering, there are many applications of magnetic materials, as:

- cores for electric transformers and rotate electrical machines,
- cores for specific coils,
- permanent magnets for microphones and telephone,
- permanent magnets for stators and rotors of synchronous machines,
- memory recorders,
- magnetic sensors, etc.

The most used magnetic materials are ferromagnetic materials, as: iron (Fe), cobalt (Co), nickel (Ni), and their alloys. But, today new and new magnetic materials are developed for applications as: storage energy, transportation, medicine and aerospace applications.

Two main functions of the magnetic materials are:

- to enhance the magnetic field (soft magnetic materials)
- to generate the magnetic field (hard magnetic materials for permanent magnets)

The physical quantities which described the magnetic state in materials are described by material laws of macroscopic and microscopic theories of electromagnetism.

<u>10.1.1. Description of magnetic state with macroscopic theory of electromagnetism</u>

The magnetic flux density***B*** *(*valid in vacuum)

$$\bar{B} = \mu_0 \bar{H} \qquad (10.1)$$

is modified inside of a material:

$$\bar{B} = \mu_0(\bar{H} + \bar{M}) \qquad (10.2)$$

which takes in account contribution of magnetization *M* in material

$$\bar{M} = \chi_m \cdot \bar{H} \qquad (10.3)$$

where:

$\mu_0 = 4\pi \cdot 10^{-7}$ H/m (Henry/meter) is the permeability of free space,

M isthe magnetization, which the magnetic state of the material, measeured in A/m,

χ_m is the magnetic susceptivity (adimensionless material parameter).

H is intensity of applied magnetic field, measured in A/m.

The term $\mu_0 \boldsymbol{M}$characterises the contribution of material to the magnetic field state, and it is named magnetic polarization $\boldsymbol{J}_{\mathbf{B}}$:

$$\bar{J_B} = \mu_0 \cdot \bar{M} \tag{10.4}$$

Thus, the law of connection between ***B***, ***H*** and ***M*** can be re-written as:

$$\bar{B} = \mu_0 \cdot \bar{H} + \bar{J_B} \tag{10.5}$$

Units of measurement for magnetic quantities and magnetic parameters are specifically:

- for magnetic flux density B is: Tor Gs(Tesla or Gauss), 1T = 10^4 Gs,
- for intensity of magnetic field strength H is: A/m (Amps/meter),
- for magnetic polarisation$J$ is: T (Tesla),
- formagnetisation$M$ is: A/m,
- for absolute magnetic permeability μ is: H/m (Henry/meter),
- for magnetic susceptivity, $\chi_m$is a dimensionless parameter,
- for relative magnetic permeability of a material, μr is dimensionless parameter,

$\chi_m = \frac{M}{H}$	$\mu - \frac{B}{H}$	$\mu_r = \frac{\mu}{\mu_0}$	$\mu_r = 1 + \chi_m$

In Table 10.1, a classification of the materials are given, in function of the values of magnetic permeability and based on purely phenomenological classification. In Table 10.2, The major magnetic parameters expressed in SI and CGS units as well as the conversion process between them.

Table 10.1.Values of magnetic permeability and classification of the materials

Classes of materials	$\mu_r = 1 + \chi_m$	Types of magnetization processes
Linear materials with diamagnetic behaviour	1-(10^{-6}-10^{-3})	Diamagnetism
Linear materials with paramagnetic behaviour	1+(10^{-6}-10^{-3})	Paramagnetism
Non-linear magnetic materials	Tens, hundreds, thousands	Ferromagnetism Ferrimagnetism Spontaneous magnetization

Table 10.2.Values of magnetic permeability and classification of the materials

Magnetic field parameters	Symbol	SI $B = \mu_0(H + M)$	CGS $B = H + 4\pi M$	CGS → SI Multiply by:
Field Intensity	H	A m^{-1}	Oe	$10^3/4\pi$= 79.58
Induction	B	T	G	10^{-4}
Magnetization	M	A m^{-1}	erg G^{-1} cm^{-3} (emu cm^{-3})	10^3
Permeability	μ	H m^{-1}	Dimensionless	$4\pi\ 10^{-7}$
Susceptibility	χ	Dimensionless	emu cm^{-3} Oe^{-1}	4π
Energy product	$(B\ H)_{max}$	kj m^{-3}	MGOe (mega-gauss-oersted)	$10^2/4\pi$= 7.958

10.1.2.Description of magnetic state with microscopic theory of electromagnetism

To justify the macroscopic state of magnetism, it is important to find the connection of the macroscopic quantity – magnetization - ***M***in term of the properties of the material at atomic level.
Microscopic theory of magnetismestablished that the magnetic state in a material is generated by the atomic electric charged particles, which move through closed trajectories.

For the case of one-electron atom (Fig. 10.1), the magnetization can be expressed as:

$$M = \frac{\Delta m}{\Delta V} = \frac{N \cdot m_{atom}}{\Delta V} = n \cdot m_{atom} \qquad (10.6)$$

Where the magnetic moment is:

$$m_m = iS \qquad (10.7)$$

In this relation, i is current intensity which defines the trajectory of the electric charge that moves along the closed Γ curve, and S is oriented surface corresponding the Γ curve,
It is demonstrated that in the case of multi-electron atom, the total magnetic momentum will be:

$$\overline{m}_m = \sum_i^Z \overline{m}_{orb,i} + \sum_i^Z \overline{m}_{spin,i} \qquad (10.8)$$

a) Orbital magnetic moment of electron

The orbital movement of the electron in one-electron atom is shown in Fig. 10.1.

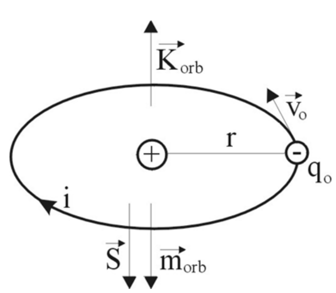


Fig. 10.1.Orbital movement of the electron and connection between kinetic orbital momentum and orbital magnetic momentum of the electron

The connection between orbital magnetic momentum and orbital kinetic momentum is described by the following relations:

-A current *i* flowing on a circular orbit will create a magnetic moment$m_m = iS$ (equation 10.7):

a. The current generated by a single electron rotating with an angular frequency ω_0 on a circular orbit:

$$\bar{\imath}_{orb} = \frac{q_0\omega_0}{2\pi} \quad (10.9)$$

b. The orbital magnetic momentum is:

$$\bar{m}_{orb} = \frac{q_0\omega_0}{2} \cdot \bar{r}^2 \quad (10.10)$$

c. The orbital kinetic momentum:

$$\bar{K}_{orb} = \bar{m}_0 r^2 \omega_0 \quad (10.11)$$

With these equations, it is obtained that:

$$\bar{m}_{orb} = \frac{q_0}{2m_0} \cdot \bar{K}_{orb} \quad (10.12)$$

Fig. 10.1 shows that orbital magnetic moment $\bar{m}_{orb}$ is a vector quantity in opposite side of kinetic momentum $\bar{K}_{orb}$ of electron in his orbital movement.
Because the orbital kinetic momentum is quantified, the orbital magnetic moment will be also quantified.

b) Spin magnetic moments of an atom

Due to the spin motion, the magnetic state is also generated by the intrinsic spin of electrons: each electron spinning on its own axis behaves as a magnetic dipole and it has a dipole momentum called the Bohr-Procopiu magneton μ_{B-P}, given by expression:

$$\bar{m}_{spin} = \mu_{B-P} = \frac{q_0 h}{4\pi\, \bar{m}_0} \quad (10.13)$$

whereq_o is electronic charge, *h* is Planck's constant, and m_0 is electron mass.

In SI units, the value of Bohr-Procopiu magneton is:

$\mu_{B-P} = 9.27 \times 10^{-24}$ A·m^2

c) Total magnetic moment of an atom with Z electrons

For the atom with Z electrons, the total magnetic momentum will be:

$$\bar{m}_{atom} = \sum_i^Z \bar{m}_{orb,i} + \sum_i^Z \bar{m}_{spin,i} \quad (10.14)$$

Note: the magnetic momentum of the multi-electronic atom is obtained by vector summation of the orbited magnetic moments and the magnetic spin moments of the electrons.

d) Classification of materials

A clasification of materials according to criterion – value of atomic magnetic moment – can be done:

- Magnetic non-polar materials, for which atomic moment is null,

$$\bar{m}_{atom} = 0$$

- Magnetic polar materials, for which atomic moment is different of zero, $\bar{m}_{atom} \neq 0$

Magnetic polar materials could be classified in:

- Materials with no ordering of magnetic moments (examples: Vanadium, Aluminum, Platinum etc.),
- Materials with parallel alignment of magnetic moments (examples: Fe, Ni, Co, Gd-gadolinium, which are included in the classes of ferromagnetic materials, ferromagnetic materials, etc.).

10.2. Materials with Magnetic Order

Section Summary: *In this section, a description of magnetic state of materials with magnetic order of the magnetic dipole of atoms is done. The classifications of materials with magnetic order is done, taking in account the parallel alignment of atomic magnetic dipoles in materials, as ferromagnetic materials and antiferromagnetic materials.*

Materials with parallel alignment of magnetic moments have many applications in electrical engineering. From the category of ferromagnetic materials, the most important ferromagnetic elements from industrial view-point are:

- iron (Fe),
- cobalt (Co),
- nickel (Ni).

Gadolinium (Gd), as a rare earth element, is also a ferromagnetic material, but only below $16\,^{o}C$ (it has less industrial application).The ferromagnetic propertiesof the transition elements Fe, Co, and Ni are due to the way the spins of the inner unpaired electronsare aligned in the crystal lattices of these elements.

Notes:

- The inner layers of individual atoms are filled with pairs of electrons with opposed spins, and so there are no resultant magnetic dipole moments due to them;
- In Fe, Co, and Ni, the unpaired inner 3*d* electronsare responsible for ferromagnetism, which these elements exhibit (Table 10.2).

Table 10.2.Unpaired 3d electrons of some transient elements

Electrons number	Atom	1s	2s	2p	3s	3p	3d	4s	Un-paired 3d electrons
21	Sc	↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓	↑↓ ↑↓ ↑↓	↑	↑↓	1
22	Ti	↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓	↑↓ ↑↓ ↑↓	↑ ↑	↑↓	2
23	V	↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓	↑↓ ↑↓ ↑↓	↑ ↑ ↑	↑↓	3
24	Cr	↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓	↑↓ ↑↓ ↑↓	↑ ↑ ↑ ↑ ↑	↑	5
25	Mn	↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓	↑↓ ↑↓ ↑↓	↑ ↑ ↑ ↑ ↑	↑↓	5
26	Fe	↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓ ↑ ↑ ↑ ↑	↑↓	4
27	Co	↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓ ↑↓ ↑ ↑ ↑	↑↓	3
28	Ni	↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓ ↑↓ ↑↓ ↑ ↑	↑↓	2
29	Cu	↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓ ↑↓ ↑↓ ↑↓ ↑↓	↑	0

In a solid sample of Fe, Co or Ni at room temperature, the spins of the 3d electrons of adjacent atoms align in a parallel direction by a phenomenon called **spontaneous magnetization**.

This parallel alignment of atomic magnetic dipoles occurs only in microscopic regions, called **magnetic domains.**
Taking in account the relative orientation of atomic magnetic dipoles, the materials with magnetic order could be:

- **Ferromagnetic materials,** in which the parallel alignment of atomic magnetic dipoles is in the same direction (Fig. 10.2.a). It is the case of Fe, Co, and Ni crystals,
- **Antiferromagnetic materials,** in which the parallel alignment of atomic magnetic dipoles is in the opposite direction (Fig. 10.2.b, c, d). The magnetic state in these materials could be:
    - With fully compensated antiferromagnetic order (Fig. 10.2.b) – as in the cases of the Manganese and Chromium crystals
    - With incompletely compensated antiferromagnetic order (Fig. 10.2.c, d) – as in the cases of ferrites, and other structures.

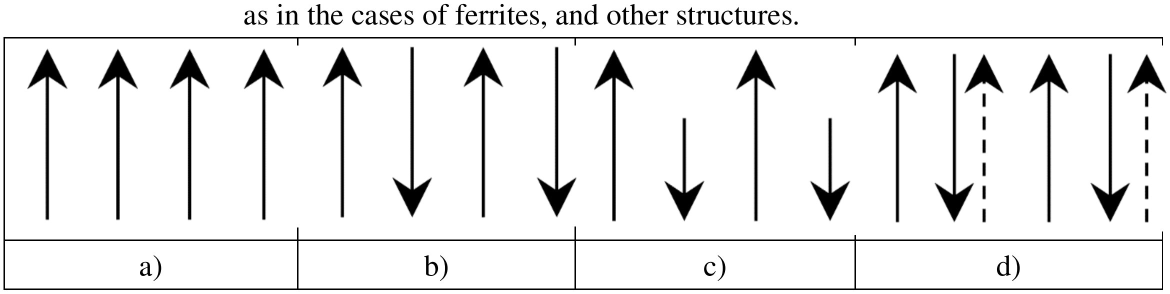


Fig. 10.2.Cases of parallel alignment of atomic magnetic dipoles (in the magnetic domains): a) ferromagnetic order; b) fully compensated antiferromagnetic order; c) and d) incompletely compensated antiferromagnetic order

Table 10.3 and Table 10.4 describe some properties and characteristics for ferro and ferri-magnetic materials. Note that ferrites are especially used in the high frequency range: coil cores, magnetic memories and amplifiers, antennae, TV transformers, etc.

Table 10.3.Performances of Fe-Ni alloys and other feromagnetic materials

Material	Composition, %		μ_{ri}	$\mu_{r\,max}$	H_c [kA/m]	B_s [T]
	Fe	***Other elements***				
Iron	99.9	-	200	5000	0.08	2.15
Iron-Silicon	96	Si-4	450	8000	0.048	1.97
Alsifer	85	Si-9	30000	120000	0.004	1.0
Permalloy 78	21.5	Ni-78.5	8000	100000	0.004	1.0
Mumetal	17	Ni-76 Cu-4 Mo-3	25000	110000	0.0016	0.8
Dynamax (B_r/B_s = 0.95)	32.7	Ni-65 Mo-2 Mn-0.3	-	1530000	$0.4\cdot10^{-3}$	1.26
Rectimphy (B_r/B_s = 0.96)	49	Ni-50	-	100000	0.011	1.6
Perminvar	20	Ni-45 Co-25 Mo-7	-	3800	0.04	1.03
Permenorm	64	Ni-36	2000	7000	0.065	1.3
Permendur	49	Co-49	800	5000	0.16	2.36
Supermendur (anisotropic)	49	Co-49 V-2	800	100000	-	2.35

Table 10.4.Manganese-zinc and nickel-cobalt ferrites performances.

Property	Soft Ferrites									
	MZ-1	**MZ-2**	**MZ-3**	**MZ-4**	**MZ-5**	**MZ-6**	**MZ-7**	**NC-4**	**NC-5**	**NC-6**
Initial permeability μ_{ri}	400	1000	1200	1250	1500	2200	1500	50	80	120
Frequency domain, *f*, kHz	0.02-1.5	-	0.004-0.2	0.001-0.2	0.001-0.1	0.001-0.1	-	10-50	3-30	15-10
Saturation magnetic induction *B* in T, la *H*=3 kA/m	0.38	-	0.38	0.42	0.38	0.40	0.30	-	-	-
Resistivity ρ, Ω·cm	10^2	-	10	10^2	10^2	10^2	-	$3\cdot10^6$	$3\cdot10^6$	$3\cdot10^6$

Curie temperature, °C	180	210	130	210	170	150	200	450	400	300
Half-finish materials	cylindrical core	Core U+I	Deflection core TV	tube cylinder	core ladle	E	U+T	Cylindrical core with adjustments		

Short Test for Lesson 10

1. Specify the role of magnetic materials. Give some examples of magnetic materials and their applications.
2. Express the magnetization in function of the atomic magnetic moment. Express the two components of the atomic magnetic moment and justify the orbital magnetic moment of the atom (mathematically, graphically and literary).
3. Make a classification of magnetic materials in function of the value of atomic magnetic moment and give some material examples.
4. Make a classification of magnetic materials in function of the orientation of electron spins. What are the similarities and differences between them?

11. Behavior of Materials in Constant Magnetic Fields

Contents

11.1. Diamagnetism of Materials

Section Summary: *In this section, a description of diamagnetism processes and the main characteristics for the materials with diamagnetic comportment are given. Langevin theory of diamagnetic susceptivity is also presented. Examples in which diamagnetic susceptivity is put in evidence are given.*

Diamagnetismrepresents the phenomenon of decreasing (weakening) the total magnetic field due to magnetic moments, supplementary induced by the external magnetic field applied, in each electronic orbit of the constituent atoms of a material.
Paramagnetismrepresents the phenomenon of increasing (strengthening) the total magnetic moment due to the orientation of the orbital and spin magnetic moments of the **polar magnetic material** on the direction of the external magnetic field.

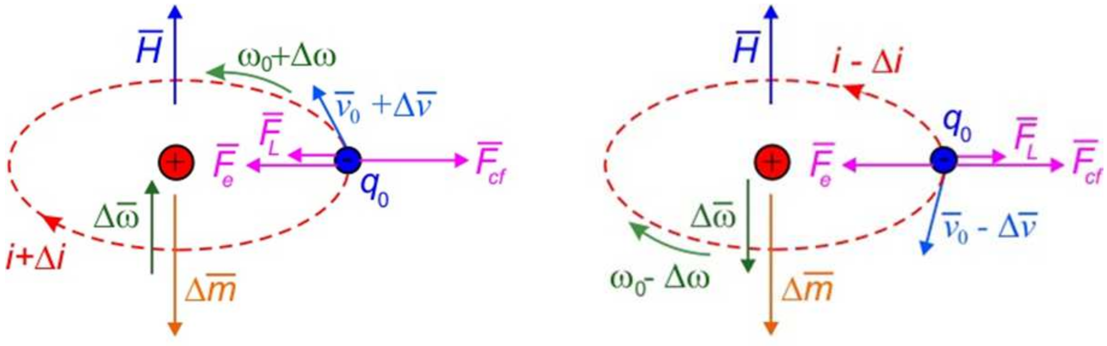
Justification of material diamagnetism and characterisation:

- The external magnetic field acts on the electrically charged particles (free or bonded particles) moving on closed trajectories, determining the apparition of a supplementary movement, thus, of a supplementary magnetic moment for each moving particle.
- As the effect opposes to the cause, the **magnetic susceptivity will take negative values**.
- The diamagnetism is **a universal phenomenon**, found in all materials introduced in a magnetic field!

Classes of diamagnetism phenomena:
- Langevin diamagnetism - movements of the bonded electrons,
- Landau diamagnetism - movements of conduction electrons.

11.1.1. Calculus of the magnetic susceptivity generated by Langevin diamagnetism

In Fig. 11.1, the case of hydrogen atom placed in a homogeneous magnetic field with the perpendicular direction on the electron orbit plane.



a) b)

Fig. 11.1.Hydrogen atom placed in a homogeneous magnetic field with the perpendicular direction on the electron orbit plane, and with angular velocity vector: a) in the same direction with intensity of magnetic field; b) in the opposite direction with intensity of magnetic field

Cases:

a) <u>In the absence of magnetic field</u>

The dynamic equilibrium of the electron in his movement on the orbit is described by:

$$\bar{F}_e + \bar{F}_{cf} = 0 \quad (11.1)$$

Where:

- the interaction force between the nucleus and the electron is:

$$\bar{F}_e = \frac{q_0^2}{4\pi\varepsilon_0 r^2} \quad (11.2)$$

- the centrifugal force is:

$$\bar{F}_{cf} = m_0 r\omega_0^2 \quad (11.3)$$

wherem_0 is the electron mass, *r* is the circular orbit radius, and ω_0 is the electron angular velocity.

b) <u>In the presence of the external magnetic field</u>

In the presence of an external magnetic field (Figure 2.2a), Lorentz force is established, thus, we obtain (note that in order to maintain the dynamic equilibrium:

$$\bar{F}_L = -q_0\bar{v} \times \mu_0\bar{H} \quad (11.4)$$

Or

$$F_L = q_0 v\mu\bar{H} = q_0\bar{\omega}_0 r\mu_0\bar{H} \quad (11.5)$$

In order to maintain the dynamic equilibrium, the modification of the angular velocity of electron must be done:

$$\bar{\omega} = \bar{\omega}_0 + \Delta\bar{\omega} \quad (11.6)$$

Dynamic equilibrium condition will be:

$$\overline{m}_0 r(\overline{\omega}_0 + \Delta\overline{\omega})^2 = \frac{q_0^2}{4\pi\varepsilon_0 r^2} + q_0 r(\overline{\omega}_0 + \Delta\overline{\omega})\mu_0\overline{H} \quad (11.7)$$

For a small variation of the angular velocity the following approximations are done:

- in the left side:

$$2\omega_0\Delta\omega \gg \Delta\omega^2 \quad (11.8)$$

- in the right side:

$$\omega_0 \gg \Delta\omega$$

The variation of angular velocity is:

$$\Delta\omega = \frac{q_0}{2\overline{m}_0}\mu_0\overline{H} \quad (11.9)$$

c) Calculus of the induced magnetic moment

From the expression of angular velocity variation, the variation of frequency:

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{q_0}{4\pi\overline{m}_0}\mu_0\overline{H} \quad (11.10)$$

The expression of the induced magnetic moment will be:

$$\Delta\overline{m}_i = q_0\Delta f\pi r^2 = \frac{q_0^2 r^2}{4\overline{m}_0}\mu_0\overline{H} \quad (11.11)$$

$$\Delta\overline{m}_i = -\frac{q_0^2 r^2}{4m_0}\mu_0\overline{H} \quad (11.12)$$

Calculus of the sum of all induced electronic moments

$$\langle\Delta\overline{m}_i\rangle = -\frac{q_0^2\mu_0\overline{H}}{4m_0}\cdot\sum_{i=1}^{Z} r_i^2 \quad (11.13)$$

The expression for the magnetization:

$$\overline{M} = n\langle\Delta\overline{m}_i\rangle = -\frac{nq_0^2\mu_0\overline{H}}{4m_0}\cdot\sum_{i=1}^{Z} r_i^2 \quad (11.14)$$

The expression for magnetic susceptivity:

$$\chi_{md} = -\frac{nq_0^2\mu_0}{4m_0}\cdot\langle\sum_{i=1}^{Z} r_i^2\rangle \quad (11.15)$$

Some conclusions on the Langevin diamagnetism are underlined here:

- Diamagnetic materials susceptivity is proportional to the volume concentration *n* of constituent atoms, to the average of the trajectory radii squares $<r^2>$ of the electrons and to the number *Z* of electrons in atom;
- Diamagnetic materials susceptivity does not explicitly depend on the temperature;
- The linear behavior of the material is underlined: when the exterior magnetic field is increased, the magnetization increases proportionally, in modulus.
- The magnetic susceptivity takes negative values, whch means that the value of magnetization vector has anti-parallel orientation relative to the exterior magnetic field strength; the magnetic field is weakened inside the material.

In Fig. 11. 2, the values of magnetic susceptivity of some diamagnetic materials are shown.

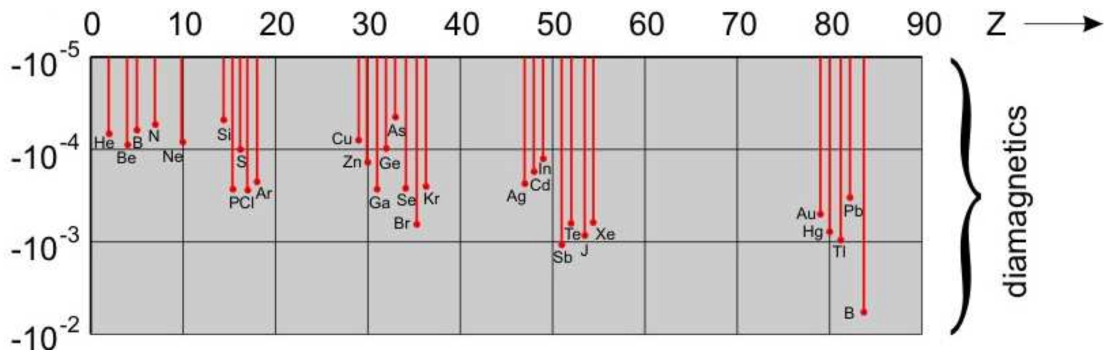


Fig. 11. 2. Magnetic susceptivity χ_m of some diamagnetic materials in function of their atomic number *Z.*

In Table 11.1, the values of magnetic susceptivity of some metals, in which diamagnetic character is predominant, are given,

Table 11.1. Magnetic susceptivity of some metals in which diamagnetic character is predominant

Metal	$\chi_m \cdot 10^6$	Corresponding ion	$\chi_m \cdot 10^6$
Cu	- 5.5	Cu^+	- 18
Ag	- 21.6	Ag^+	- 31
Au	- 29.6	Au^+	- 44.8

11.2. Paramagnetism of Materials

Section Summary: *In this section, a description of paramagnetism processes and the main characteristics for the materials with paramagnetic comportment are given. Langevin theory of paramagnetic susceptivity is also presented. Examples in which diamagnetic susceptivity is put in evidence are given.*

Paramagnetismporceses are encountered in the case of the polar magnetic substances, in which

$$\overline{m}_{atom} \neq 0$$

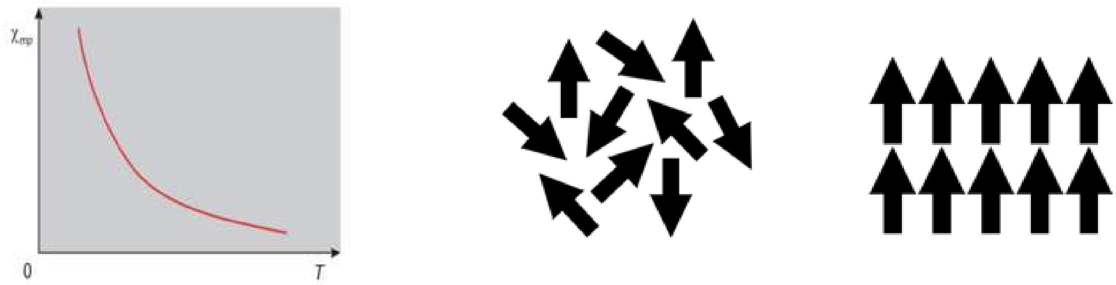
The corresponding magnetic susceptivity takes positive values, that has the significance that the magnetic field in these materials are reinforcement when a magnetic field is applied. There are two classes of paramagnetism porcesses:

- Langevin paramagnetism -it is generated by bound charges,
- Pauli paramagnetism - it is generated by conduction electrons.

The paramagnetic susceptibility of systems of bound electrons dependswith the inverse of the temperature. Thus:

$$M = \chi_{pm} H = C \cdot \frac{H}{T} \qquad (11.16)$$

which is called Curie's law (C is a constant). The graphical representation of this dependence is shown in Figure 2.3. Example of elements are: Al, Ba, Ca, O, Pt, Na, Sr, U, Mg, Tc (artificial), Dy, W.



a) b) c)

Fig. 11.3.Paramagnetism: (a)The Curie law characteristic of the paramagnetic susceptibility depending on the temperature; (b), (c) The magnetic moments in a paramagnetic material in the presence and absence of an applied field.

For the transitional elements belonging to the group of iron (Z = 21 ÷ 28), palladium group (Z = 39 ÷ 45), platinum group (Z = 71 ÷ 78), that are in a paramagnetic state, a deviation from Curie law appears, determined by the action of the crystalline potential on the atomic magnetic moments. In this case, Curie-Weiss law is applied:

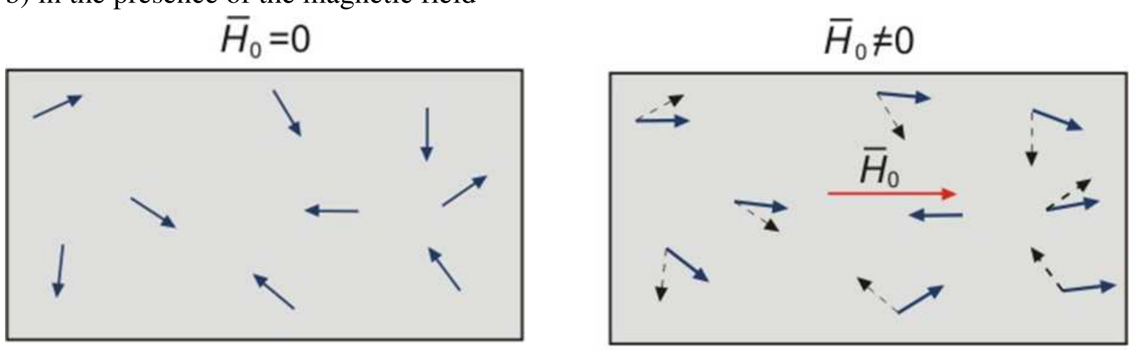
$$\chi_{mp} = \frac{C}{T-T_c} \tag{11.17}$$

a) <u>The classical theory of paramagnetism of the bonded charges, elaborated by Langevin</u>

It considers that in a polar magnetic material, there are magnetic dipoles, with a volume concentration *n*, and a permanent magnetic moment $\boldsymbol{m}_{\mathbf{p}}$.

In Fig. 11.4, the distribution of the magnetic moments in the absence and in the presence of the magnetic field is described.

b) in the presence of the magnetic field



a) b)

Fig. 11.4. Distribution of permanent magnetic moments: a) in the absence of magnetic field; b) in the presence of the magnetic field

When a magnetic field is applied, the permanent magnetic moments are oriented in the direction and sense of the magnetic field.

- The average value of the atomic magnetic moments is:

$$\overline{m}_{avg\,\mathrm{Ox}} = \frac{N_{\uparrow\uparrow}\overline{m}_p - N_{\uparrow\downarrow}\overline{m}_p}{N_{\uparrow\uparrow}+N_{\uparrow\downarrow}} \qquad (11.18)$$

Where:

- The number of magnetic moments homo-parallel oriented and anti-parallel are:

$$N_{\uparrow\uparrow} = const \cdot e^{\left(\frac{m_p \mu_0 H}{\mathrm{kT}}\right)} \qquad (11.19)$$

$$N_{\uparrow\downarrow} = const \cdot e^{\left(-\frac{m_p \mu_0 H}{\mathrm{kT}}\right)} \qquad (11.20)$$

- The projection of the average magnetic moment on the field direction is obtained:

$$m_{med\,Ox} = m_p th\frac{m_p \mu_0 H}{kT} \cong \frac{m_p^2 \cdot \mu_0 H}{kT} \qquad (11.21)$$

- The expression for the magnetization is computed with relation for magnetization, and compared with the temporary magnetization law:

$$\begin{cases} M = n \cdot m_{medox} = \frac{n \cdot m_p^2\, \mu_0}{kT} H \\ M_t = \chi_{mp} H \end{cases} \qquad (11.22)$$

- The expression of the paramagnetic susceptivity is obtained:

$$\chi_{mp} = \frac{n \cdot m_p^2\, \mu_0}{kT} \qquad (11.23)$$

b) <u>Conclusions on the Langevin paramagnetism</u>

- The magnetic susceptivity in the case of a paramagnetic behaviour is directly proportional with the volume concentration n of the magnetic moments, with the square of the permanent magnetic moment mp of the structural unit (atom, molecule, basic cell in the case of crystals, etc.) and with 1/T.
- The susceptivity takes small positive values: the field is strengthened in paramagnetic materials.

  In the paramagnetic materials are included: the oxygen, materials that include the –OH group, alkaline metals, ferromagnetic materials at high temperatures, etc.

The paramagnetic susceptivity depends on temperature:

$$\chi_{mp} = \frac{const.}{T} \qquad (11.24)$$

In the case of paramagnetic materials, when the temperature increases, the magnetic susceptibility decreases.

In Fig. 11.5, the values of magnetic susceptivity for crystals of some chemical elements relative to the atomic number Z is shown.

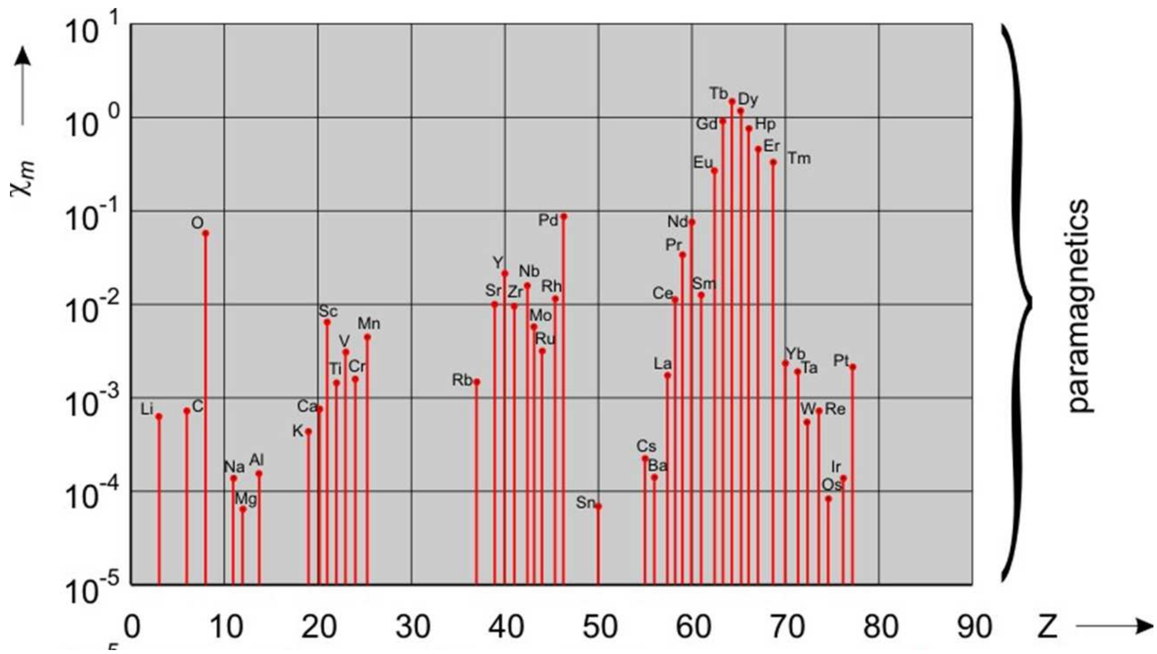


Fig. 11.5. Magnetic susceptivity for crystals of some chemical elements relative to the atomic number Z

It can be noted that the values of materials in which paramagnetism is predominant is about$\mu_{rp} = 1 + (10^{-5} \div 10^{-4})$.

11.3. Superparamagnetism of Materials

Section Summary: *In this section, a description of superparamagnetism of specific materials and the main characteristics for these processes are given. Examples, and applications of superparamagnetism are pointed.*

Superparamagnetismis the process in which, in sufficiently small ferromagnetic or ferrimagnetic particles, magnetization can randomly flip direction under the influence of temperature in a time called Néel relaxation time.
Néel relaxation is regarded as the internal flipping of the magnetic moment inside a stationary Superparamagnetic Iron Oxide Nanoparticle (SPION). The SPIONs usually have a diameter between 20 to 150 nm and most of them are suspended in a liquid. Also, due to the inherent anisotropy, each SPION tends to align with an easy axis.

The energy barrier between two opposite directions of the easy axis is function of the magnetic anisotropy constant *K* and particle volume *V*:

$$\Delta E = KV \quad (11.25)$$

Thus, as the particle size decreases, the relaxation time decreases exponentially.

If it also takes into account that the particles are suspended in a liquid, a further relaxation path occurs (Brownian relaxation). In such a case, the high energy anisotropy barrier may be avoided by a flip of the particle under the influence of an external magnetic field.

Several other parameters are involved: instead of hysteresis losses, frictional losses are defined by viscosity of the carrier liquid (dynamic viscosity) η and the hydrodynamic volume V_h of the particle.

Total relaxation time is:

$$\frac{1}{\tau} = \frac{1}{\tau_N} + \frac{1}{\tau_B} \quad (11.26)$$

Where:

- Néel relaxation time is given by:

$$\tau_N = \tau_0 \cdot e^{\frac{KV}{kT}} \tau_0 \sim 10^{-9} \mathrm{s} \quad (11.27)$$

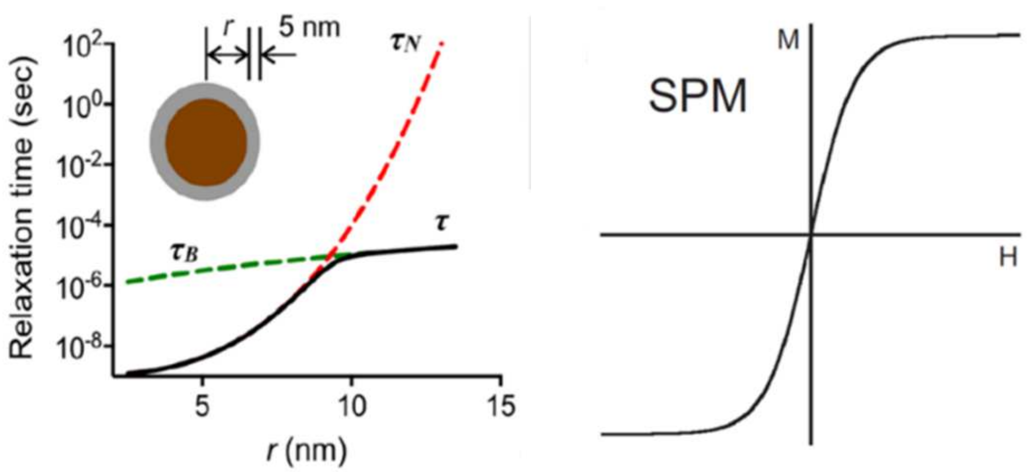
- Brownian relaxation time is given by:

$$\tau_B = \tau_0 \frac{3\eta V_h}{kT} \quad (11.28)$$

The Brownian relaxation phenomenon is characterized in frequency of the Brownian relaxation:

$$f_B = \frac{k_B T}{6\eta V_h} \quad (11.29)$$

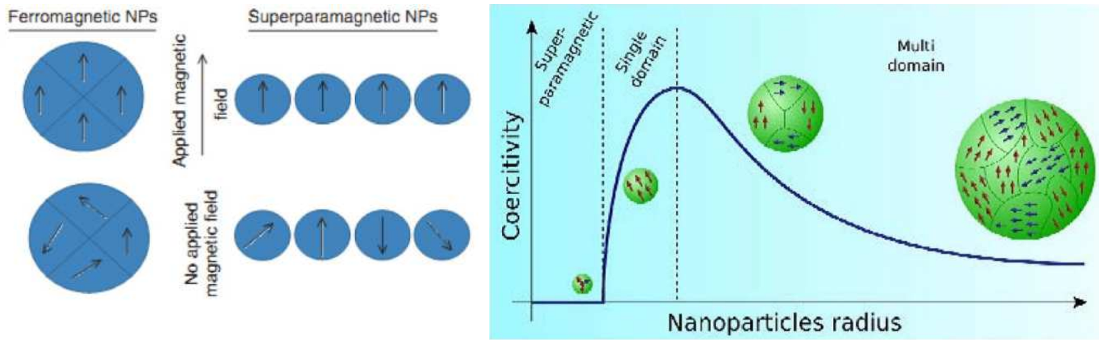
In Fig. 11.6.a, the Néel ($\square_N$) and the Brownian ($\square_B$) relaxation times for Fe_3O_4 MNPs particles is shown in function of the radius of particles, with 5 nm surface nonmagnetic coating. The effective relaxation time $\square$ is shown in a black line. A typical magnetization curve for magnetic nanoparticles is shown in Fig. 11.6.b.



a) b)

Fig. 11.5.Caracteristics of superparamagnetism materials: a) Néel ($\square_N$), Brownian ($\square_B$) and total relaxation times for Fe_3O_4 magnetic nanoparticles in function of the particle radius; b) magnetization curve for magnetic nanoparticles

Magnetization behavior of ferromagnetic and superparamagnetic nanoparticles under an external magnetic field is shown in Fig. 11.6. In Fig. 11.6.a, the domains of a ferromagnetic and superparamagnetic nanoparticle (NPs) under the presence of an external magnetic field are described (notice that superparamagnetic NPs exhibit no net magnetization due to rapid reversal of the magnetic moment). In Fig. 11.6.b, a schematic illustration of the coercivity-size relationship between the NP size and domain structures is shown, where the superparamagnetism and critical size thresholds can be observed.



a) b)

Fig. 11.6. Comparative magnetization behavior of ferromagnetic and superparamagnetic nanoparticles under an external magnetic field: a) domain structure of a ferromagnetic and superparamagnetic nanoparticle (NPs) under the presence of an external magnetic field; b) coercivity-size relationship between the NP size and domain structures

The superparamagnetismphenomena isutilized in many applications:

- Targeted drug delivery
- Magnetic resonance imaging
- Ferrofluids
- Magnetic hyperthermia and thermoablation
- Magnetofection (concentrate particles containing vectors to target cells in the body)
- Bioseparation
- Biosensing
- Therapeutic and diagnostic agents (when combined with biosensing systems).

Short Test for Lesson 11

1. Specify the behaviour of Landau diamagnetism in the presence and absence of an externally applie magnetic field (equations and justification).
2. What is the meaning of Curie temperature in paramagnetic materials? And what effect it has on the magnetic behaviour of the material?
3. Describe the bonded charges theory of paramagnetism (Langevin paramagnetism). Specificy equations and justify results.
4. Explain the superparamagnetic behaviour of materials (theory and equations).
5. How the superparamagnetic behaviour of materials can be applied? Give examples for some use cases and briefly explain implementation of superparamagnetic behaviour for a specific application.

12. Characteristics and Losses in Soft Ferromagnetic Materials

Contents

12.1. Characteristics of Soft Magnetic Materials

Section Summary: *In this section, there are described the main characteristics of soft ferromagnetic materials: non-linearity and saturation, hysteresis cycle, remanence and coercivity, and Curie temperature.*

Soft magnetic materials (ferromagnetics) are materials with homo-parallel order of the atomic magnetic moments.

These materials have many applicationsin the electrical and electronic engineering, due to their property of concentrating and amplifying the magnetic field within them (Fig. 12.1).

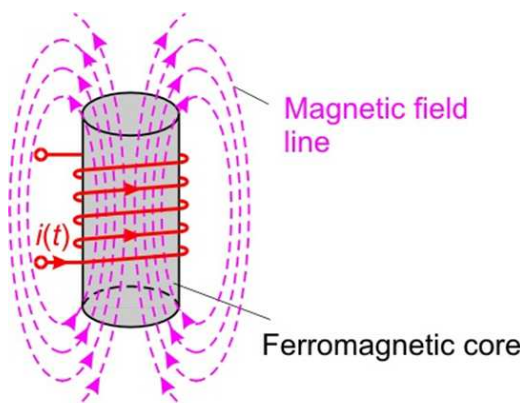


Fig. 12.1. Concentrating of the magnetic field lines in the soft magnetic materials

Specific properties of ferromagnetic materials are:

a. Non-linearity and Saturation
b. Hysteresis cycle
c. Remanence and Coercivity
d. Pronounced temperature dependence of the magnetic characteristics.

a. Non-linearity and Saturation

Ferromagnetic materials are non-linear materials: the characteristic curve B=f(H) is not a straight line (Fig. 12.2.a).Also, the magnetic permeability, defined as the slope of curve$B$=f(H) has different values (Fig. 12.2.b).

Due to the specific form of the curve of the first magnetization (obtained through the constant increase in the intensity of the magnetic field, for a material demagnetized in the initial state), the value of the magnetic permeability modifies in accordance to the value of the intensity of the magnetic field: from an initial state, the permeability reaches a maximal point and then tends downwards down to 1.

In high magnetic fields, a limitation phenomenon (saturation) of the magnetization state is produced.From a macroscopic point of view, ferromagnetic materials are characterized through a family of magnetization curves, given bty the dependencies of B=f(H), which have the same shape as M=f(H) dependencies, in the case of low to average magnetic fields. At magnetization with a continuous current, the first magnetization curve is obtained, while in variable magnetic fields, non-linear dependenciesoccur between the magnetic induction and the intensity of the applied magnetic field.

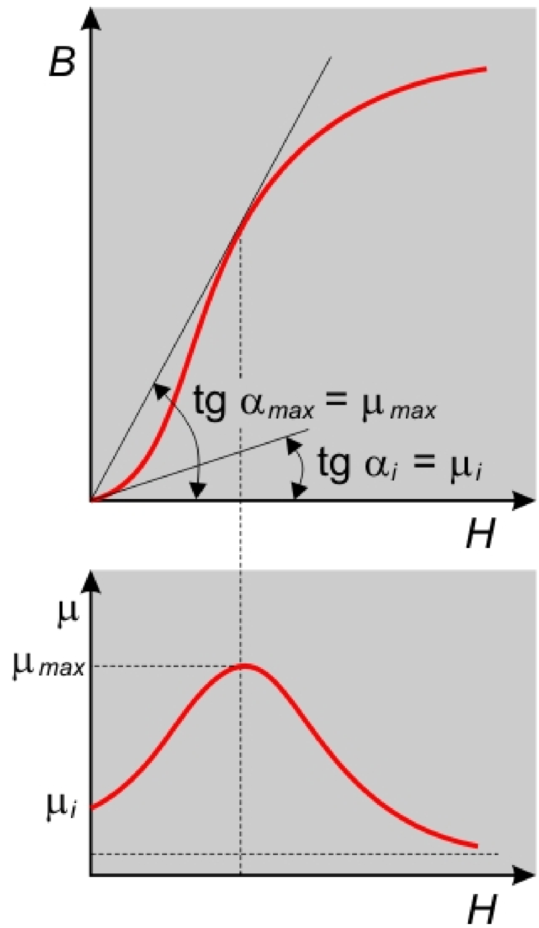


Fig. 12.2.Non-linearity and Saturation in ferromagnetic materials: a) magnetic characteristic *B*=f(*H*); magnetic permeability curve μ=f(*H*).

In Fig. 12.2, *B* is magnetic induction, in T (Tesla) and *H* is intensity of magnetic field, in A/m (Amper/meter), μ is magnetic permeability, in H/m(Henry/meter).

The characterization of a ferromagnetic material around a certain state, defined with a pair of values (B, H), is done with different expressions for relative magnetic permeability:

- Relative static permeability (varies with the increase in magnetic field):

$$\mu_{\mathrm{r\ st}} = \frac{B}{\mu_0 H} \tag{12.1}$$

- Relative incremental permeability (is defined by the magnetization curve slope to the axe of the abscise, for a low variation of H in the opposite direction):

$$\mu_{r\,\Delta} = \lim_{\Delta H \to 0} \left(\frac{\Delta B}{\Delta H}\right) \tag{12.2}$$

- Relative reversal permeability (is proportional with the hysteresis reversible minor cycle slope whose peak is in the point coordinated (B,H):

$$\mu_{r\,\text{rev}} = \lim_{\Delta H' \to 0} \left(\frac{\Delta B'}{\mu_0 \Delta H'}\right) \tag{12.3}$$

- Relative dynamic permeability (is proportional with the average curve of the hysteresis cycle described around the characteristic point of the considered state):

$$\mu_{r\,i} = \lim_{\Delta H \to 0} \left(\frac{\Delta B}{\mu_0 \Delta H}\right), \tag{12.4}$$

- Relative permanent permeability (is proportional with the hysteresis cycle curve whose basis is on the curve of demagnetization of a permanent magnet):

$$\mu_{r\,p} = \frac{1}{\mu_0}\frac{\Delta B}{\Delta H} \tag{12.5}$$

b. Hysteresis cycle

The ferromagnetic behavior is dependent on the history of magnetic stress: the ferromagnetic materials have a hysteresis cycle.

Thus, with a cyclic variation of the magnetic field, a typical curve, called hysteresis cycle is obtained (Fig. 12.3).

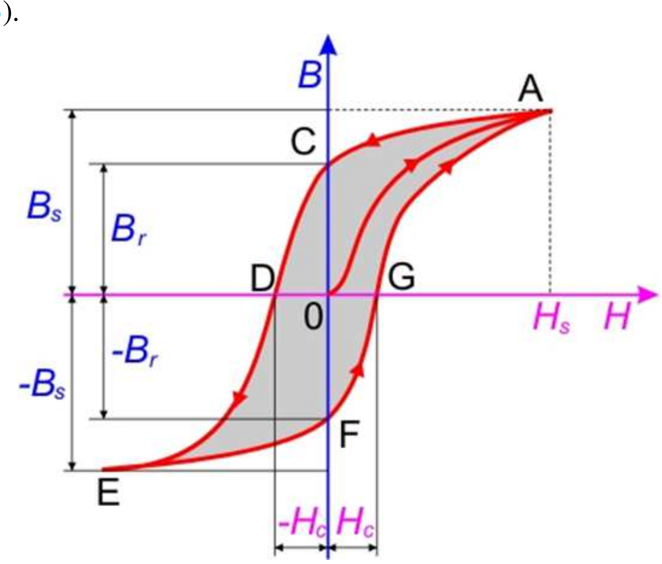


Fig. 12.3. First magnetization curve (OA curve) and hysteresis curve (ACDEFGA curve) in ferromagnetic materials

In a ferromagnetic material, for different magnetization regimes, different hysteresis curves are obtained.

The state of magnetization is described by the quantities: **M** –magnetization and/or magnetic polarization $\boldsymbol{J}_B$, which are connected by the relation:

$$\bar{J}_B = \mu_0 \cdot \bar{M} \tag{12.6}$$

When a magnetic field of intensity H is applied, total state of magnetization in material is described by magnetic induction:

$$\bar{B} = \mu_0 \cdot (\bar{H} + \bar{M}) \quad (12.7)$$

or

$$\bar{B} = \mu_0 \cdot \bar{H} + \bar{J}_B \quad (12.8)$$

In Fig. 12.4, family of hysteresis cycles for a soft ferromagnetic material is shown. There is a maximum hysteresis cycle and inside, minor hysteresis cycles.

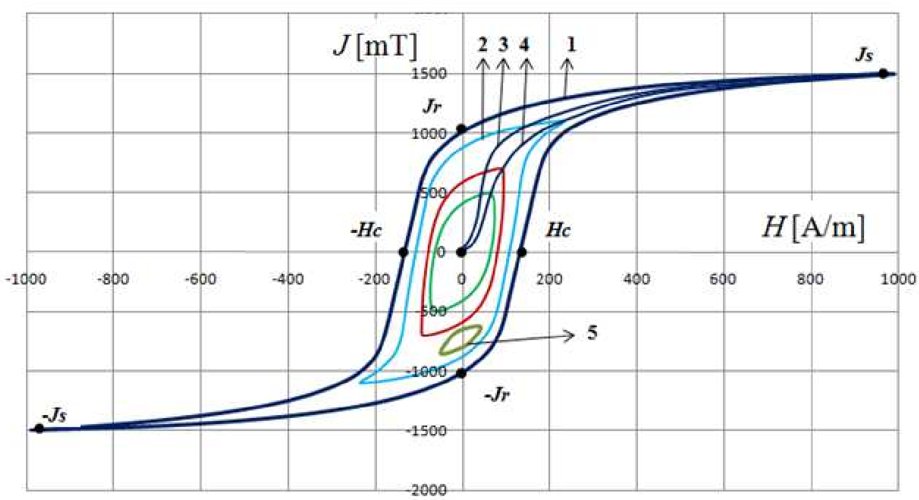


Fig. 12.4. Hysteresis cycles in soft magnetic materials: 1 – maximum (limit) curve; 2 - minor curves; 3 - dynamic curve; 4 - first magnetization curve; 5 - non-symmetric minor curve

c. Remanence and Coercivity

In Fig. 12.4, the remanence and coercivity are described.

- Point A corresponds to magnetic induction of saturation B_s
- Point C corresponds to remanence magnetic induction B_r
- Point D corresponds to coercive magnetic field intensity H_c

Remanence magnetic induction is obtained for the case when the intensity of magnetic field is cancelled.

Coercive magnetic field has the significance of the required magnetic field for which the magnetic remanence is cancelled.

The width of the hysteresis cycle serves as the criterion of classification of the ferromagnetic materials. Thus, there are (Fig. 12.5):

- Soft magnetic materials – materials for which H_c << 1kA/m
- Hard magnetic materials – materials for which H_c >> 1kA/m.

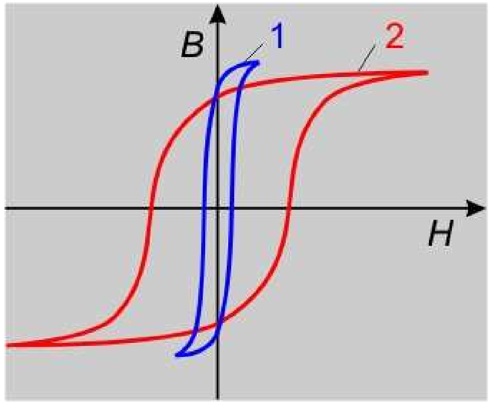


Fig. 12.5. Maximum hysteresis cycles for: 1 – soft magnetic materials; 2 – hard magnetic materials

d. Pronounced temperature dependence of magnetic characteristics

If the temperature in the ferromagnetic materials increases, then the thermal agitation energy has the tendency to reduce the state of magnetic ordering. Remanence magnetic induction Br decreases with increasing the temperature (Fig. 12.6).

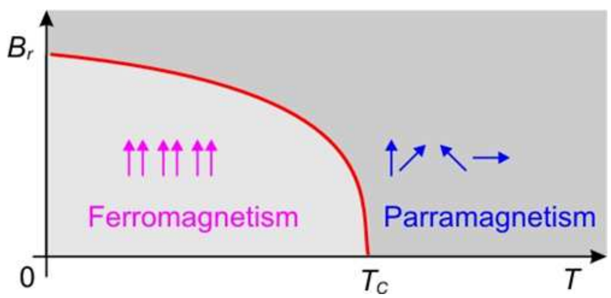


Fig. 12.6. Temperature dependence of the remanence magnetic induction

Curie temperature is the temperature at which the material passes in paramagnetic state.
In Fig. 12.7, the modification of iron hysteresis cycles near the Curie temperature is shown.

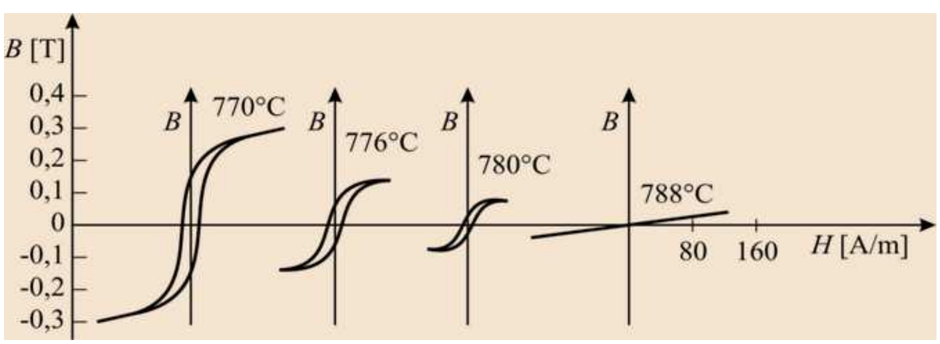


Fig. 12.7. Modification of iron hysteresis cycles near the Curie temperature

Above the Curie temperature, which for iron is T_c= 788°C, the hysteresis disappears, the iron behaves paramagnetic.

In Table 12.1, the values of Curie temperature for some ferromagnetic materials are given

Table 12.1. Curie temperature values for some ferromagnetic materials

Material	Gd	Ni	Fe	Co
Curie Temperature [^{0}C]	16	358	770	1131

12.2. Losses in Magnetic Materials

Section Summary: *In this section, there are described the magnetic hysteresis losses, eddy current losses and supplementary magnetic losses which are developed in soft magnetic materials.*

A magnetic material, introduced in a constant or variable external magnetic field, consumes magnetic field energy for magnetization, because many of the magnetization processes are irreversible.

More types of losses can be highlighted:

a. Hysteresis losses
b. Eddy current losses
c. Supplementary losses

a. Hysteresis losses

Losses due to magnetic hysteresis are determined by the energy spent by the external magnetic field during the magnetization cycle.

The magnetic dissipated energy on the volume unit during a complete hysteresis cycle has the following expression:

$$w_h = \oint BdH \qquad (12.9)$$

Hysteresis losses are numerically equal to the area of the cycle surface of the magnetic hysteresis cycle. To calculate this integral, it is necessary to know the analytical expression of the hysteresis cycle loop.

$$w_h = S_{cycle}\left[\frac{W}{m^3}\right] \qquad (12.10)$$

With the Rayleigh model of the hysteresis cycle, for low magnetic fields, the volume density of magnetic energy is:

$$w_{mg} = \mu_0 \nu_R \cdot \frac{4H_m^3}{3}\left[\frac{J}{m^3}\right] \qquad (12.11)$$

Where:

μ_0=4π.10^{-7} H/m -is magnetic permeability of vacuum,

H_m-is the maximum intensity of magnetic field applied,

υ_R-is Rayleigh coefficient, specific for each material.

For a period T of magnetizing cycle, respectively, for the frequency $f=1/T$ of magnetization, it will result that the density of re-magnetization losses per unit of volume:

$$p_h = \frac{w_{mg}}{T} = w_{mg} \cdot f = \mu_0 \nu_R \cdot f \cdot \frac{4H_m^3}{3}\left[\frac{W}{m^3}\right] \tag{12.12}$$

and

$$p_h = \frac{\mu_0 \nu_R \cdot 4}{3} \cdot \frac{B_m^3}{(\mu_0 \mu_{rR})^3} \cdot f \tag{12.13}$$

whereμ_{rR} is the Rayleigh relative permeability, and B_m is maximum value of magnetic induction.

The specific losses can be expressed in W/kg, considering the material density d_m:

$$p_h = \mu_0 \nu_R \cdot f \cdot \frac{4H_m^3}{3d_m}\left[\frac{\mathrm{W}}{\mathrm{kg}}\right] \tag{12.14}$$

Observations:

For magnetization regimes with a sinusoidal variation in time, the experimental relation of Steinmetz can be used:

$$p_h = const \cdot f \cdot B_m^n, \tag{12.15}$$

where index n takes values between 1.2 and 1.8.

Thus, hysteresis magnetic losses density (in W/kg or W/m^3) increase with the magnetization frequency$f$ and magnetic induction B.

In magnetic fields of non-sinusoidal form, the losses due to hysteresis loop increase, due to the contribution of associated harmonics. In this case, we can the add all the harmonics by serial decomposition of the waveforms for H, and B, respectively.In order to reduce the hysteresis losses, it is necessary to reduce the area of the hysteresis cycle. For this purpose, it is necessary for both the remanent induction and coercive field to have lower values.

b. Eddy current losses

Eddy current losses are due to the currents induced into the materials which have finite conductivity when they are placed in a time variable magnetic field.

An example is considered here: a coil with ferromagnetic core supplied in the alternative current (Fig. 12.8).

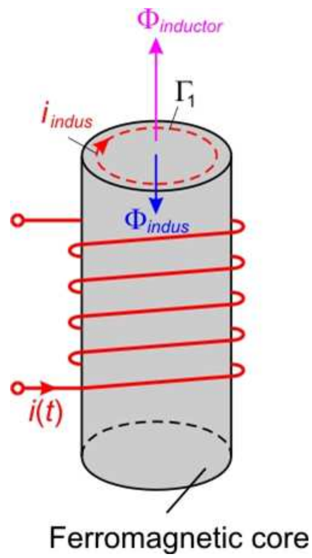


Fig. 12.8. Model for evaluation of eddy current losses in a ferromagnetic core

The electromotive forces induced by the variable flux in the considered piece establish electric currents, which produce Joule-Lenz losses in it. These currents, called eddy currents, modify the induction dictribution in the ferromagnetic piece. By taking the case of a coil with a ferromagnetic core (Figure 12.8.), we can express the losses through eddy currents in the Γ circuit. In other words,the variable magnetic field in time induces in the Γ circuit the currents, called eddy currents *i,* which produce losses.The losses through eddy currents p_{ct} can be evaluated:

$$p_{ct} = \sum R_i \cdot i_{induced}^2 = \sum \frac{u_{e\Gamma_i}^2}{R_i} = const. \cdot \left(\frac{d\Phi_{inductor}}{dt}\right)^2, \quad (12.16)$$

where$R_i$is the electric resistance of frames Γ_i traced in the ferromagnetic material, and $u_{e\Gamma i}$ is the electromotive voltage induced by the magnetic field $\Phi_{inductor}$.

The specific losses on the mass unit, in aelectrotechnic sheet, in a harmonic regime of low frequency magnetization, are obtatined with theory of electromagnetism, as

$$p_{ct} = \frac{\pi^2}{6\rho d_m} \Delta^2 \cdot f^2 \cdot B_m^2 [\text{W/kg}], \quad (12.17)$$

where:

ρ-is the sheet resistivity,

d_m -is the mass density of the sheet,

Δ -is the sheet's thickness,

f -is the frequency of the magnetic field,

B_m -is the amplitude (magnitude) of the magnetic induction in the transversal section of the sheet.

This relation signifies:

- the effect of the frequency on the losses: the specific losses due to eddy currents increase with the square of magnetization frequency;
- the effect of the material resistivity ρ and of the thickness Δ.

c. Supplementary magnetic losses

The additional magnetic losses are determined by:

- Magnetic resonance phenomena;
- Relaxation phenomena due to impurities and defects;
- Viscosity phenomena at the limits between the crystalline grains, etc.

d. Total losses in magnetic materials

Total specific losses in magnetic materials are given by the sum of the magnetic hysteresis losses, eddy current losses and supplementary magnetic losses:

$$p_{tot} = p_h + p_{ct} + p_{supl} \qquad (12.18)$$

In the case of magnetization in harmonic fields at industrial frequencies, preponderant are the hysteresis losses and the eddy current losses:

$$p_{tot} = p_{Fe} = af + bf^2 \qquad (12.19)$$

Example: The total specific losses variation according to the induction at frequencies of 30 Hz, 50 Hz şi 60 Hz, for hot-rolled Fe-Si sheets of 0.35 mm, is shown in Fig. 12.9.

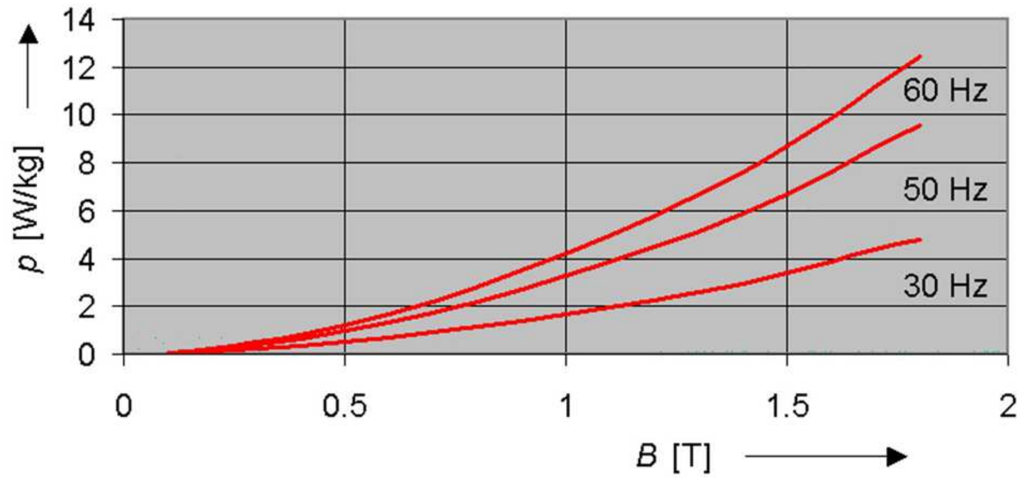


Fig. 12.9. Total specific losses variation with the magnetic induction and frequency for hot-rolled Fe-Si sheets of 0.35 mm.

Short Test for Lesson 12

1. Describe the four specific properties of soft magnetic materials.
2. Explain the occurrence of the hysteresis cycle behaviour in soft magnetic materials. What parameters of the material are influenced by the hysteresis cycle behaviour?
3. Define the hysteresis magnetic losses (literary and mathematically)
4. Define the eddy current magnetic losses (literary and mathematically).
5. Give the expression of total losses in magnetic materials (literary and mathematically).

13. Soft Ferromagnetic Materials - Magnetic Domain Structures and Applications

Contents

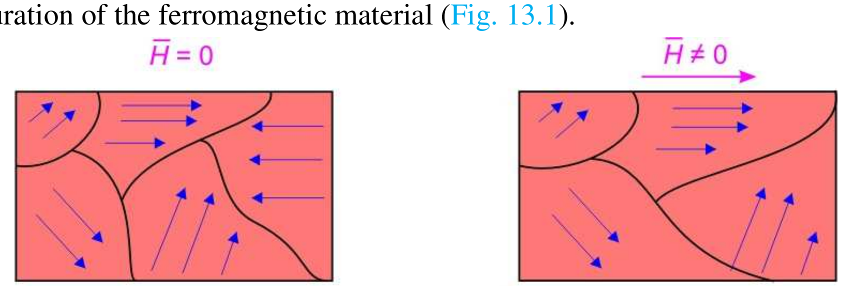
13.1. Theory of Magnetic Domains in Ferromagnetic Materials

Section Summary: *In this section, the main aspects of theory of magnetic domains and, in particular, the structure of magnetic domains in ferromagnetic materials are described. Some argumentation about the formation of magnetic domains, and properties of Bloch walls between magnetic domains are pointed.*

Soft magnetic materials (ferromagnetics) are materials with homo-parallel order of the atomic magnetic moments. The properties of this type of materials can be justified with magnetic domain theory.

13.1.1. Structure of magnetic domains in ferromagnetic materials

The domain structure of a ferromagnetic material is determined by the energetic configuration of the ferromagnetic material (Fig. 13.1).



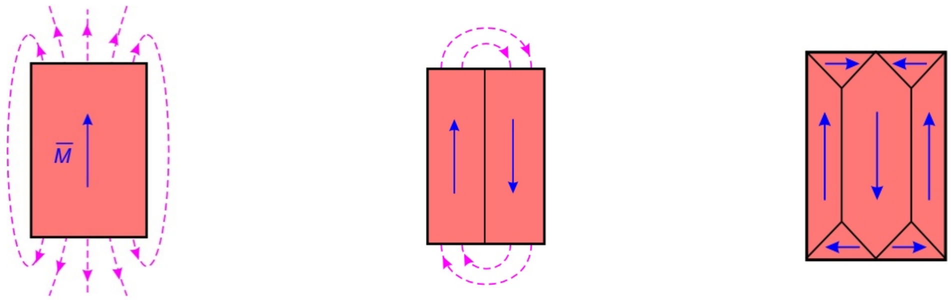
a) b)

Fig. 13.1. Magnetic domain structure in ferromagnetic materials: a) in the absence of magnetic field; b) when an magnetic field is applied

In Fig. 13.1, it is observed that when the magnetic field is applied, the volume of the magnetic domains that have the magnetic moments oriented in a direction close to the direction of the applied magnetic field increases.

13.1.2. Why the magnetic domains structure is favorable?

The case of a ferromagnetic single-crystal is considered (Fig. 13.2). In hypothesis that all magnetic moments are oriented in the same direction, the state of magnetization is described by Fig. 13.2.a.



a) b) c)

Fig. 13.2. Magnetic domain structure in ferromagnetic single-crystal, with the following hypotheses: a) single magnetic domain; b) two magnetic domains; c) multi-domains structure

Analyzing the structures described in the Fig. 13.2, it can justify that only the multi-domain structure can describe the non-magnetic state in ferromagnetic materials. The multi-domain structure can be formed when a minimum energy state is achieved.

The total energy in the ferromagnetic crystal is formed by:

- The magnetostatic energy;
- The exchange energy;
- The magnetocrystalline anisotropy energy (anisotropy energy);
- The magnetostrictive energy;
- Zeeman energy.

Magnetostatic energy is potential energy of magnetic moments in the field they have created. The demagnetizing factor is an essential contribution to its definition.

The exchange energy is a quantum term essential for defining the ferromagnetic state and is responsible for the spin alignment process. This implies an energy use in order to change the magnetization direction which can be compensated by thermal energy. At high temperatures (Curie temperature), the phase transition ferro-paramagnetic occurs.Magnetostatic energy is potential energy of magnetic moments in the field they have created. The demagnetizing factor is an essential contribution to its definition.

Magnetocrystalline anisotropy defines preferential magnetization direction over the crystallographic axes.

Magnetoelastic energy produced the change in magnetization due to mechanical tension (magnetostriction).

Zeeman energy represents a potential energy term of the magnetic moment in magnetic field.

The stability is obtained with the condition of minimum of these forms of energy assembly.

Observation:

Note that the magnetic behaviour of bulk materials is different than that of magnetic thin films. Usually, for a sufficiently small thickness of the layer (in many cases, lower than

3·10^{-4} mm, the stable structure consists of a single magnetic domain with the magnetization parallel in the film plane, but for a greater thickness, the magnetization is perpendicular on the film plane (a multi-domain structure can arise).

In Figure 13.3 Bloch andNéel type domain walls are illustrated. On the inside of a Bloch domain wall, the magnetization rotates perpendicularly to the film plane.Note that in Néel type magnetization remains in the film plane, rotating around an axis which is perpendicular on the film plane.   Due to the dependency of the magnetostatic energy to the thickness of the film, a transition between Bloch and Néel walls will exist (cross-tie type wall).

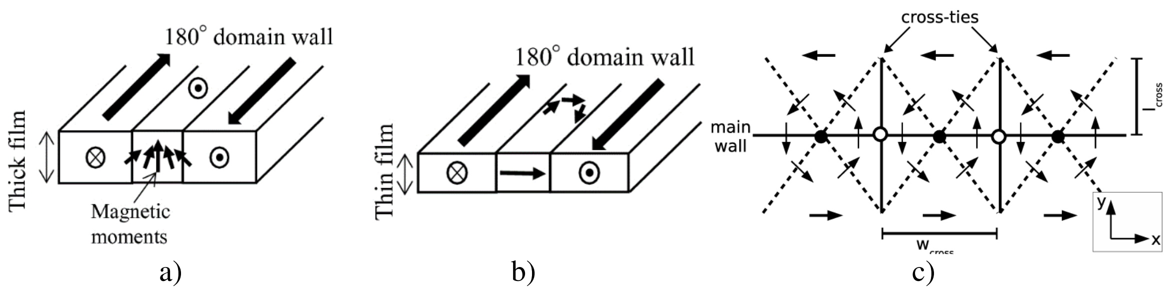


Fig. 13.2.Domain walls in magnetic thin films: (a) Bloch wall; (b) Néel wall; (c) Cross-tie type wall 180ºwall (The empty circle notation signifies that the magnetization M enters perpendicularly on the film plane).

13.1.3. Bloch walls between magnetic domains

In description of the magnetic properties of ferromagnetics, important are the Bloch walls between the magnetic domains (Fig. 13.4).

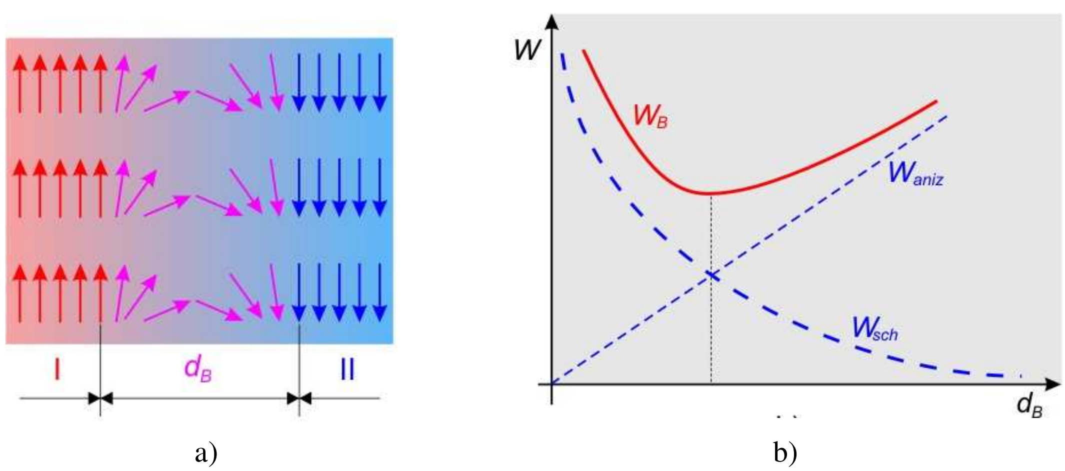


Fig. 13.4. Magnetic domain walls: a) orientation of magnetic moments in Bloch wall of thickness d_B; b) dependence of Bloch wall's magnetic anisotropy energy $W_{aniz}$and the exchange energy W_{sch} on the wall thickness d_B.

In Fig. 13.4.a, it is described the model for the region between two magnetic domeins I, and II, with the magnetic moment oriented at 180 degrees.

In Fig. 13.4.b, it is described that the Bloch wall corresponds to the state of the equilibrium between magnetic anisotropy energy $W_{\text{aniz}}$and the exchange energy W_{sch}.

Notes:

- Narrow and flexible Bloch walls allow easier modification of magnetic domains - This corresponds to the type of soft magnetic materials.
- The broad and immobile Bloch walls do not allow the modification of magnetic domains - This corresponds to the type of hard magnetic materials.

13.2. Structure of Domains in Magnetization Process

Section Summary: *In this section, the structure of domains in magnetization processes is described. The structures of magnetic domains for first magnetization curve and for hysteresis curve are detailed.*

13.2.1. Structure of magnetic domains for first magnetization curve

For the case of magnetization of a ferromagnetic single-crystal in a constantly increasing magnetic field of intensity *H*, initially in a de-magnetized state, in Fig. 13.5, the stucture of magnetic domains is shown.

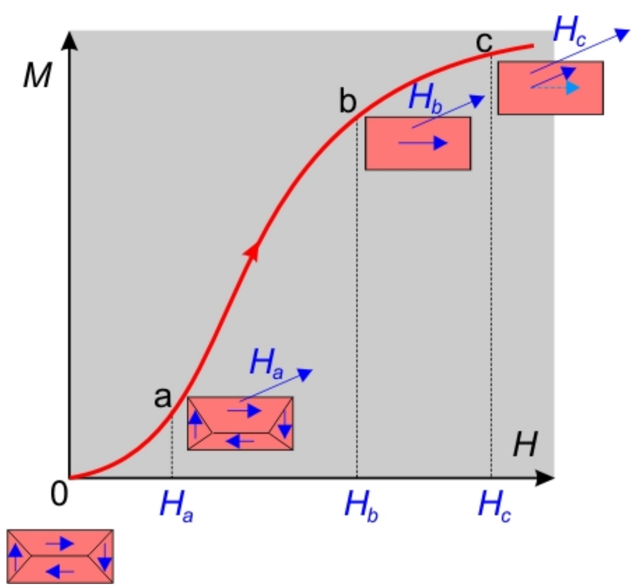


Fig. 13.5. The magnetic domain structures for a ferromagnetic single-crystal in a constantly increasing magnetic field of intensity *H*

In Fig. 13.5, it is shown the modification of form and structure of the magnetic domains for the first magnetization curve (0-a-b-c).

- For small values of the magnetic field ($H<H_a$), the domain structure corresponding to the demagnetized state is preserved.
- When the intensity H of the magnetic field increases ($H_b>H>H_a$), the area of the magnetic domain that has the direction of magnetization close to the direction of the magnetic field increases.
- In state b, corresponding to the magnetic field of intensity H_b, the single-crystal contains a single-magnetic domain, whose magnetization is oriented following the axis of easy magnetization. The point b defines technical state of saturation**.**
- In high values of magnetic field ($H_c>H>H_b$) the ferromagnetic single-crystal is characterized by existence of one magnetic domain, with magnetization oriented after the easy magnetization axis.
- In very high values of magnetic field ($H>H_c$) the ferromagnetic single-crystal is characterized by existence of one magnetic domain, with magnetization oriented after the direction of magnetic field.

<u>13.2.1. Structure of magnetic domains for hysteresis curve</u>

In order to justify the hysteresis cycle, it is considered the de-magnetization process of ferromagnetic single-crystal, magnetized to saturation, the state *c* in Fig. 13.5.

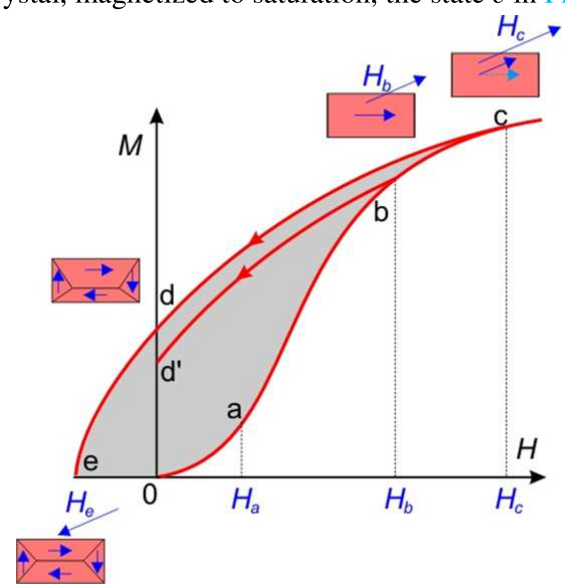


Fig. 13.5.Domain structure for demagnetizing region of the hysteresis curve.The width of the cycle depends on the volume and elasticity of the magnetic domains.

In direct dependence on the thickness and mobility of the Bloch walls, there are:

- **hard magnetic materials**(high coercive fields), with non-elastic and high thickness of Bloch walls;
- **soft magnetic materials** (low coercive fields), with thin elastic and high mobilities of Bloch walls.

13.3. Applications of Soft Magnetic Materials

Soft magnetic materials are present in many applications: magnetic data storage devices, magnetic cores for electrical transformers, electromagnets, coils, relays, stator and rotor for electrical machines, magnetic sensors etc. In many sectors of activities, such as vehicle industry, their use is vital (see some examples inFig. 13.6).

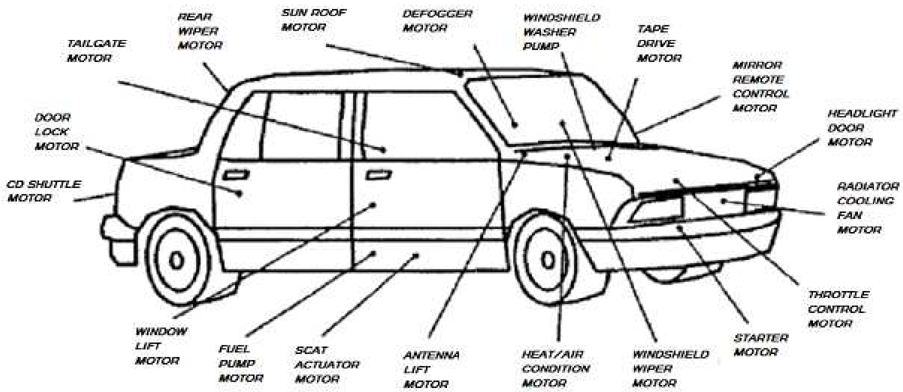


Fig. 13.6.The use of magnetic materials in hybrid and electrical vehicle industry.

Short Test for Lesson 13

1. Describe the four specific properties of soft magnetic materials.
2. Define the hysteresis magnetic losses (literary and mathematically).
3. Define the eddy current magnetic losses (literary and mathematically).
4. Give the expression of total losses in magnetic materials (literary and mathematically).
5. What are the magnetic domains? What is the Bloch wall? How can be classified the ferromagnetic materials according to the thickness and mobility of the Bloch walls?
6. Draft a single crystal with magnetic domains in demagnetized state, respectively at saturation magnetized state. Justify the shape of the first magnetization curve and the shape of hysteresis cycle, using the magnetic domains theory.
7. Draw the curves of a hysteresis cycles specific to a ferromagnetic material (the first magnetization curve, the maximum limit curve and a minor symmetrical and asymmetrical curve).

14. Hard Magnetic Materials - Properties and Applications

Contents

14.1. Properties of Hard Magnetic Materials

Section Summary: *In this section, the main characteristics and material parameters of hard magnetic materials are described. The demagnetizing curve and magnetic energy density curve are defined and examples of characteristics are given.*

Hard magnetic materials keep their magnetization state after cancelation of external magnetic field action.

The main characteristics of hard magnetic materials are:

- Large magnetic hysteresis loops (Fig.14.1),
- High remanence inductions B_r, usual values are from 0.2 T to 2.5 T,
- High coercive fieldsH_c, usual values are from 4 to 1000 kA/m,
- High values of the quality index, denoted by $(BH)_{max}$, which has usual values from 50 to 300 kJ/m^3.

In Fig. 14.1, the magnetic hysteresis loop difference between the soft and hard magnetic materials is shown. The maximum limit for coercive field for which a magnetic material is considered as soft magnetic material is H_{cmax} = 1 kA/m.

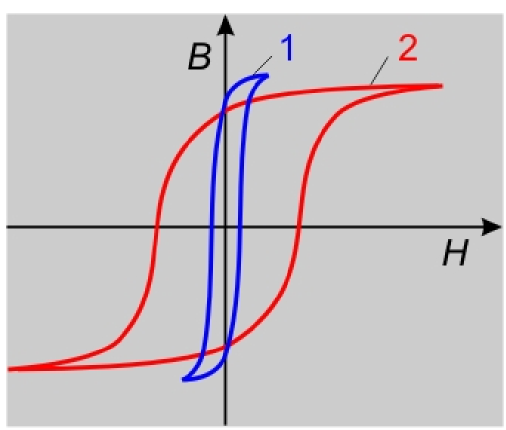


Fig. 14.1. Hysteresis loops: a) for soft magnetic materials b) for hard magnetic materials

In applications of hard magnetic materials, of great importance is demagnetizing curve situated in the second quadrant of hysteresis cycle (Fig. 14.2).

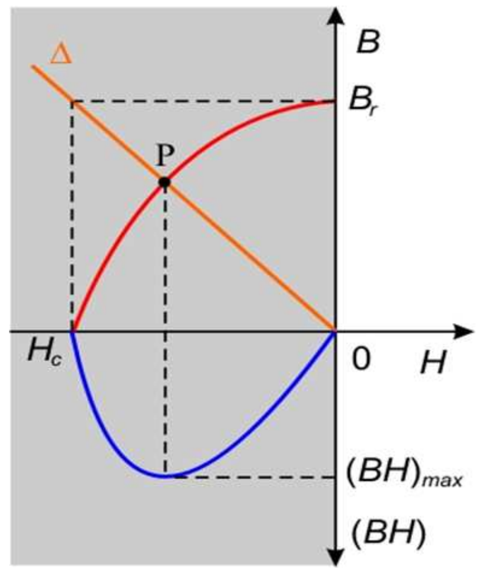


Fig. 14.2. Demagnetizing curve B=f(H) and magnetic energy density curve (BH)=$f(H)$ in hard magnetic materials

Demagnetization curve is the corresponding portion of the magnetic hysteresis loop B = f(H) of the second quadrant, in which the working point of the permanent magnet P(H_m, B_m) is defined.

Magnetic energy density curve (BH)=f(H) represent the dependence of (BH) product on magnetic induction B and intensity H of applied magnetic field.

Quality index is a material parameter which corresponds to the maximum value of magnetic energy density stored by the magnet, measured in kJ/m^3.

Examples of quality index values for some hard magnetic materials:
- for martensitic steels:

$(BH)_{max}=(B_r H_c)/2 = (1.2\text{T x } 4\text{kA/m})/2 \sim 2.4 \text{ kJ/m}^3$

- for barium ferrite:

$(BH)_{max}=(B_r H_c)/2 = (0.3\text{T x } 200 \text{ kA/m})/2 \sim 30 \text{ kJ/m}^3$

- for neodymium-iron-boron:

$(BH)_{max} \sim 300 \div 400 \text{ kJ/m}^{3.}$

Examples of magnetic characteristics for a neodymium-iron-boron hard magnetic material are given in Fig. 14.3 – for demagnetizing curve, and in Fig. 14.4 – for hysteresis cycle B=f(H).

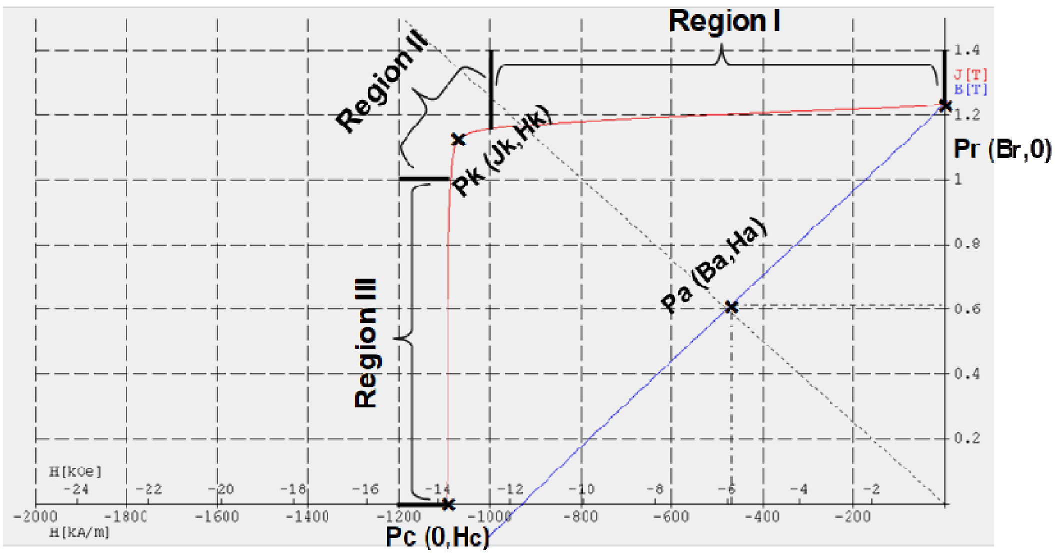


Fig. 14.3. Demagnetizing curve for aNdFeB permanent magnet sample

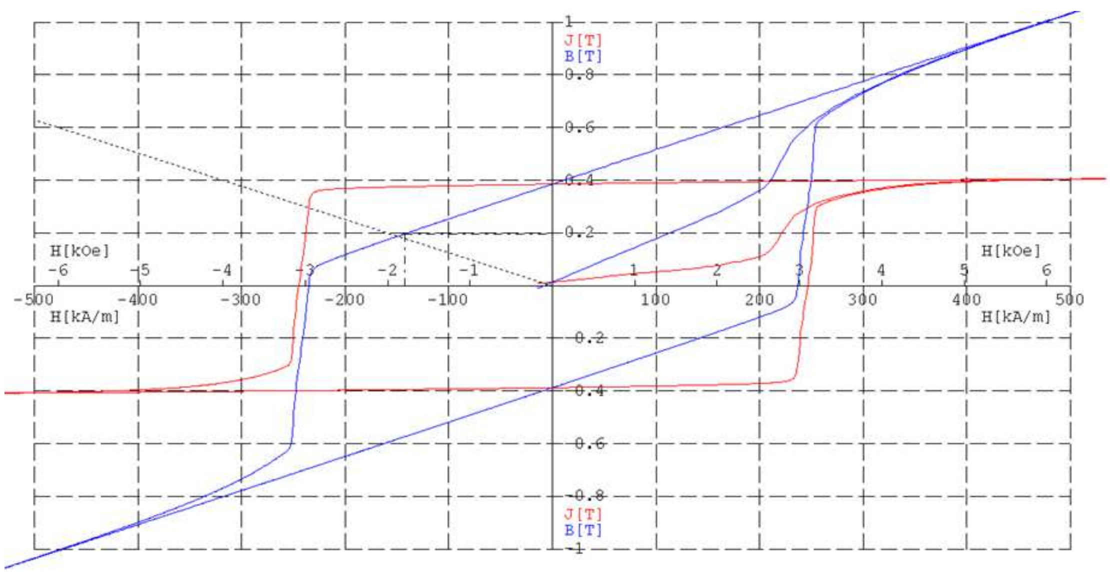


Fig. 14.4. Hysteresis cycles B=f(H) and J=f(H) for a NdFeB permanent magnet sample.

In Fig. 14.4, the hysteresis cycles indicate the variation of magnetic induction B and variation of magnetic polarization J_B when magnetic field applied described a full cycle of variation. Note that the values of the coercive field is around 200 kA/m for this type of magnetic material.

14.2. Permanent Magnet Operation Point and Design Selection Criteria

Section Summary: *In this section, the elements of establishment of the operation point for permanent magnets are given. Dynamic operation regime for a rotate magnet is described. Selection criteria for permanent magnets are defined and some examples of designing the permanent magnets are also pointed.*

14.2.1. Establishment of the operation point of permanent magnets

The structure of a magnetic circuit with permanent magnet is shown in Fig. 14.5.

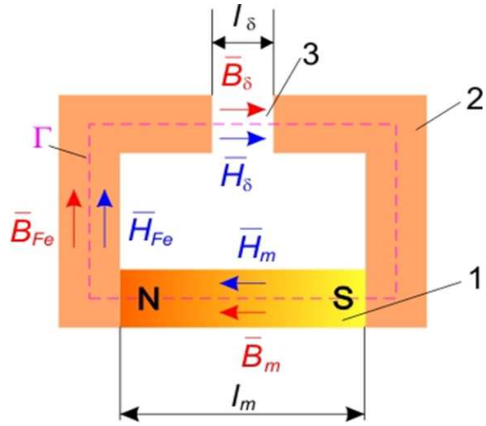


Fig. 14.5. Structure of a magnetic circuit with permanent magnet: 1 – had magnetic material; 2 – soft magnetic material; 3 – air gap.

In Fig. 14.5, the permanent magnet, noted 1, made of hard magnetic material, is the magnetic field source. The magnetic pieces, noted 2, made of soft magnetic material, are role for directing and concentrating the magnetic field lines in the air gap of the magnetic circuit (aperture), noted 3.
Magnetization process of the permanent magnet and establishing the operating point on $B=f(H)$ curve is shown in Fig. 14.6.

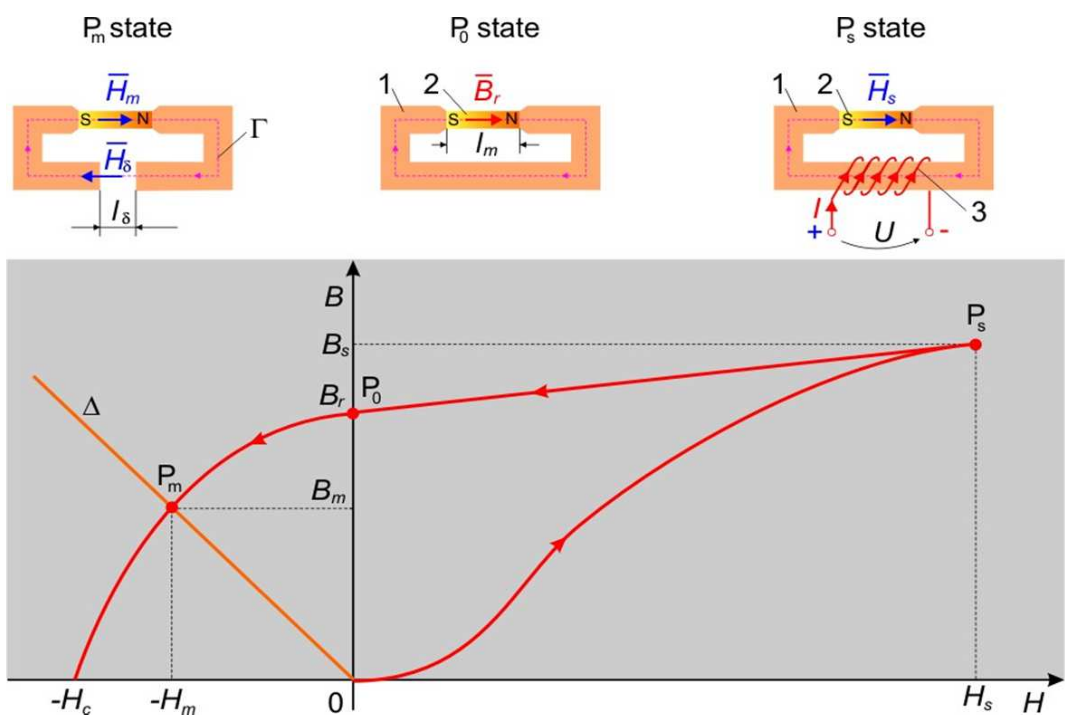


Fig. 14.6. Magnetization process of the permanent magnet and establishing the operating point on $B=f(H)$ curve.

The states through which a hard magnetic material passed until it is used as a permanent magnet are described in Fig. 14.6. The magnetic states of the magnet are changed in function of the applied magnetic field and with the structure of magnetic circuit.

- Saturation state (noted by Ps) corresponds to the magnetization state at saturation, which is obtained by placing the hard magnetic material in a magnetizer coil system where a magnetizing current passes. Note: in order to use the hard magnetic material as a permanent magnet, the material is first magnetized in high magnetic fields ($H > 5\times H_c$). Magnetic circuit is a closed circuit.
- Remanence state (noted by P_0) corresponds to the remanence state, when the permanent magnet is placed in a closed magnetic circuit. In the magnetizer coil system the magnetizing current is cancelled. The magnetic circuit is also a closed circuit.
- Working state (noted by P_m) represents the operating state of permanent magnet, when the permanent magnet is introduced in the open magnetic circuit, provided with air gap.

14.2.2. Mathematical modelfor establishing the operating point

The notations from Fig. 14.5 are used for establishing the operating point of a permanent magnet.
With the magnetic circuit law applied on the Γ curve:

$$\oint_{\Gamma} \bar{H} \cdot d\bar{l} = 0 \quad (14.1)$$

$$\int_{l_m} \bar{H}_M \cdot d\bar{l} + \int_{l_\delta} \bar{H}_\delta \cdot d\bar{l} = 0 \quad (14.2)$$

it results the expression of the magnetic field intensity in the permanent magnet:

$$\bar{H}_m = -\frac{l_\delta}{l_m} \cdot \bar{H}_\delta \quad (14.3)$$

Or

$$\bar{H}_\delta = -\frac{l_m}{l_\delta} \cdot \bar{H}_m \quad (14.4)$$

The sign minus specifies that the magnetic field strength in the magnet H_m has an opposite direction relative to the field strength in the air gap H_δ. This magnetic field ($-H_\delta$) is therefore called demagnetizing field.

With the magnetic flux law, applied on the surfaces of magnet S_m and of air gap S_δ:

$$\int_{S_m} \bar{B} \cdot d\bar{S} = \int_{S_\delta} \bar{B} \cdot d\bar{S} \quad (14.5)$$

Thus, the expression of the magnetic induction in the air gap is obtained:

$$\bar{B}_m = \bar{B}_\delta \frac{\bar{S}_\delta}{\bar{S}_m} \quad (14.6)$$

whereB_m and B_δ represent the magnetic inductions in the permanent magnet and the air gap, respectively, and S_m and S_δ are the cross sections of the permanent magnet and air gap.

The analytic expression of the load line Δ is obtained:

$$\bar{B}_m = -\frac{l_m}{l_\delta} \cdot \frac{\bar{S}_\delta}{\bar{S}_m} \cdot \mu_0 \bar{H}_m \tag{14.7}$$

Or

$$\bar{B}_m = -\pi_m \mu_0 \bar{H}_m \tag{14.8}$$

where magnetic circuit permeance is:

$$\pi_m = \frac{l_m}{l_\delta} \cdot \frac{\bar{S}_\delta}{\bar{S}_m} \tag{14.9}$$

The slope of the load line depends of the geometry of magnetic circuit (Fig. 14.7).

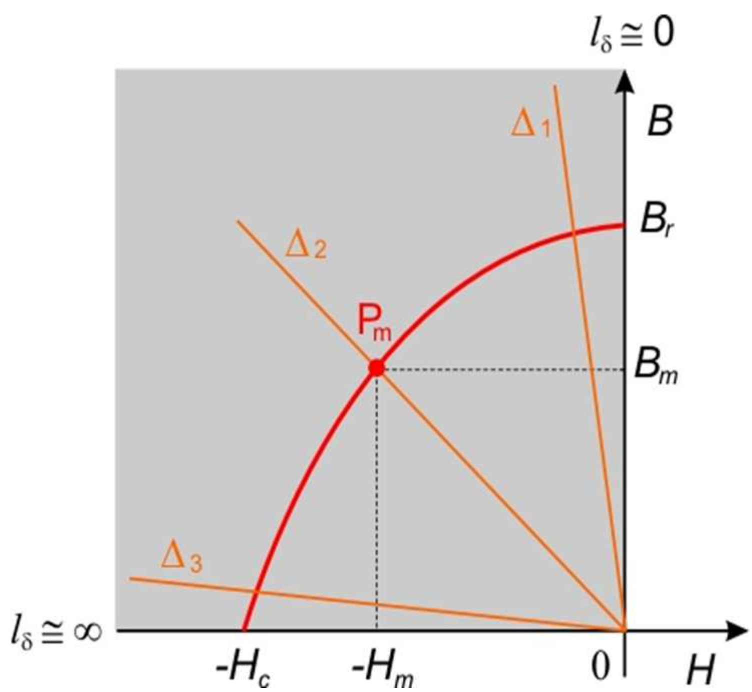


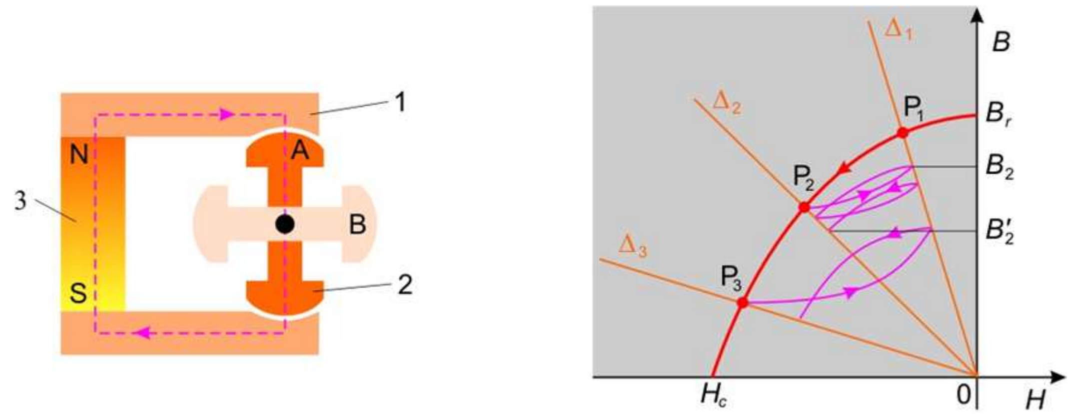
Fig. 14.7. Influence of the air gap length on the permanent magnet Δ load lines

The operating point $P_m$is obtained by intersecting the load line Δ with the *B*=*f*(*H*) characteristic of the magnet.

- A larger air gap corresponds to a lower slope of the load line (Δ_2) so that the operating point is displaced towards greater values of the demagnetizing field.
- For δ=0, operation point P_m reaches the remanence point (Δ_1).
- For δ very high (open circuit), operation point P_m tends to the coercivity point (Δ_3).

14.2.3. Dynamic operation regime for a rotate magnet

In Fig. 14.8, the dynamic operation regime for a rotate magnet is described.



a) b)

Fig. 14.8. Dynamic operation regime for a rotate magnet: a) magnetic circuit structure (1 - soft magnetic material, 2- rotative piece, 3 -permanet magnet as source of magnetic field); b) demagnetization curve of permanent magnet and variation of load curve when rotative piece closes or opens the magnetic circuit

In Fig. 14.8, load line Δ_1 corresponds to a maximum of the permeance, for the case when the magnetic core is completely introduced in the circuit. The state of magnet is described by P_1 point.

During the rotation of the piece, the air gap increases and the slope of the load line diminishes. Thus, if the core is rotated with 90º, the air gap permeance reaches the minimum value, and the reluctance is maximal, the operating point of the magnet reaches the P_2 position, corresponding to the load line Δ_2.

If the rotation continues, the air gap permeance tends again to a maximum value corresponding to recoil line Δ1, but the working point and the recovery, respectively are not made at the initial position, the magnetic induction reaching only B_2 value.

14.2.4. Designing and selection criteria for permanent magnets

In magnet circuit, various designing optimization criteria are applied.
An efficient optimization criterion is the energy criterion: the magnetic circuit is dimensioned so that the operating point P_m of the magnet to coincide with the maximum specific energy point.
Example, for a simple magnetic circuit, as in Fig. 14.5, the dimensions of the magnetic circuit will be:

$$\bar{l}_m = \bar{l}_\delta \cdot \frac{\bar{H}_\delta}{\bar{H}_m} \quad (14.10)$$

$$\bar{S}_m = \bar{S}_\delta \cdot \frac{\bar{B}_\delta}{\bar{B}_m} \quad (14.11)$$

The volume of the magnet will be:

$$V_m = V_\delta \cdot \frac{\bar{H}_\delta \cdot \bar{B}_\delta}{\bar{H}_m \bar{B}_m} \quad (14.12)$$

Note: the volume of magnetic material necessary for obtaining a certain magnetic field in the air gap decreases as the product $(H_m B_m)$ increases.

When the magnet operates in a point characterized by maximum energy density $(H_m B_m)_{max}$, it results:

$$V_{mmin} = V_\delta \cdot \frac{\mu_0 H_\delta^2}{(\bar{B}_m \bar{H}_m)_{max}} \quad (14.13)$$

An optimal magnetic circuit dimensioning corresponds to a correct choice of the magnet and to establishing of air gap permeance value such that the operating point is placed on the B = f(H) curve, corresponding to quality index $(H_m B_m)_{max}$.

14.3. Materials for Permanent Magnets – Evolution and Applications

Section Summary: *In this section, the main evolution of the research regarding the obtaining the hard magnetic materials with appropriate properties for a large scale of applications is presented. Examples are given for the performances of synthesized permanent magnets of neodymium-iron-boron, and a comparative analysis for demagnetizing curves and magnetic energy density curves of different hard magnetic materials is made.*

14.3.1. Evolution of the materials for permanent magnets

Permanent magnets, as sources for magnetic fields, have been developed along with the first experiences related to magnetism, but a great breakthrough in research to obtain new materials for permanent magnets took place at the end of the 19th century and the beginning of the 20th century, when the applications of electromagnetism diversified a lot.

Today, in the automotive industry alone, there are entire sectors of permanent magnet applications.

In theTable 14.1, some types of magnetic material developed between 1880 and 2000 are presented.

Table 14.1. Types of materials for permanent magnets and specific characteristics obtained during 1988-2000

Period / year	$(BH)_{max}$ [kJ/m³]	Material	Types
1880-1917	1-4	Carbon steel (1-1.2%)	-
1917-1930	4-10	Cobalt steel	01 MK, K5
1940-1950	10-40	Alnico	Alnico 5
1950-1955	20-40	Hard ferrites	Spinal 1
1960	40-80	Alnico	Alnico 9
1970	150-200	Cobalt-Rare earths	$SmCo_5$
1980	200-250	Cobalt-Rare earths	Sm_2Co_{17}
1980-1996	250-320	Iron-Neodymium-Boron	$Nd_2Fe_{14}B$

Progress in development of hard magnetic materials with high magnetic energy density have ben achieved.

The evolution of magnetic quality index $(B_m H_m)_{max}$, in the years 1900-2000, is shown in Fig. 14.9.

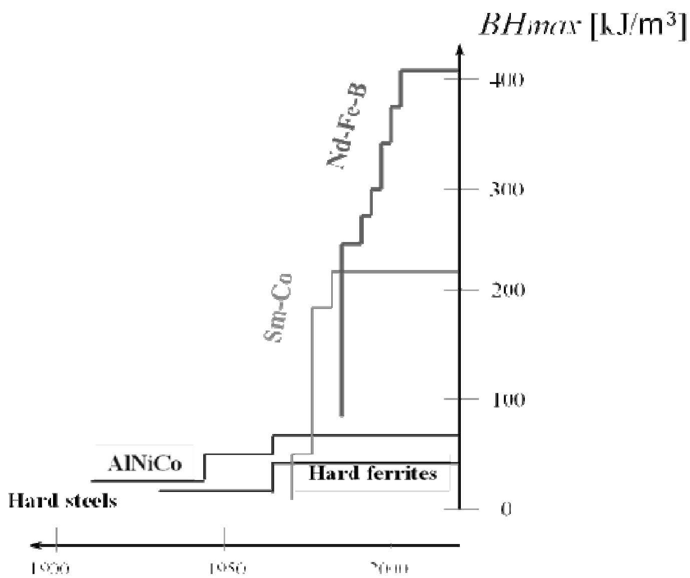


Fig. 14.9. Evolution of magnetic quality index $(B_m H_m)_{max}$, in the years 1900-2000, for different types of hard magnetic materials

A spectacular evolution was also obtained in assuring the desired values of remanence magnetic induction $B_r$and coercive magnetic field H_c (Fig. 14.10).

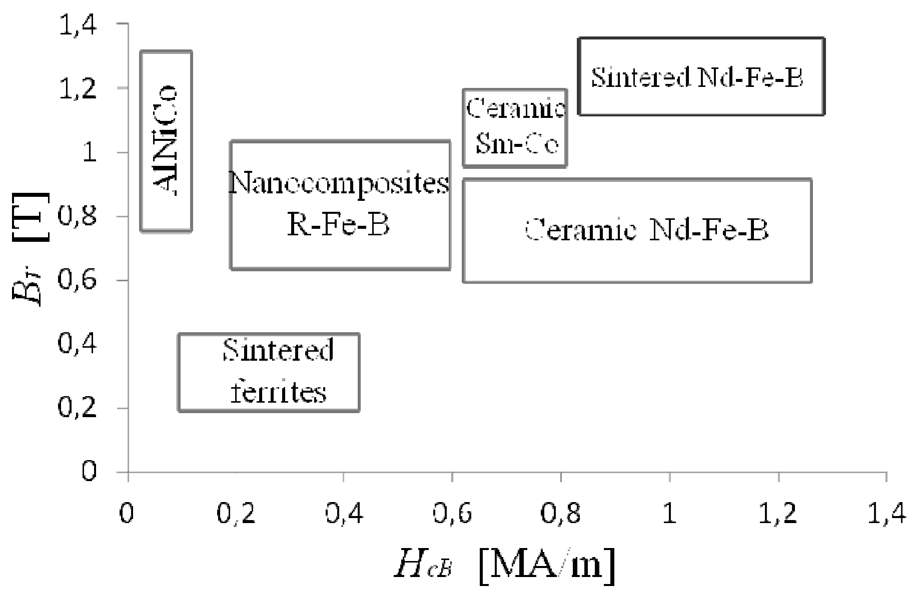


Fig. 14.10. Hard magnetic materials and their specific properties: remanence magnetic induction B_r and coercive magnetic field H_c

Sintered permanent magnets of neodymium-iron-boron (www.linkupmagnets.com) have been continuously improved, to ensure the desired values of remanence magnetic induction $B_r$and coercive magnetic field H_c (Fig. 14.11).

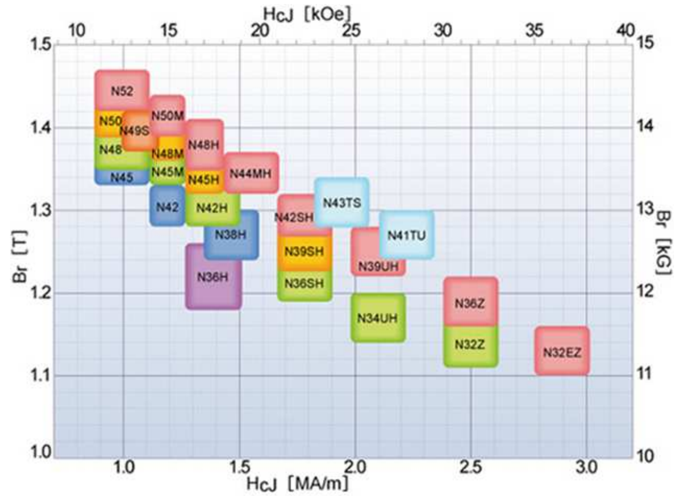


Fig. 14.11. Remanence magnetic induction B_r and coercive magnetic field H_c for some types of permanent magnets of synthesized neodymium-iron-boron

In Table 14.2, the main characteristics of sintered permanent magnets of synthesized neodymium-iron-boron are given (www.linkupmagnets.com)

Table 14.2. Characteristics of synthesized permanent magnets of neodymium-iron-boron

Magnetic Properties Of Sintered Neodymium Magnet

Series	Grade	Max Operating Temp. ℃	Remanence Br mT(KG)	Coercivity Hcb KA/m(KOe)	Intrinsic Coercivity Hcj KA/m(KOe)	Maximum Energy Product (BH)max KJ/m3 (MGOe)
N Series	N35	80℃	1170-1220(11.7-12.2)	≥868 (≥10.9)	≥955 (≥12)	263-287(33-36)
	N38	80℃	1220-1250(12.2_12.5)	≥899 (≥11.3)	≥955 (≥12)	287-310(36-39)
	N40	80℃	1250-1280(12.5-12.8)	≥907 (≥11.4)	≥955 (≥12)	302-326(38-41)
	N42	80℃	1280-1320(12.8-13.2)	≥915 (≥11.5)	≥955 (≥12)	318-342(40-43)
	N45	80℃	1320-1380(13.2-13.8)	≥923 (≥11.6)	≥955 (≥12)	342-366(43-46)
	N48	80℃	1380-1420(13.8-14.2)	≥923 (≥11.6)	≥955 (≥12)	366-390(46-49)
	N50	60℃	1400-1450(14.0-14.5)	≥796 (≥10.0)	≥876 (≥11)	382-406(48-51)
	N52	60℃	1430-1480(14.3-14.8)	≥796 (≥10.0)	≥876 (≥11)	398-422(50-53)
	N55	60℃	1470-1520(14.7-15.2)	≥796 (≥10.0)	≥876 (≥11)	414-438(52-56)
M Series	N35M	100℃	1170-1220(11.7-12.2)	≥868 (≥10.9)	≥1114 (≥14)	263-287(33-36)
	N38M	100℃	1220-1250(12.2-12.5)	≥899 (≥11.3)	≥1114 (≥14)	287-310(36-39)
	N40M	100℃	1250-1280(12.5-12.8)	≥923 (≥11.6)	≥1114 (≥14)	302-326(38-40)
	N42M	100℃	1280-1320(12.8-13.2)	≥955 (≥12.0)	≥1114 (≥14)	318-342(40-43)
	N45M	100℃	1320-1380(13.2-13.8)	≥955 (≥12.5)	≥1114 (≥14)	342-366(43-46)
	N48M	100℃	1360-1430(13.6-14.3)	≥1027 (≥12.9)	≥1114 (≥14)	366-390(46-49)
	N50M	100℃	1400-1450(14.0-14.5)	≥1033 (≥13.0)	≥1114 (≥14)	382-406(48-51)
	N52M	100℃	1430-1480(14.3-14.8)	≥1033 (≥13.0)	≥1114 (≥14)	398-414(50-52)
H Series	N33H	120℃	1130-1170(11.3-11.7)	≥844 (≥10.6)	≥1353 (≥17)	263-287(31-34)
	N35H	120℃	1170-1220(11.7-12.2)	≥868 (≥10.9)	≥1353 (≥17)	263-287(33-36)
	N38H	120℃	1220-1250(12.2-12.5)	≥899 (≥11.3)	≥1353 (≥17)	287-310(36-39)
	N40H	120℃	1250-1280(12.5-12.8)	≥923 (≥11.6)	≥1353 (≥17)	302-326(38-41)
	N42H	120℃	1280-1320(12.8-13.2)	≥955 (≥12.0)	≥1353 (≥17)	318-342(40-43)
	N45H	120℃	1300-1360(13.0-13.6)	≥963 (≥12.1)	≥1353 (≥17)	326-358(43-46)
	N48H	120℃	1370-1430(13.7-14.3)	≥955 (≥12.5)	≥1353 (≥17)	366-390(46-49)
	N50H	120℃	1400-1450(14.0-14.5)	≥955 (≥12.5)	≥1353 (≥17)	382-406(48-51)
SH Series	N30SH	150℃	1080-1130(10.8-11.3)	≥804 (≥10.1)	≥1592 (≥20)	233-247(28-31)
	N33SH	150℃	1130-1170(11.3-11.7)	≥844 (≥10.6)	≥1592 (≥20)	247-271(31-34)
	N35SH	150℃	1170-1220(11.7-12.2)	≥876 (≥11.0)	≥1592 (≥20)	263-287(33-36)
	N38SH	150℃	1220-1250(12.2-12.5)	≥907 (≥11.4)	≥1592 (≥20)	287-310(36-39)
	N40SH	150℃	1240-1280(12.5-12.8)	≥939 (≥11.8)	≥1592 (≥20)	302-326(38-41)
	N42SH	150℃	1280-1320(12.8-13.2)	≥987 (≥12.4)	≥1592 (≥20)	318-342(40-43)
	N45SH	150℃	1320-1380(13.2-13.8)	≥1003 (≥12.6)	≥1592 (≥20)	342-366(43-46)
UH Series	N30UH	180℃	1080-1130(10.8-11.3)	≥812 (≥10.2)	≥1990 (≥25)	223-247(28-31)
	N33UH	180℃	1130-1170(11.3-11.7)	≥852 (≥10.7)	≥1990 (≥25)	247-271(31-34)
	N35UH	180℃	1180-1220(11.8-12.2)	≥860 (≥10.8)	≥1990 (≥25)	263-287(33-36)
	N38UH	180℃	1220-1250(12.2-12.5)	≥876 (≥11.0)	≥1990 (≥25)	287-310(36-39)
	N40UH	180℃	1240-1280(12.4-12.8)	≥899 (≥11.3)	≥1990 (≥25)	302-326(38-41)
	N42UH	180℃	1280-1320(12.8-13.2)	≥899 (≥11.3)	≥1990 (≥25)	318-342(40-43)
	N45UH	180℃	1320-1380(13.2-13.8)	≥899 (≥11.3)	≥1990 (≥25)	334-358(42-45)
EH Series	N28EH	200℃	1040-1090(10.4-10.9)	≥780 (≥9.8)	≥2388 (≥30)	207-231(26-29)
	N30EH	200℃	1080-1130(10.8-11.3)	≥812 (≥10.2)	≥2388 (≥30)	223-247(28-31)
	N33EH	200℃	1130-1170(11.3-11.7)	≥836 (≥10.5)	≥2388 (≥30)	247-271(31-34)
	N35EH	200℃	1170-1220(11.7-12.2)	≥876 (≥11.0)	≥2388 (≥30)	263-287(33-36)
	N38EH	200℃	1220-1250(12.2-12.5)	≥899 (≥11.3)	≥2388 (≥30)	287-310(36-39)
	N40EH	200℃	1240-1280(12.4-12.8)	≥939 (≥11.6)	≥2388 (≥30)	302-326(38-41)
	N42EH	200℃	1280-1320(12.8-13.2)	≥939 (≥11.6)	≥2388 (≥30)	320-340(38-41)
AH Series	N30AH	220℃	1020-1080(10.2-10.8)	≥804 (≥10.1)	≥2786 (≥35)	223-239(28-30)
	N33AH	220℃	1140-1170(11.4-11.7)	≥844 (≥10.6)	≥2786 (≥35)	247-263(31-33)
	N35AH	220℃	1170-1210(11.7-12.1)	≥876 (≥10.9)	≥2786 (≥35)	263-279(33-36)
	N38AH	220℃	1220-1250(12.2-12.5)	≥876 (≥10.9)	≥2706 (≥34)	279-310(36-39)

The high quality of Nd-Fe-B permanent magnets was obtained by continuous improvement of the manufacturing processes of magnet production. In Fig. 14.12, the stages of manufacturing synthesized permanent magnets are described.

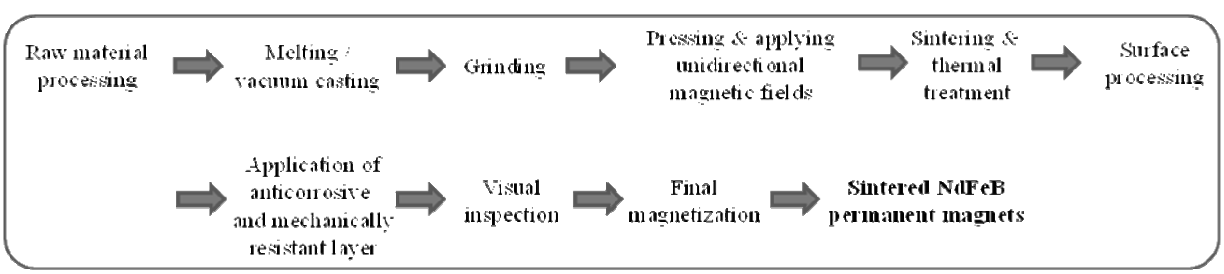


Fig. 14.12. Manufacturing stages for synthesized permanent magnets obtaining

In Table 14.3, Fig. 14.13 and Fig. 14.14, the comparative analyses of the hard magnetic material performances are given.

Table 14.3. Characteristics of hard magnetic materials

Type of PM	Chemical composition	H_c [kA/m]	B_r [T]	$(BH)_{max}$ [kJ/m^3]	Resistivity [Ωm × 10^{-4}]	Curie temp. [°C]
Hard Ferrite	$MO \cdot 6Fe_2O_3$ (M = Ba or Sr or their combination)	150÷290	0.23÷0.41	8÷32	1	450
AlNiCo	Al, Ni, Co, Cu, Fe, Ti, other (Si, Nb, Zr)	37÷151	0.67÷1.35	11÷72	47÷75	810÷860
Rare Earth	PR Co$_5$ (PR = Sm,Pr)	600÷720	0.83÷0.95	130÷180	53	1380
	PR_2MT_{17} (PR = Sm; MT = Fe, Cu, Co, Zr, Hf)	480÷840	1÷1.16	190÷240	86	1520
	PR_2MT_{14}B (PR = Nd, Pr, Dy; MT = Fe, Co)	760÷1030	1÷1.41	190÷400	160	590

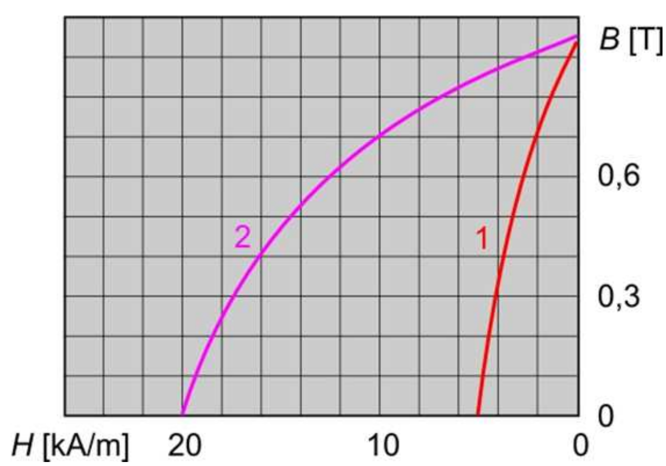


Fig. 14.13. Demagnetization curves for carbon steel magnets (curve 1) and 36% cobalt alloyed carbon steel magnets (curve 2)

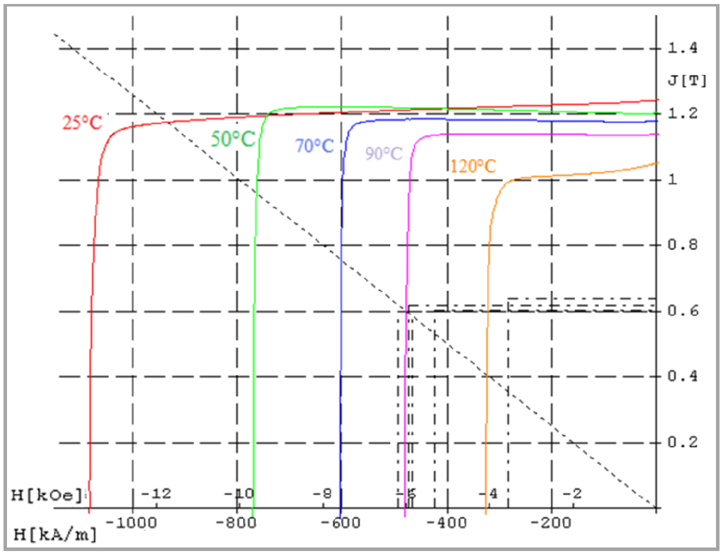


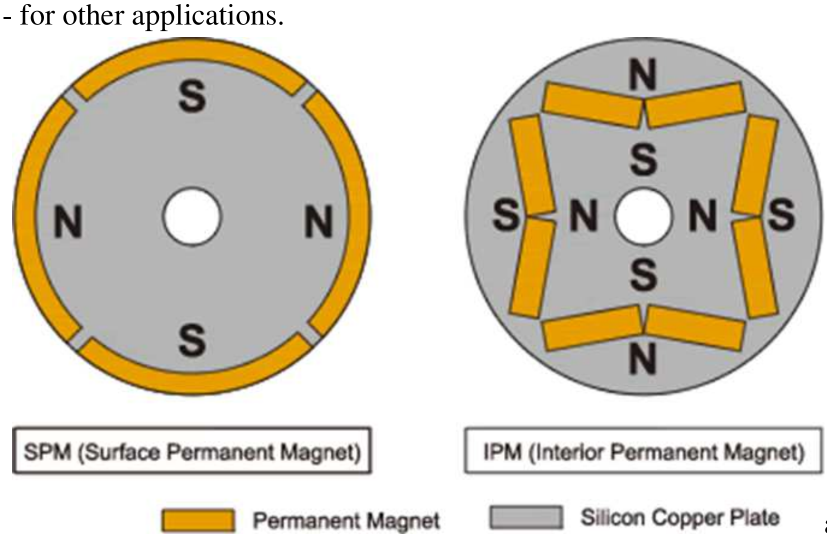
Fig. 14.14. The demagnetization curves for NdFeB permanent magnets in the temperature range of 25÷120ºC

14.3.2. Permanent magnet applications

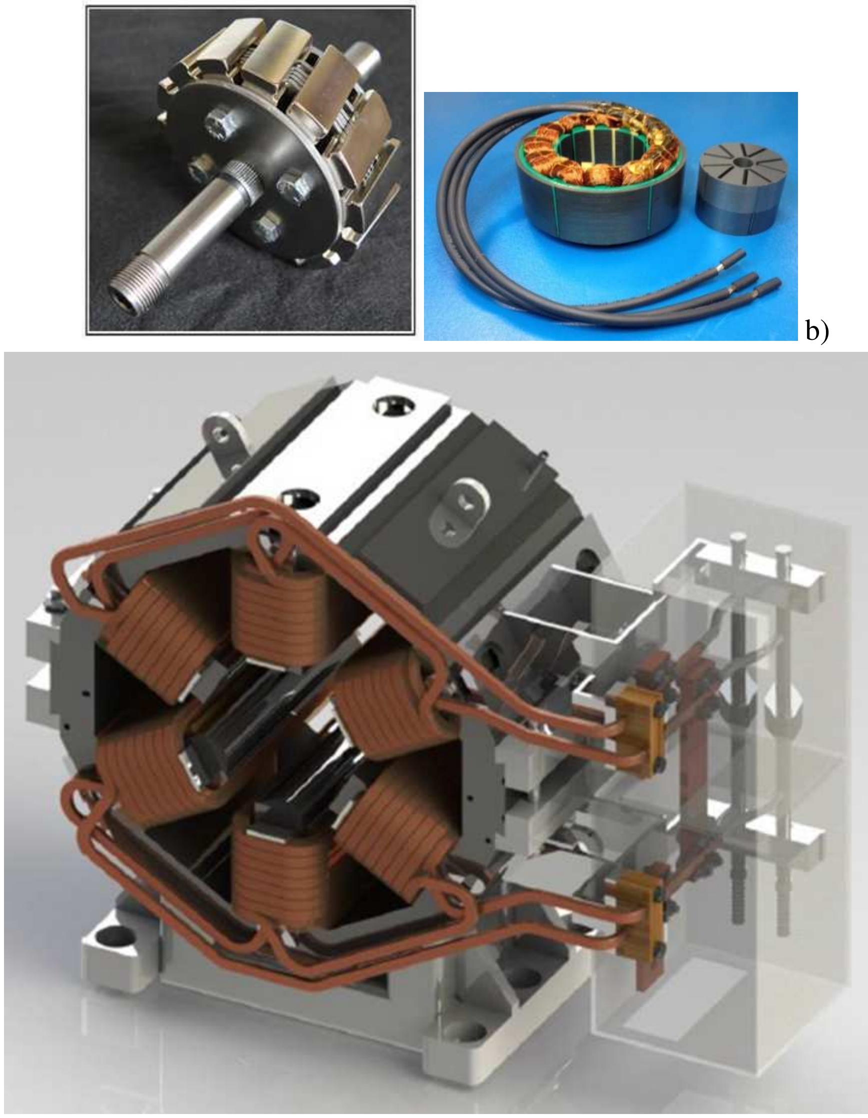
Permanet magnets are found in all areas where self-contained magnetic field sources are required.

Today, for the automotive sector, the share of permanent magnet production is continuously increasing, the distribution being as follows:

20% - for electric motors and generators,
20% - for automotive board electronic devices,
15% - for audiovisual devices (speakers, antennae, etc.),
10% - for electrical motors from auxiliary equipment,
8% - for informatics,
7% - for other applications.



a)



b)

c)

Fig. 14.15. The permanent magnet applications: a), b) PM motors; c) Hexapolar electromagnet for particle accelerator

Short Test for Lesson 14

1. Which are the magnetic parameters of hard magnetic materials?
2. Express some differences between soft and hard magnetic materials.
3. Give some examples of hard magnetic material classes, according to their evolution.
4. Draw the demagnetization characteristic of a permanent magnet.
5. Draw the structure of a magnetic circuit with permanent magnet.
6. Express the mathematical model for establishing the operating point of a permanent magnet.
7. Describe the energy criterion for optimal designing of magnetic circuits with permanent magnets.

15. Homework in Materials for Electrical Engineering

15.1. Homework Description, Proposed Equipment and Timetable

Aim:

Knowledge and explanation of the basic properties and functions of electrical and electronic materials in correlation with factors which influence their behaviour under different external stresses: electrical, thermal, mechanical, etc.

Working steps:

For electrical or electronic equipment, established at the beginning of the course, the following elements will be detailed:

1. Purpose and usefulness of the equipment (*with addition of found bibliographical resources*);
2. Description of the component parts (*accompanied by a draft or image - as in Fig. 1*);
3. Exemplification of active (conductive/ semiconductive, magnetic, dielectric) and passive materials (electro-insulating, mechanical, thermal, etc.) used in the construction of constituent elements (*drawing of a Table with the component parts and used materials*);
4. Description of the performance of an active material and of a passive material from the structure of the considered equipment (*the material types will be established together with tutor*);
5. Estimation of the effects of electrical, thermal, mechanical, environmental stresses to which the materials used in the construction of the constituent elements are subjected (*the calculation of the material parameters must be exemplified in detail, with their measurement units, as in the example given below*).

Proposed equipment for analysis:

1. Low power transformer (S_n< 1 kVA)
2. Three-phase transformer with oil cooling
3. Squirrel-cage induction motor
4. Permanent magnet synchronous motor
5. Electromagnet
6. Induction coil
7. Electronic converter
8. Photovoltaic panel
9. Submersible water pump
10. Electrical fuse with high breaking power
11. Ultra-fast safety fuse
12. Electric surge arrester
13. SF6 switch circuit breaker
14. Oil switch breaker
15. Three-polar contactor

16. Three-phase electric cable
17. Superconducting cable
18. Capacitor for power factor compensation
19. Supercapacitor
20. Rheostat
21. Thermocouple thermometer
22. Thermometer with semiconductor diode
23. Motion sensor
24. Electricity meter
25. Magnetometer
26. Wattmeter
27. Ammeter
28. Level gauges
29. Flowmeter
30. Magnet for particle accelerators
31. Other home or industrial appliances (at the choice of the students)

Remarks:

1. The work will be developed as teamwork (2-3 students per group).
2. Other types of electrical or electronic equipment can be proposed for homework by the students, in addition to the ones proposed by the teacher, with the teacher permission.
3. The evaluation is done during and at the end of the course, by presentation in front of the class.

Timetable for homework completion:

Work stage	Description	W1	W2	W3	W4	W5	W6	W7	W8	W9	W10	W11	W12	W13	W14
I	The choice of equipment studied	X	X												
II	Part I: Description of equipment. Relevant bibliography.			X	X	X	X	X							
III	Part II: Finding and noting the material parameters and calculations. Conclusions.								X	X	X	X	X		
IV	Presentation of the homework.													X	X

15.2. Example of Good Practice

Electrical materials applied in RHEOSTATS[1]

Part I – Description of the Equipment

What is a rheostat?

A rheostat is a variable resistor which is used to control current. Rheostats are able to vary the electric resistance in a circuit without current interruption. The construction is very similar to the construction of potentiometers. It uses only two connections, even when 3 terminals (as in a potentiometer) are present. The first connection is made to one end of the resistive element and the other connection to the wiper (sliding contact). In contrast to potentiometers, rheostats have to carry a significant current. Therefore,they are primarily constructed as wirewound resistors. Resistive wire is wound around an insulating ceramic core and the wiper slides over the windings.

Rheostats were often used as power control devices, for example to control light intensity (dimmer), speed of motors, heaters, and ovens. They are not typically used for this function anymore. This is because of their relatively low efficiency. In power control applications, they have been replaced by switching electronics. As a variable resistance, they are often used for tuning and calibration in circuits. In these cases they are adjusted only during fabrication or circuit tuning (preset resistor). In such cases trimpots are often used, wired as a rheostat. But dedicated two-terminal preset resistors also exist.

Activer part of a rheostat is made up of high resistivity material, like, nickel-chromium iron alloy closely wound over a circular tube. These are available both in a single tube and double tube. Inter-turn insulation is provided to avoid short-circuiting of turns. The tube of rheostat is made of insulating material, like ceramics.

The origin of the word rheostat comes from the Greek words “rheos” and “statis”. When these words are combined, they mean “flow control device” or “current control device”.

How does a rheostat work?

The basic principle that rheostat use is Ohm’s law, which states that current is inversely proportional to resistance for a given voltage. This means the current decreases as the resistance increases or increases as the resistance decreases.

[1]Homework was developed by the students Sutu Andrei-Teodor and MasarRazvan-Marius, (andrei.sutu@student.unitbv.ro, razvan.masar@student.unitbv.ro) in 2021-2022 academic year.

Current enters the rheostat through one of its terminals, flows through the wire coil and contact, and exits through the other terminal. Rheostats do not have polarity and operate the same when the terminals are reversed.

The structure of the rheostat is given in Fig. 15.1.Like the potentiometer, the rheostat consists of three terminals: terminal A, terminal B, and terminal C. However, only two terminals are used: A and B or B and C.

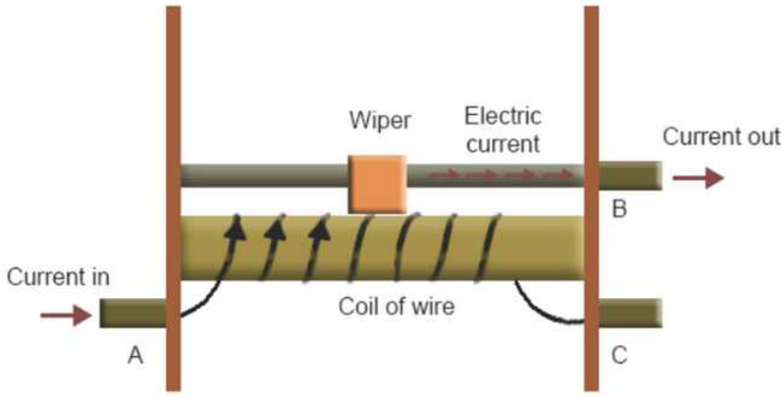


Fig. 15.1. Structure and operation principle of a rheostat

Two of the terminals are connected to the opposite ends of a resistive element, and the third terminal is located in between the other two terminals.Terminal B is the fixed terminal attached to the dial. A and C terminals are also connected to the resistive element.

Moving along the resistive element, the wiper changes the resistance of the rheostat. The resistive element in the rheostat is made from a wire coil or a thin carbon film. The resistance of the rheostat depends on the length of the resistance wire through which the electric current flows.

Symbol of rheostat

The American standard and the international standard symbol of rheostat are shown in Fig. 15.2.

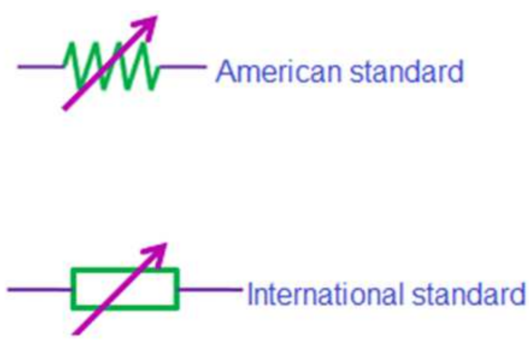


Fig. 15.2. Symbols of rheostat

The zigzag lines with three terminals represent the American standard symbol of rheostat and the rectangular box with three terminals represents the international standard symbol of a rheostat.

Working Principle of Rheostat

A rheostat is based on Ohm's law, which is given by:

$$\boldsymbol{R} = \frac{\boldsymbol{U}}{\boldsymbol{I}} \qquad (15.1)$$

where$R$ is resistance, U is voltage at terminals of rheostat, and I is intensity of the electric current passing through the rheostat.
From the above law, it can be seen that resistance $R$is inversely proportional to current intensity I. This means that an increase in resistance decreases the current and vice-versa.

Also, according to the following formula

$$R = \rho.\frac{L}{S} \qquad (15.2)$$

where ρ is the resistivity of the resistive wire of rheostat, L is the length and S is the area of cross-section of resistive wire, it results that the rheostat resistance is directly proportional to length of wire. Therefore, resistance increases as the length of the wire (i.e. number of turns) increases.

Types of Rheostats

1. Linear Rheostat

This type of rheostats has a linear resistive path. The sliding terminal glides over this path. There are two fixed terminals; however only one of the two is used. The other terminal is connected to the slider (Fig. 15.3).

Fig. 15.3. Linear rheostat

These are mostly used in laboratory applications. Mostly wire-wound resistive path along a linear cylinder shaped material is used.

2. Rotary Rheostat

With full justice to its name, a rotary rheostat has a rotary resistive path. These are mostly used in power applications. These rheostats have a shaft on which the wiper is mounted. Wiper is nothing but the sliding contact for a rotary rheostat, which can rotate over ¾ of a circle. The function and working principle are the same for both types of rheostats.
The Fig. 15.4 shows a rotary rheostat.

Fig. 15.4. Rotary rheostat

3. Preset Rheostat

When rheostats are used in a printed circuit board, they are used as trimmers or preset rheostats (Fig. 15.5). Trimmers are small rheostats, mostly used in calibration circuits. In practice, trimmers with two terminals are available; those with three terminals are generally used as rheostats with two terminals.

Fig. 15.5. Preset rheostat

What is the difference between rheostat and potentiometer?

- Both, rheostat and potentiometer are a kind of resistor, but there is a difference in usage between them.
- Potentiometers use all three terminals; whereas rheostats use only two terminals.
- Potentiometers divide the voltage, instead rheostats that adjust the current.
- Since rheostats are larger in structure than potentiometers, they limit higher currents.
- A rheostat cannot be utilized like a potentiometer, whereas a potentiometer can be utilized like a rheostat.

Where is the rheostat used?

The rheostat is still used in some applications, although its usage area has decreased in recent years due to reasons such as low efficiency and consumption of energy, and it has been replaced by switching electronic devices, semiconductor elements, and potentiometers.

- The rheostat is generally used in applications where high current is required: microwave ovens, refrigerators, mixers, fans, power tools, etc.
- In dimmer switches, rheostats are used to change the intensity of the light. If the resistance of the rheostatsis increased, the electric current flows less through the bulb, and the brightness of the light decreases. Similarly, if we reduce the resistance of the rheostats, more electric current flows through the bulb, and the light brightness increases.
- Rheostats are used to increase or decrease the speed of an electric motor.
- It is used in the buttons where the temperature setting of electric stoves is made. It is used in all applications similar to kitchen appliances that have heating elements that the temperature needs to be increased or decreased.
- It is used to increase or decrease the signal volume in devices such as TV, radio.
- It is used as gauge resistance in laboratories.
- It is used for resistance measurements with the bridge measurementmethod.

In Table 15.1, the rheostat component parts and used materials are presented.

Table 15.1. Rheostat component parts and used materials

Crt. no.	Component parts	Material
1	Contacts	Iron Alloy, Copper
2	Coil of wire	Nickel-Chromium, Iron Alloy
3	Support tube	Asbestos (or any non-conductive material)
4	Casing (where is used)	Iron Alloy, Aluminum, Non-conductive materials
5	Wiper	Carbon-Graphite (enrichment with conductive metals, e.g. copper)
6	Wiper's rod	Nickel, Iron Alloy

Part II – Effects of thermal, electrical and magnetic stresses on the material parameters

Application A. Copper for electrical contacts

A copper piece is considered to be used used for electrical contacts, circular in shape, with diameter D = 12 mm and length L = 2 cm. If the operating temperature of the copper contact is 50°C, compared to the normal temperature of 20°C, establish:

1 How much does this copper piece (piece mass) weigh?

2 How much does the length of the copper contact piece change due to the phenomenon of thermal expansion?

3 How much does the resistivity of the cooper piece change?

4 What is the value of the electrical resistance of the cooper contact piece and how much does this value change (in percentage)?

5 What is the loss of electrical power due to the Joule Lenz effect, if a current I = 50 A passes through this piece?

Application B. Nichrom as material for resistive wire

For the resistive wire of a rheostat Nichrom type resistive material is proposed to be used.

1. What lengths of resistive wire are needed to build a rheostat with resistance R = 30 Ω, if nichrome resistive wires are available, with diameters of: 0.4 mm; 0.6 mm; 1.25 mm; 2.5 mm?
2. Compared to the temperature of 20^{o}C considered in designing, what are the lengths of the resistive wires, if the rheostat operates at a temperature of 55^{o}C?
3. What is the value of power dissipated in the resistive wires if the current passing through the rheostat is I = 10 A, in the cases when the rheostat is maintained at temperature: a) T_0= 20^{o}C; b) T= 55^{0}C.
4. For the same diameter D=1.2 mm, compare the length required to obtain R= 30 Ω.with nichrome and with constantan.

Application C. Electrical insulating ceramics as mechanical supportfor resistive wire

The resistive wire of the rheostat is considered to be wound on an electrically insulating ceramic cylinder made by alumina (Al_2O_3).

For obtaining the characteristics of this electrical insulating material, a rectangular sample was prepared, with dimensions: 20 cm x 40 cm and thickness g = 5 mm. Establish:

1. Mass of this sample.

2. How much does the thickness of the sample increase if the working temperature increases from 20^{o}C to 150^{o}C?

2. Volume resistance (volume) of the sample.

2. Capacitance of the capacitor formed with this sample as dielectric.

3. Loss angle tangent (tgδ) for this material, with a parallel RC scheme (f=50 Hz).

4. How much does the thickness of the sample increase if the working temperature increases from 20^{o}C to 55^{o}C?

5. If the capacitor is used at f = 50 Hz and the voltage applied is U = 400 V, estimate the value of the dielectric losses.

6. Does this insulation breakdown in the event of a voltage pulse with a peak value of 35 kV? Argue!

Solutions

For these three materials, the characteristic parameters are searched and are noted in Table 15.2.

Table 15.2. Parameters for studied materials

Material	**Material parameter**	**Parameter Value**
Copper	Density	d_m=8960 kg/m^3
	Thermal expansion coefficient	α_l=16.7x10^{-6} 1/^{o}C
	Resistivity at 20^{o}C	ρ_0=1.72 x10^{-8}Ωm
	Resistivity variation coefficient with temperature	α_ρ= 3.9 x 10^{-3}1/^{o}C
	Melting point	T_m = 1084 ^{o}C
Nichrome	Density	8400 kg/m^3
	Thermal expansion coefficients	α_l=14 x 10^{-6} 1/^{o}C
	Resistivity at 20^{o}C	ρ_0= 150 x 10^{-8}Ωm
	Resistivity variation coefficient with temperature	α_ρ= 4 x 10^{-4}1/^{o}C
	Melting point	T_m = 1400 ^{o}C
Constantan	Density	d_m =8860 kg/m^3
	Thermal expansion coefficient	α_l =15 x 10^{-6} 1/^{o}C
	Resistivity at 20^{o}C	ρ_0=49 x 10^{-8} Ωm
	Resistivity variation coefficient with temperature	α_ρ =8 x 10^{-6} 1/^{o}C
	Melting point	T_m = 1200 ^{o}C
Ceramics (Alumina -Al_2O_3)	Density	d_m =3900 kg/m^3
	Thermal expansion coefficient	α_l=8.1 x10^{-6} 1/^{o}C
	Maximum use temperature	T_m = 2273 ^{o}C
	Volume resistivity at 20^{o}C	ρ_V= 1 x 10^{12} Ωm
	Vacuum permittivity	ε_0=8.854x10^{-12} F/m
	Relative permittivity	ε_r=9
	Breakdown strength	E_{str}=16.7 kV/mm

Solution for application A

1. $r = D/2 = 6$ mm,

$$V = \pi \cdot r^2 \cdot L = \pi \cdot (6 \cdot 10^{-3})^2 \cdot 2 \cdot 10^{-2} = 2.262 \cdot 10^{-6}\ m^3 = 2.262\ cm^3$$

$$m = d_m \cdot V = 8960 \cdot 2.262 \cdot 10^{-6} = 2.0267 \cdot 10^4 \cdot 10^{-6}\ kg = 2.0267 \times 10^{-3}\ kg = 20.267\ g.$$

2. $L(T) = L(T_0) \cdot [1 + \alpha_1 \cdot (T - T_0)]$

$$\Delta L = L(T) - L(T_0) = L(T_0) \cdot \alpha_1 \cdot (T - T_0) = 2 \cdot 10^{-2} \cdot 16.7 \cdot 10^{-6} \cdot (50 - 20) =$$

$$= 1.002 \cdot 10^3 \cdot 10^{-8}\ m = 1.002 \cdot 10^{-5}\ m = 10.02\ \mu m$$

3. $\rho(T) = \rho(T_0) \cdot [1 + \alpha_\rho \cdot (T - T_0)]$
$\rho(T) = 1.72 \cdot 10^{-8} \cdot [1 + 3.9 \cdot 10^{-3} \cdot (50 - 20)] = 1.72 \cdot 10^{-8} \cdot 1.117\ \Omega m =$
$= 1.921 \cdot 10^{-8} \Omega m$
$\Delta\rho = \rho(T) = \rho(T_0) = 1.921 \cdot 10^{-8} - 1.72 \cdot 10^{-8} = 0.201 \cdot 10^{-8}\ \Omega m$

4. $A = \pi r^2 = \pi \cdot (6 \cdot 10^{-3})^2 = 0.0001131\ m^2 = 1.131 \cdot 10^{-4} m^2$
If the temperature influence on the area of the piece is not considered:
$R(T) = \rho(T) \cdot \frac{L(T)}{A}$
$R(T) = 1.92 \cdot 10^{-8} \cdot 2 \cdot \frac{10^{-2}}{(1.13 \cdot 10^{-4})\Omega} = 3.39 \cdot 10^{-6}\ \Omega = 3.39\ \mu\Omega$
$R(T_0) = 1.72 \cdot 10^{-8} \cdot 2 \cdot \frac{10^{-2}}{(1.13 \cdot 10^{-4})\Omega} = 3.04 \cdot 10^{-6}\ \Omega = 3.04\ \mu\Omega$
$\Delta R_\% = \frac{[R(T) - R(T_0)]}{R(T_0)} \cdot 100 = [3.39 - 3.04] \cdot \frac{100}{3.04} = 15.5\ \%.$

5. $P = R \cdot I^2$
$P = 3.39 \cdot 10^{-6} \cdot 50^2 = 8.475 \cdot 10^3 \cdot 10^{-6}\ W = 8.475 \cdot 10^{-3}\ W = 8.475\ mW.$

Solution for application B

1. $R(T_0) = \rho(T_0) \cdot L(T_0)/A \Rightarrow L(T_0) = A \cdot R(T_0)/\rho(T_0)$

$\rho_{NiCr}(T_0) = 1.5 \cdot 10^{-6} \Omega m, \qquad L(T_0) = 30\ m$
Length L_i at T_0 temperature will be:
$L(T_0)_i = A_i \cdot \frac{30}{1.5 \cdot 10^{-6}} = A_i \cdot 20 \cdot 10^6\ m$
$L(T_0)_1 = \pi \cdot (0.4 \cdot 10^{-3})^2 \cdot 20 \cdot 10^6 = 10.05\ m$
$L(T_0)_2 = \pi \cdot (0.6 \cdot 10^{-3})^2 \cdot 20 \cdot 10^6 = 22.61\ m$
$L(T_0)_3 = \pi \cdot (1.25 \cdot 10^{-3})^2 \cdot 20 \cdot 10^6 = 98.17\ m$
$L(T_0)_4 = \pi \cdot (2.5 \cdot 10^{-3})^2 \cdot 20 \cdot 10^6 = 392.17\ m$

2. $L(T)i = L(T_0)_i \cdot [1 + \alpha_1(T - T_0)]$

$L(T)_i = L(T_0)i\ x \cdot [1 + 14 \cdot 10^{-6} \cdot (55 - 20)] = L(T_0)_i \cdot 1.00049$

$L(T)_1 = 10.055\ m;\ \Delta L1(T) = 4.9\ mm$

$L(T)_2 = 22.621\ m;\ \Delta L2(T) = 11.0\ mm$

$L(T)_3 = 98.22\ m;\ \Delta L3(T) = 48.1\ mm$

$L(T)_4 = 392.89\ m;\ \Delta L4(T) = 192.4\ mm$

3. $P_i = R(T_0) \cdot I^2$, where $R(T_0)_I = \rho(T_0) \cdot \frac{L(T_0)_i}{A_i}$

$P(T_0)_1 = P(T_0)_2 = P(T_0)_3 = P(T_0)_4 = 30 \cdot 10^2 = 3 \cdot 10^3\ W = 3\ kW$

Increasing the temperature, will produce a varation of the resistivity and of the length of wires. It is negklected the variation of transversal area with temperature. It results:

$$R(T)_i = \rho(T) \cdot \frac{L(T)_i}{A_i}$$

$$= \rho(T_0) \cdot \left[1 + \alpha_\rho \cdot 1 + (T - T_0)\right] \cdot L(T_0)_i \cdot \frac{[1 + \alpha_1 \cdot (1 + (T - T_0)]}{A_i}$$

$$R(T)_i = 1.5 \cdot 10^{-6} \cdot [1 + 4 \cdot 10^{-4} \cdot (55 - 20)] \cdot [1 + 14 \cdot 10^{-4} \cdot (55 - 20)] \cdot \frac{L(T_0)_i}{A_i}$$

$$R(T)i = 1.5 \cdot 10^{-6} \cdot 1.014 \cdot 1.00049 \cdot \frac{L(T_0)_i}{A_i} = 1.5217 \cdot \frac{L(T_0)_i}{A_i}$$

where:

$$\frac{L(T_0)_i}{A_i} = \frac{L(T_0)_i}{\pi \frac{(D_i)^2}{4}} = \frac{4}{\pi} \cdot \frac{L(T_0)_i}{(D_i)^2} = 1.273 \cdot \frac{L(T_0)_i}{(D_i)^2}$$

Finally:

$$R(T)i = 1.5217 \cdot 1.273 \cdot \frac{L(T_0)_i}{(D_i)^2} = 1.9371 \cdot \frac{L(T_0)_i}{(D_i)^2}$$

It resuls:

$R(T)_1 = 30.418\ \Omega$ $\Delta R(T)_1 = 0.418\ \Omega$

$R(T)_2 = 30.415\ \Omega$ $\Delta R(T)_2 = 0.415\ \Omega$

$R(T)_3 = 30.426\ \Omega$ $\Delta R(T)_3 = 0.426\ \Omega$

$R(T)_4 = 30.427\ \Omega$ $\Delta R4(T) = 0.427\ \Omega$

It results:

$$P(T)_1 = P(T)_2 = P(T)_3 = P(T)_4 = R(T) \cdot I^2 = 30.42 \cdot 10^2 = 3042\ W$$

4. For Constantan

$$L(T_0) = A \cdot \frac{R(T_0)}{\rho(T_0)}$$

$$L(T_0)cst = (\pi \cdot (0.6 \cdot 10^{-3})^2 \cdot \frac{30}{49 \cdot 10^{-8}} = 69.20\ m$$

$$L(T_0)_{NiCr} == 22.621\ m$$

$\Delta L = L(T_0)_{cst} - L(T_0)_{NiCr} = 46.57 \text{ m}$

$R(T)_{cst}$

$$= 49 \cdot 10^{-6} \cdot [1 + 8 \cdot 10^{-6} \cdot (55 - 20)] \cdot [1 + 15 \cdot 10^{-6} \cdot (55 - 20)] \cdot \frac{L(T_0)_{cst}}{A_{cst}}$$

$$R(T)_{cst} = 49 \cdot 10^{-6} \cdot 1.00028 \cdot 1.000525 \cdot \frac{L(T_0)_{cst}}{A_{cst}} \approx R(T_0)_{cst}$$

$P(T)_{cst} \approx P(T_0)_{cst} = 30 \cdot 10^2 = 3 \text{ kW}$

Solution for application C

1. $V = l \cdot L \cdot g$
   $V = 20 \cdot 10^{-2} \cdot 40 \cdot 10^{-2} \cdot 5 \cdot 10^{-3} = 4 \cdot 10^{-4} \text{ m}^3 = 400 \text{ cm}^3$
   $m = d_m \cdot V = 3900 \cdot 4 \cdot 10^{-3} \text{ m}^3 = 1.56 \cdot 10^4 \cdot 10^{-3} \text{ kg} = 15.6 \text{ kg}.$

2. $g(T) = g(T_0) \cdot [1 + \alpha_1 \cdot (T - T_0)]$
   $\Delta g(T) = g(T) - g(T_0) = g(T_0) \cdot \alpha_1 \cdot (T - T_0)$
   $\Delta g(T) = 5 \cdot 10^{-3} \cdot 8.1 \cdot 10^{-6} \cdot (150 - 20) = 5265 \cdot 10^{-9} =$
   $= 5.265 \cdot 10^{-6} \text{ m} = 5.265 \text{ µm}.$

3. $R_V = \rho_V \cdot \frac{g}{A}$, where $A = l \cdot L = 40 \cdot 20 = 800 \text{ cm}^2 = 8 \cdot 10^{-2} \text{ m}^2$

   $R_V = 10^{12} \cdot 5 \cdot \frac{10^{-3}}{8 \cdot 10^{-2}} = 62.5 \cdot 10^9 \ \Omega = 62.5 \text{ G}\Omega$

4. $C = \varepsilon_0 \cdot \varepsilon_r \cdot \frac{A}{d}$

   $$C = 8.854 \cdot 10^{-12} \cdot \frac{9 \cdot 8 \cdot 10^{-2}}{5 \cdot 10^{-3}} = 0.274 \cdot 10^{-9} \text{ F} = 1.274 \text{ nF}$$

5. $\tan\delta = \frac{1}{2\pi f \cdot R \cdot C}$

   $$\tan\delta = \frac{1}{2\pi \cdot 50 \cdot 62.5 \cdot 10^9 \cdot 1.274 \cdot 10^{-9}} = 0.3999 \cdot 10^{-4} = 3.99 \cdot 10^{-5}$$

6. $P = \omega \cdot \tan\delta \cdot C \cdot U^2$
   $P = 2\pi \cdot 50 \cdot 3.99 \cdot 10^{-5} \cdot 0.274 \cdot 10^{-9} \cdot 400^2 = 0.549 \cdot 10^{-6} \text{ W} = 0.549 \text{ µW}$

7. The electric field intensity which stresses the sample is: E=U/d
   $E = 35 \cdot 10^3 / (5 \cdot 10^{-3}) = 7.0 \text{ x } 10^6 \text{ V/m} = 7 \text{ MV/m} = 7 \text{ kV/mm}$

Ceramic has a dielectric strength $E_{str} = 16.7 \text{ kV/mm}$

Because: $E < E_{str}$, the dielectric does not electrically breakdown at 35 kV.