PERTURBATIVE POTENTIAL EVALUATION OF THE ELECTRIC CURRENT

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Abstract: The paper presents the research on the disruptive influences of electrical current variation - it has been analytically determined how measurement resistances can influence these variations. The variance relationships of the current ratios were determined on the basis of the theory of variable electrical circuits for the special case of an inductive coupling. It was studied also the effect of measuring resistors as a function of frequency on the signal integrity.

Key words: inductive coupling, electromotive force, electromagnetic compatibility.

1. Introduction

Disturbing electromagnetic phenomena are characterized by diversity and dynamism. There are different approaches to understanding, evaluating and measuring them. The paper [1] mainly uses the Mathcad-Simulink simulation tool. The present paper uses analytical calculation based on electromagnetic field laws [2] and electrical circuit theory to evaluate the effects of different methods of measuring electric current on signal integrity. The influence of the geometry of the circuit loops and the distance between them on the coupling of the circuits by magnetic field were presented especially from a qualitative point of view in [3]. In analyzing a disturbing phenomenon, it must be extracted the essential information from the point of view of electromagnetic compatibility. The most important information about a magnetic disturbance is represented by the signal waveform meaning the current amplitude variation that it generated. The waveform is the information that is obtained by visualization with an oscilloscope or a spectrum analyzer. The signal in question is an electric current signal whose disturbing potential depends on its variation over time.

An ideal measurement method is one that, through the act of measurement, does not affect in any way the measured phenomenon, preserving its integrity. Two methods of current measurement are analyzed: by using a current probe and by visualizing a voltage drop across a current resistor which is evaluated from a disturbing point of view. The

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choice of measurement or visualization method from a practical point of view is dictated by the limits of the available equipment. The disturbing potential evaluation of the electric current can be performed for two adjacent inductively coupled circuits. In the case of two neighboring electrical circuits, circuit 1 being the source of the magnetic disturbance and circuit 2 the “victim”, the electrical diagram is presented in Figure 1. For a quantitative assessment, the following values of the self inductance of the victim loop and of the mutual inductance were considered: self inductance $L_2 = 0.9$ µH; mutual inductance $M = 0.2$ µH; resistance $R_2 = 50$ Ω. According to the electromagnetic induction law, the voltage induced in the second circuit is given by Equation (1). Equations (2) and (3) lead to a relationship between the generating current of the disturbance (source), $i_1$ and the current $i_2$, induced by inductive coupling in the disturbed circuit (victim):

$$R_2 i_2 = -\frac{d}{dt}(\Phi_2 - \Phi_{nt}),$$

$$R_2 i_2 = -j \omega L_2 i_2 + j \omega M I_1 \Rightarrow i_2 = \frac{j \omega M}{R_2 + j \omega L_2} I_1 \Rightarrow$$

$$\frac{i_2}{I_1} = \frac{j \omega M (R_2 - j \omega L_2)}{R_2^2 + \omega^2 L_2^2} = \frac{\omega^2 M L_2}{R_2^2 + \omega^2 L_2^2} + \frac{j \omega^2 M R_2}{R_2^2 + \omega^2 L_2^2} \Rightarrow \left| \frac{i_2}{I_1} \right| = \sqrt{\left( \frac{\omega^2 M L_2}{R_2^2 + \omega^2 L_2^2} \right)^2 + \left( \frac{\omega^2 M R_2}{R_2^2 + \omega^2 L_2^2} \right)^2}. \quad (3)$$

Fig. 1. Inductive coupling between two adjacent circuits

The $i_2/i_1$ ratio depending on the frequency $f$ is shown in the Figure 2.

Fig. 2. Graph of the $i_2/i_1$ ratio as a function of frequency
It is observed that for sinusoidal currents, the $I_2/I_1$ ratio increases with the increasing frequency up to a saturation value (Figure 2). In the case of the waveform of a current pulse (Figure 3), a steep current generates greater disturbances compared to a current that has a smoother slope. The disturbing current $i_2$ reaches values of about 10% of the amplitude $i_1$ for smooth slopes of the disturbing current pulse and 20% of the amplitude $i_1$ at steep fronts of the disturbing current pulse.

The 50 Hz signal generates inductively coupled, insignificant disturbances. Circuit loops capture electromagnetic disturbances, so their electrical characteristics must be optimized in order to minimize the disturbing current $i_2$. The property of the current $i_2$ to generate an opposite direction magnetic field to the external one can be used to shield the sensitive electrical or electronic systems that it borders.

As a result of the analysis of the inductive coupling between a source circuit of the disturbance, located near a victim circuit of the disturbance, in order to reduce the magnetic disturbance effects, the mutual inductance between the two circuits must be as small as possible and the inductance of the circuit victim to be considerable.

1. Experimental Evaluations of the Inductive Coupling

   Evaluations performed with an experiment proposed in [4] which highlights the importance of frequency in the current distribution in the case of the coupling phenomenon of current loops thru magnetic field, is shown in Figure 4. The sinusoidal signal generator is connected to a load of 50 Ω through a one meter long cable. There are two current return paths from the load to the signal source:
   - a short path from a geometric point of view, physically made of a short, thick conductor between points D and A. This path is called the reference and the current flowing through it will be noted with $I_r$;
   - a long path from a geometric point of view, physically made by the second
conductor with a length of one meter of the source-load cable, the DGFA route. The current flowing through it will be noted with $I_2$.

Two inductive current probes are connected to the oscilloscope, displaying the waveforms and amplitudes of the $I_r$ and $I_2$ currents on the two return paths. For this experiment, the current probes were replaced with two measuring resistors with the value of $1 \Omega$. These resistors were chosen of low value to not influence the amplitude of the current through the load. Also, the value of $1 \Omega$ makes a one-to-one ratio, voltage-current, for example a measured voltage of 100 mV will correspond to a current of 100 mA.

The equivalent electrical circuit is that of Figure 5, where $L_1-R_1$, $L_2-R_2$ and $R_r$ are the inductances and resistances of the connecting conductors between the signal generator and the load.

Between the cable conductors there is a mutual inductance $M$. The measuring resistors $R_{2m}$ and $R_{rm}$ are mounted with a terminal at the generator reference ground. The experiment showed that for low frequencies, the current will choose the geometrically short path and very low resistance ($I_r > I_2$). From a certain frequency upwards, the situation will be reversed ($I_r < I_2$), thus the current will choose the path of minimum impedance. The $I_r/I_2$ ratio will decrease to a value at which it will remain constant even if the frequency continues to increase. Table 1 shows the results obtained when using two cables of different types (two-wire and coaxial). The current values are peak-to-peak values.

### Experimental data obtained by measurement

<table>
<thead>
<tr>
<th>Cable type</th>
<th>$f$ [kHz]</th>
<th>$k$</th>
<th>$I_r/I_2$</th>
<th>$I_r/I_1$</th>
<th>$I_2/I_1$</th>
<th>$I_1$ [mA]</th>
<th>$I_2$ [mA]</th>
<th>$I_r$ [mA]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-wire</td>
<td>1000</td>
<td>0.55</td>
<td>0.8</td>
<td>0.44</td>
<td>0.55</td>
<td>135</td>
<td>75</td>
<td>60</td>
</tr>
<tr>
<td>Coaxial</td>
<td>1000</td>
<td>0.81</td>
<td>0.23</td>
<td>0.18</td>
<td>0.81</td>
<td>185</td>
<td>150</td>
<td>35</td>
</tr>
</tbody>
</table>

Figure 6 shows the currents $I_r$ (yellow sinusoid) and $I_2$ (blue sinusoid) for the two cable types.

The $I_2/I_1$ ratio gives the measure of the mutual impedance, $M$, and noting this ratio with $k$, we can consider it as a cable quality coefficient. The coaxial cable has a much
better quality, the current thru the reference being 4.2 times lower than \( I_2 \) current. The higher the mutual inductance \( M \) is, \( I_2 \) and \( I_1 \) have closer values and \( I_r \) approaches zero.

![Image of oscilloscope readings](image)

*Fig. 6. Visualization with oscilloscope of voltage forms for a two-wire cable (a); or for a coaxial cable (b)*

In terms of the disturbance potential that is present in the AFGDA loop (Figure 5), if the current \( I_r \) has high values, the magnetic induction \( B_2 \) will be considerably lower than \( B_1 \) and will not be able to compensate it and in the loop there will be a high disturbance magnetic potential. When \( I_r \) is small, \( I_2 \) and \( I_1 \) have very close values, \( B_2 \) will compensate \( B_1 \) and in the AFGDA loop there will be an insignificant disturbing potential.

Therefore, a first conclusion refers to the distribution of currents in frequency through reference loops with different geometries, coupled by magnetic field. At high frequencies, the current distribution is done in such a way as to create a minimum magnetic flux in the AFGDA loop.

### 2. Evaluation of Current from the Perspective of Disturbing Potential

The voltage drop method affects with very little the current amplitude through the load, gives correct information about the current waveform and also does not seem to affect the integrity of the measured signal. But an analytical assessment of the effect of measuring resistors as a function of frequency will identify how they affect signal integrity.

Given the wiring diagram in Figure 5, it was considered: \( U_g = 10 \, \text{V}, \, R_g = R_L = 50 \, \Omega, \, R_{rm} = R_{2m} = 1 \, \Omega, \, L_1 = L_2 = 1000 \, \text{nH} \). The conductor resistance, \( R_1 \) and \( R_2 \) were computed for different frequencies: a) \( f = 1 \, \text{kHz} \), b) \( f = 150 \, \text{kHz} \), c) \( f = 1000 \, \text{kHz} \). With these values, for the considered frequencies, the resistance of the conductors has the values of 21.53 m\( \Omega \), 38.18 m\( \Omega \) and, respectively, 87.69 m\( \Omega \). The resistance \( R_r \) has a constant value and was computed with the usual formula, depending on the resistivity of the material (copper), the length of the conductor and the surface of it, already known.

As the frequency increases, the conductor penetration depth decreases and the current flows on the outer surface of the conductor, causing the resistance to increase.
• Evaluation of currents in case when the measuring resistances $R_{rm}$ and $R_{2m}$ are neglected

Applying Kirchhoff's first law and the law of electromagnetic induction on the two closed loops for the same induced voltage, making the ratio of currents, there can be obtained the relations (4) and (5) for high frequencies:

$$\frac{l_2}{l_1} = \frac{R_r + j\omega M}{R_r + R_2 + j\omega M} \cdot \frac{R_r + j\omega}{R_r + R_2 + j\omega}, \quad k = \frac{M}{L_2}, \quad \omega = \frac{R_2 + j\omega}{M + j\omega}.$$

$$\frac{l_1}{l_1} = \frac{R_2 + j\omega(L_2 - M)}{R_r + R_2 + j\omega L_2}, \quad \frac{l_1}{l_2} = \frac{R_2 + j\omega(L_2 - M)}{R_r + j\omega M}. \quad \frac{l_2}{l_2} = \frac{R_2 + j\omega(1-k)}{R_r + j\omegaM}.$$

(4)

$$\frac{l_1}{l_1} = k, \quad \frac{l_2}{l_1} = 1 - k, \quad \frac{l_2}{l_2} = \frac{1}{k} - 1.$$

(5)

Fig. 7. Variation of frequency ratios $I_2/I_1$ (a), $I_1/I_1$ (b) and $I_3/I_1$ (c) as a function of frequency, obtained by analytical calculation, resistors $R_{rm}$ and $R_{2m}$ are not in circuit.
With the considered data it can be calculated $R_1$, $R_2$ and $R_r$ for the frequencies of interest, and then be determined the currents.

Figure 7 shows the graph of current variation, if the measuring resistances are not taken into account.

The current ratio $I_r/I_2$ shows that at low frequencies almost all the current flows through the short conductor of low resistance, but as the frequency increases there are noticeable changes in the current return path through the reference. At the frequency of 1 kHz the ratio decreases, which indicates a current flow through the long conductor. At a frequency of about 65 kHz, the ratio reaches saturation, remaining constant even if the frequency continues to increase, the decrease of the ratio showing that most of the current $I_1$ flows through the long conductor.

The $I_r/I_1$ ratio decreases with the increasing frequency, indicating a small percentage of the $I_r$ current, from the value of $I_1$, which passes through the short conductor.

- **The effect of measuring resistances on the current ratio**

The experimental results obtained by the method of measuring the voltage drop on two measuring resistors with a value of 1 Ω, confirm that their introduction in the circuit, at a first evaluation, does not affect the phenomenon evaluated qualitatively and to a very small extent quantitatively.

The graph of the variation of the current ratios is shown in Figure 8.

![Graph of current ratios](image)

**Fig. 8.** Varying ratios of the currents $I_r/I_2$ (a), $I_r/I_1$ (b) and $I_2/I_1$ (c) depending on frequency for the case where in the circuit are also resistors $R_{rm}$ and $R_{2m}$
A results comparison between the analytical calculation presented in Figure 7 and the experimental results provided by the voltage drop method showed differences in terms of saturation frequency, which means the frequency from which the $I_2/I_1$ ratio does not decrease. The experimental results showed that the presence of resistances causes the saturation to occur at much higher frequency values. Also, in the domain of frequencies lower than 1 MHz, the graphs $I_2/I_1$ from Figure 7 and Figure 8 shows significant differences.

3. Conclusions

The research method was the same as in [5]: experiment-analytical calculation-experiment the experimental results being compared with those obtained by analytical calculation.

The analysis showed that each measurement method evaluated affects the integrity of the information provided.

The current probe measuring method informally affects the measurement result; the waveform of the current is not visualized but it is the waveform of its derivative. The user of this information, by understanding the phenomenon, allows considering a class of signals and not the concrete signal. It is a suitable method only for sinusoidal signals, but also in this case, the saturation phenomenon of the measuring probe must be taken into account.

In the case of the voltage drop method, what is viewed on the oscilloscope is the current waveform itself, but the integrity of the information provided in the low frequency area is affected.

References