Bulletin of the *Transilvania* University of Braşov
Series II: Forestry • Wood Industry • Agricultural Food Engineering • Vol. 5 (54) No. 1 - 2012

MATHEMATICAL MODELING OF THE TRACTOR-GRADER AGRICULTURAL SYSTEM CINEMATIC DURING LAND IMPROVING WORKS

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Abstract: *This paper studies the influence of the pulled leveling machine on the general dynamics of the wheel tractor. The obtained data are employed in the design of the transmission and of other tractor sub-assemblies, representing the disturbance factors of the mathematical and dynamic models set up of establishing the strain on various components. It is shown that the rear axle of the tractor is loaded up to 53%, while the front one is released to values below the admissible minimum (20% of the static load). The loading coefficient of the rear wheels can reach* $A_2 = 0.76°$*, while the longitudinal stability angle tilts during climbing and worsens by about 57% in relation to the solo tractor.*

Key words: *agricultural tractors on wheels, general dynamic, graders.*

1. Symbols List

Z_{01}, Z_{02} - static reactions on the front and rear tractor axels;
Z_1, Z_2 - dynamic reactions on the front and rear tractor axels;
G_t, G_m - tractor weight, grader weight respectively;
F_a, F_{jt}, F_m, F_f, F - forces of: air resistance; tractor inertia; driving; rolling resistance; resistance on the working movement of the technical system;
r_m - dynamic radius of the driving wheel;
M_{r1}, M_{r2} - rolling resistance moments on the front and rear tractor wheels;
L, a, b, c, h_c, h_t, h_a - geometrical dimensions, according to Figure 1;
f - rolling resistance coefficient;
α, β - longitudinal and transversal angles of the slopes;
γ - angle made by the force F with the horizontal;
k_u - soil leveling resistance coefficient;
A - working surface (in soil) of the grader blade.

2. Introduction

Graders are working machines which cut the high irregularities of the terrain using blades or bottomless buckets which transport trough creeping the released material to lower places [2]. From the technical system formation point of view of the graders are towed machines, with rigid or articled frame (Figure 1). In Figure 1 we presented the technical system scheme, made from U-650 M tractor and the towed grader NT-2.25, at ramp ascension (a) and moving on transversal slopes (b) [3].

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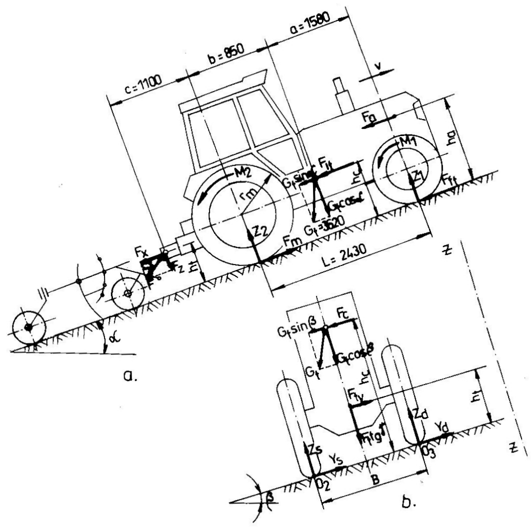

Fig. 1. *Movement on longitudinal ramps* (*a*) *and transversal slopes* (*b*) *of the tractor-grader agricultural system*

The traction resistance opposed by the grader is calculated with the relation:

$$F = k_u \cdot A + G_m \cdot f \,. \quad (1)$$

Under the dynamic aspect, the grader influences the technical system trough F forces magnitude and the γ angle, on which forces are made with an axel parallel with the soil [1].

For this technical system there will be considered in all the cases, the following constant values: tractor weight (G_t = 36200 N), tractor wheel base (L = 2430 mm), gravity center position (b = 850 mm, $L - b = a$ = 1580 mm, h_c = 900 mm) and tractor gauge (B = 1920 mm).

Variable values will be considered the ones which are related to the rolling resistance conditions and lateral slip, grader weight, grader gravity center position from the rear tractor axel, traction resistance force and its position in relation to the horizontal surface.

The simulation of the technical system dynamics is made in MATHCAD language, using mathematical models, corresponding to different functional situations and characteristic elements of the general dynamics.

3. Mathematical Modeling and Analysis of Tractor Axels' Reactions

As a reference element in analyzing the axels reactions of the tractors which form the technical systems of land melioration machines, are the static reactions resulted from the tractor weight repartition on the two axels, on horizontal terrain:

$$Z_{01} = \frac{G_t \cdot b}{L} = 12700\,[N], \quad (2)$$

$$Z_{02} = \frac{G_t \cdot (L-b)}{L} = 23600\,[N]. \quad (3)$$

The dynamic reactions on the front and rear wheels Z_1 and Z_2 are calculated with relations:

$$Z_1 = \frac{G_t \cdot b \cdot \cos\alpha - (G_t \cdot \sin\alpha + F_{jt}) \cdot h_c}{L} - \frac{F_a \cdot h \cdot a + F_t \cdot h_t + F_t \cdot c \cdot \mathrm{tg}\,\gamma + M_{r1} + M_{r2}}{L}, \quad (4)$$

$$Z_2 = \frac{G_t \cdot (L-b)\cdot \cos\alpha + (G_t \cdot \sin\alpha + F_{jt})\cdot h_c}{L} + \frac{F_a \cdot h_a + F_t \cdot h_t + F_t \cdot (L+c)\cdot \text{tg}\,\gamma}{L} + \frac{M_{r1}+M_{r2}}{L}. \quad (5)$$

The variations of the dynamic reactions on those two tractor axels are calculated with the relations:

$$\Delta Z_1 = Z_1 - Z_{01}; \quad \Delta Z_2 = Z_2 - Z_{02}. \quad (6)$$

At the study of dynamic reactions it has been considered the case of a tractor moving on a horizontal terrain with constant speed, with the traction force parallel to the soil and neglecting air resistance.

Results are presented in Table 1, from which we can find the influences of the main factors (F, γ and f) on the variation of tractor axels loads.

Table 1

Reaction forces on the tractor wheels at constant speed on a horizontal surface, with the traction force parallel with the soil surface

Variables		Z_{01},	Z_{02},	Z_1,	ΔZ_1		Z_2,	ΔZ_2		Observations
Symbols	Values	[%]	[%]	[N]	[N]	[%]	[N]	[N]	[%]	
F, [N]	5000	12700 100	23600 100	9860	–2800	–23	26340	+2800	12	$\gamma = 30°$ $f = 0.1$
	15000			6480	–6220	–49	29470	+6370	27	
	35000			590	–1260	–96	36140	+12600	53	
γ, [°]	0			8920	–3740	–29	27280	+3710	15.9	$F = 25000$ $f = 0.1$
	14			6230	–6470	–51	29970	+6370	27	
	30			3320	–9930	–78	32870	+9930	42	
f	0.06			3200	–9460	–74	33000	+9460	40	$F = 25000$ $\gamma = 30°$
	0.1			2800	–9900	–78	33510	+9910	42	
	0.2			1560	–1110	–87	34640	+11000	47	

In the conditions of maintaining all the technical system parameters constant, the resistant traction force from the grader loads on the rear axel of the tractor with 12% for F = 5000 N, with 27% for F = 15000 N and with 53% if F would have the value F = 35000 N.

If traction resistance remains constant, but the angle γ is modified, an angle which this force makes with the horizontal, the load on the rear axel increases by 15.9% for $\gamma = 0$ and F = 25000 N, from $F = 0$, with 27% for $\gamma = 14°$ and by 42% for $\gamma = 30°$.

For the rolling resistance coefficient f = 0.06, F = 25000 N and $\gamma = 30°$, the variation of the loads on the tractor axels towards the unloaded tractor situation, shows a rear axle load increasing by 40% and a download of the front axel by 74%. Increasing the value of f to f = 0.1 makes the rear axle load increase by 42% and the front axel to discharge by 78%. In the hypothetical case where F = 35000 N, the rear axle load is of 47%, and the front axel download of 87%.

4. Mathematical Modeling and Analyses of Loading Coefficients for Tractor Wheels

Tractor axels loading at the leveling work need to consider also the tire lift, expressed through wheel loading coefficients λ.

For the static position of the tractor these coefficients have the values given by the relation:

$$\lambda_{fs} = \lambda_{01} = \frac{b}{L} = 0.35, \quad (7)$$

$$\lambda_{ss} = \lambda_{02} = \frac{L-b}{L} = 0.65. \quad (8)$$

For the situation when the tractor is moving with traction force on a horizontal terrain, the coefficients λ_f and λ_s have the values:

$$\lambda_f = \frac{b}{L} - \frac{F_t \cdot h_t + f \cdot r_m \cdot G_t}{G_t \cdot L}, \quad (9)$$

$$\lambda_s = \frac{L-b}{L} + \frac{F_t \cdot h_t + f \cdot r_m \cdot G_t}{G_t \cdot L}. \quad (10)$$

The changing of these coefficients in the conditions of technical system movement on a plain terrain, with the variable force F of the grader, but arranged in an angle of 30° from the horizontal and a rolling resistance coefficient $f = 0.01$, is represented in Figure 2. We found a decrease of the dynamic loading coefficient of the steering wheels $\lambda_f = \lambda_1$ (curve 3) compared with the coefficient $\lambda_{fS} = \lambda_{01}$ (curve 1) of the same wheels, from the wheels $\lambda_f = \lambda_1 = 0.305$ from $F = 5000$ N, to the value $\lambda_f = \lambda_1 = 0.231$ for $F = 35000$ N.

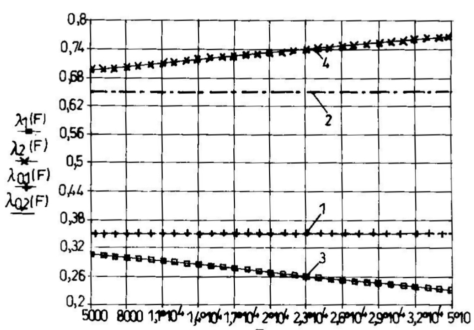

Fig. 2. *The variation of tractor wheels loading coefficients during leveling work*

At the same time, an increase of the driving wheel loading coefficient (curve 4) from the value $\lambda_s = \lambda_2 = 0.695$ for $F = 5000$ N, to the value $\lambda_s = \lambda_2 = 0.769$ for $F = 35000$ N (curve 2 represents λ_{ss}) is noticed.

5. Mathematical Modeling and Analyses of the Longitudinal Stability of the Tractor

For the determination of the critical angles of static stability at boarding ramp (α_{02max}) and ramp descending (α_{01max}), the tractor is considered slowed in both situations and the following relations are applied:

$$\alpha_{02\max} = \text{arctg}\frac{b}{h_c} = 43.363\,[^\circ], \quad (11)$$

$$\alpha_{01\max} = \text{arctg}\frac{L-b}{h_c} = 60.33\,[^\circ]. \quad (12)$$

The critical angle of tractor dynamic stability at boarding ramp is calculated with the relation:

$$\text{tg}\,\alpha_{cr} = \frac{b - (\varphi - f)\cdot(h_t + c\cdot \text{tg}\,\gamma)}{h_c - (h_t + c\cdot \text{tg}\,\gamma)}, \quad (13)$$

if the angle γ has a significant value with the relation:

$$\text{tg}\,\alpha_{cr} = \frac{b - (\varphi - f)\cdot h_t}{h_c - h_t}, \quad (14)$$

if angle $\gamma = 0$.

The case of this agricultural system is not considered relevant in determining the critical angles of longitudinal dynamic stability at ramp descending, as a result of his significantly improving with the towed technical system.

In Table 2 the angle variation of the towed technical system longitudinal stability is presented according to the resistance force F of the grader movement.

In the case of the tractor-grader agricultural system's movement with a traction resistance F parallel with the soil surface ($\gamma = 0$), the influence of the F force on back overthrow ($\Delta\alpha_2$) is in the limit of $\Delta\alpha_2 = -18.93\%$, if $F = 20000$ N and of $\Delta\alpha_2 = -9.2\%$, if $F = 5000$ N. However, if $\gamma = 30°$, $\Delta\alpha_2 = -57.27\%$,

Table 2

The angle variation of the towed technical system longitudinal stability according to the resistance forces F of the grader movement

Variables		α_{01},	α_{02},	α_1,	$\Delta\alpha_1$		α_2,	$\Delta\alpha_2$		Observations
Symbols	Values	[%]	[%]	[°]	[°]	[%]	[°]	[°]	[%]	
F, [N]	5000	60.3 100	43.3 100	63.7	+3.4	+5.6	37.77	–5.53	–12.7	G_t = 36200 N
	10000			65.6	+5.3	+8.7	34.70	–8.6	–19.8	h_t = 0.25
	15000			67.3	+7.0	+11.6	31.40	–11.91	–27.5	*f* = 0.1
	20000			68.75	+8.45	+14.0	27.83	–15.47	–35.7	γ = 15°
F, [N]	5000			65.46	+5.16	+8.5	36.0	–7.3	–16.8	G_t = 36200 N
	10000			68.47	+8.17	+13.5	30.78	–12.52	–28.9	h_t = 0,25
	15000			70.85	+10.55	+17.5	24.94	–18.36	–42.4	*f* = 0.1
	20000			72.78	+12.48	+20.9	18.5	–24.8	–52.3	γ = 30°
F, [N]	5000			62.0	+ 1.7	+2.8	39.3	–4.0	–9.2	G_t = 36200 N
	10000			62.5	+2.2	+3.6	38.0	–5.3	–12.3	h_t = 0.25
	15000			63.0	+2.7	+4.4	36.6	–6.7	–15.5	*f* = 0.1
	20000			63.5	+3.2	+5.3	35.1	–8.2	–18.9	γ = 0°

for F = 20000 N and $\Delta\alpha_2$ = –16.8%, for F = 5000 N.

More suggestive is the representation from Figure 3 of the γ angle influence over the longitudinal tractor sliding stability angles, in which 1 represents the position of longitudinal static stability critical angle at slopes descending (α_{01max}); 2 - the position of the static stability critical angle at ramps ascending (α_{02max}); 4 - the position of dynamic stability angle at ramps ascending, if γ = 0, and 3 - the position of longitudinal dynamic stability critical angle to sliding if γ = 0...30°. It is noted that if the angle γ increases, the rear axel is getting loaded and the dynamic sliding stability is improved.

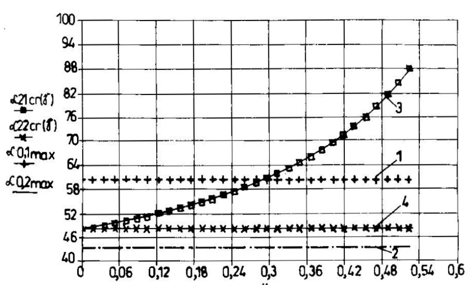

Fig. 3. *The influence of angle γ over the static and dynamic stability angles at movement on ramps and slopes*

6. The Mathematical Modeling and Analyze of Tractor Transversal Stability

The static stability at agricultural system overthrow on transversal slopes is calculated with the relation:

$$\beta_{max} = \text{arctg}\frac{0.5 \cdot B}{h_c}. \quad (15)$$

For the maximum value of the tractor U-650M track width (B = 1.92 m) the static stability overthrow angle has the variation represented in Figure 4, dependent only upon the tractor gravity center position h_c. It is noticed that for h_c = 0.6 m, β_{max} = 58°, and for h_c= 1 m, β_{max} = 43°, reaching at β_{max} = 35°, for h_c = 1.4 m.

The tractor dynamic stability at lateral overthrow shall be ensured if:

$$\frac{G_t}{g}v^2 R \le G_t\frac{0.5B\cdot\cos\beta - h_c\sin\beta}{h_c}, \quad (16)$$

in which the first part (the centrifugal force) is dependent on the speed v (for G_t and R constant on a tractor), and the second part is dependent upon angle β of slope inclination.

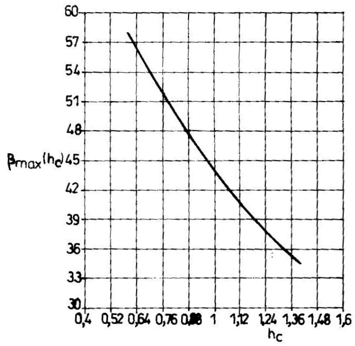

Fig. 4. *The variation of tractor overthrow static stability angle depending on the position of the tractor gravity center* h_c

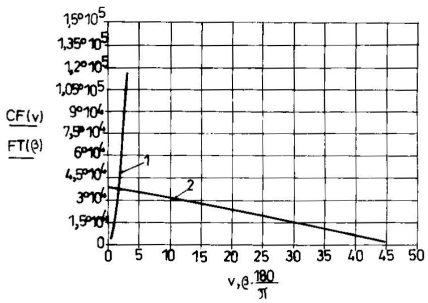

Fig. 5. *The variation of dynamic stability at lateral overthrow*

The condition required by the relation (13) is graphically represented in Figure 5, where on the abscissa, the movement speed *v* and the slope angle β are represented, and on the ordinate, the centrifugal force and the other part of the inequation are rendered.

It is noticed that the agricultural system's stability is assured only if the movement speed $v < 2$ m/s (curve 1), and the transversal slope angle $\beta < 30°$ (curve 2), a different situation from the one studied in the case of stability at static overthrow.

7. Conclusions

1. The moving resistance force *F* of the operating tractor-grader agricultural system modifies the loads on the tractor axels by almost +27% on rear axel and by –49% an front axel (at *F* = 15000N), the angle γ loads the rear axel by almost 26% and unloads the front axel by 60%, and the rolling resistance coefficient causes the rear axel loading by a maximum of 7% and a front axel unloading of 13%.

2. The front axel unloading can not be accepted with more than –80%, because the agricultural system's movement directions can not be safely controlled.

3. The dynamic loading coefficients of the tractor wheels have similar variations with the axels loads, in the sense that at the front wheels (steering wheels) those coefficients decrease from 0.35 to 0.231 (–34%) and their increase on the rear wheels from 0.65 to 0.769 (+18.3%) because of force F and in smaller proportions if only the influences of γ and f are taken into consideration.

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