Abstract: The paper presents a steering box for the rear axle of a four-wheeled vehicle where the rear wheels are steered both in the same and opposite direction, improving manoeuvrability and stability while driving at both low and high speeds. The steering box of the rear axle consists of a simple linkage mechanism with one contour that has two driving inputs that can achieve the requirements of an integral steering. By analysing the system functionality the paper highlights the quality of the single contour linkage, with two driving elements that meets the requirements of an integral steering.

Key words: linkage, steering box, integral steering, four-wheel steering.

1. Introduction

The integral steering is one of the systems used for the improvement of vehicle manoeuvrability and stability [2], [3] by controlling the rear wheels in two ways. At low speeds, the rear wheels are turned in the opposite direction (Figure 1a) of the front wheels - increasing the manoeuvrability by decreasing the turning circle of the car, while at higher speeds the rear wheels are turned in the same direction (Figure 1b) as the front wheels, ensuring a high response on the road in lane-changing and in bands [4], [7].

Fig. 1. Steering radius for 2 steering axle

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Figure 2 presents a particular case (Honda) [5] (similar in paper [1]) of steering angles within the integral steering, by using only the steering wheel angle as the control input.

According to Figure 2 the maximum values for the rear wheels angle are $\theta_r \approx 1.5^\circ$ (same direction), respectively $-9^\circ$ (opposite direction), for $\theta_f \equiv 8-10^\circ$, respectively $\theta_f = 30^\circ$ of the front steering angle. Passing from one position to another - same/opposite direction - the rear wheels will be in the initial position at $\theta_r \approx 15^\circ$ for the front wheels and $\theta_w \equiv 220^\circ$ for the steering wheel.

For this mechanism the crank $OA$ must have its initial position along the base line $OB$, having the crank for phase 1 in inferior (Figure 4a) or superior (Figure 4b) position. Thus, from phase 1 to phase 2 the rear wheels are turned in the same direction to the front ones, phase 2-3 the wheels are returned to the neutral position (when $\varphi_i = 180^\circ$), and then to be turned in the opposite direction of the front ones - phase 3-4.

Fig. 2. Diagrams of the steering angle within the integral steering [Honda]

2. Linkage Mechanism with One Driving Input

Various types of linkages for operating the lever/rack are proposed in papers [6], [8], but without tangible results for their position.

In Figure 3 is presented a simple mechanism with one contour to which the crank 1 - the driving element - performs rocking strokes to the central element 3, and which can be designed for the rear steering boxes.

The displacement $S_3$ is defined as $S_3 = l \sin \varphi_3$ and the angle $\varphi_3$ is determined by $\varphi_i \rightarrow \varphi_3(\varphi_i)$.

Fig. 3. Linkage mechanism with one contour for integral steering

Fig. 4. Functionality of the mechanism with one contour within the integral steering

The maximum steering angle is reached when the crank is in phase 4, with the
angle $\phi_i = 270^\circ - \phi$ for 4a, respectively $\phi_i = 270^\circ + \phi$ for 4b, where $\phi = \arcsin r/b$.

In the diagram $S_3(\phi_1)$ from Figure 4 is displayed with continuous line the crank in inferior position (a) and with dotted line in superior position (b). The guiding mechanism being geometrically defined by two parameters $r$ and $b$, has two optimization possibilities, especially that intervenes the length $l$ of the lever.

A full race $\phi_i = 270 \pm \phi$ of the crank correspond to the maximum rotation of the steering wheel in one direction, and it make usually two revolutions (i.e. $\phi_s = 720^\circ$), as such the intermediation from front to back through gears is absolutely necessary for obtaining the corresponding ratio.

2.1. Conclusions

The main drawbacks of this simple mechanism within the integral steering are that they can obtain the shape of the function $\theta_3(\phi_s)$, but does not meet the requirements in terms of numerical values.

Thus this mechanism:
- generate identical values for the phases 1-2 and 3-4 (same/opposite direction related to the front wheels), reducing the vehicle handling.
- for the phases 1-2-3 (same direction to the front wheels) run on a rotation angle of the crank excessively high compared with the steering angle in the opposite direction, having also negative consequences for vehicle handling.

However the simple mechanism from Figure 3 can be considered as a solution for the integral steering if it is added a displacement to the joint of the mechanism base in the race direction, having a mechanism with two driving elements.

3. Linkage Mechanism with Two Inputs

The guiding mechanism RRTR from Figure 3, by having the position presented in Figure 4, and introducing the sliding block 7, the joint $B$ (Figure 5a) is displaced from position $B_0$ successively $B_{0,1,2,3,4,5}$ simultaneously with the rotation of joint $A$ from position $A_0$ to $A_{1,2,3,4,5}$ (Figure 5b), this can achieve the displacement of point $M$ like the one presented in Figure 5c.

![Fig. 5. The RRTR mechanism with two driving elements (RRTRT)](image-url)
The displacement of $x_M$ is based on two parameters $\phi_1/x_B$, - having two driving elements - and is determined by the Eq. (1):

$$x_M = r \sin \phi_1 + (l - S_2) \sin \phi_3,$$

in which:

$$S_2 = AB = \sqrt{r^2 + x_B^2 + b^2 - 2rx_B \sin \phi_1 - 2rb \cos \phi_1},$$

$$\phi_3 = \arctg \frac{r \sin \phi_1 - x_B}{b - r \cos \phi_1}.$$  

The geometrical parameters $r$ and $b$ to which is added the length $l$ of the rocker 3, and the correlation of the input kinematical parameters $\phi_1/x_B$ make the function $x_M(\phi_1)$ to have both the shape and the numerical values that meets the requirements of the integral steering.

### 3. Numerical Calculation for the Rear Steering Box with Linkage Mechanisms

The numerical calculation will consider the guiding mechanism mentioned above, and graphically analysed in Figure 5.

The input data for the numerical application were imported from Dacia Nova, and due to the large volume of information (Eq., Figs.), they will not be presented in this paper.

#### Input parameters

<table>
<thead>
<tr>
<th>$\phi_1$ [°]</th>
<th>0</th>
<th>32.2°</th>
<th>64.4°</th>
<th>96.6°</th>
<th>128.8°</th>
<th>161°</th>
<th>193.2°</th>
<th>216°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_1$ [°]</td>
<td>0</td>
<td>80.5°</td>
<td>161°</td>
<td>241.5°</td>
<td>322°</td>
<td>402.5°</td>
<td>483°</td>
<td>540°</td>
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</tbody>
</table>

The function calculation of the guiding mechanism position is achieved by the equation of the contour OABCO (Figure 6a):

$$ru_1 + S_3u_3 - x_Bi - hj = 0,$$

where:

$$\vec{u}_1 = i \sin \phi_1 - j \cos \phi_1,$$

$$\vec{u}_3 = -i \sin \phi_3 + j \cos \phi_3.$$
Attains:

\[ S_3 = \sqrt{r^2 + x_B^2 + h^2 - 2rx_B \sin \phi_1 + 2rh \cos \phi_1} \]

\[ \cos \phi_1 = \frac{1}{S_3} (r \cos \phi_1 + h) \]  \hspace{1cm} (7)

\[ \Delta x_B = r \frac{\Delta \phi_1}{1.5} \]  \hspace{1cm} (8)

\[ x_M = x_B + l \sin \phi_3 \]  \hspace{1cm} (9)

To which are given the following values:

- \( r = OA = 30 \text{ mm} \)
- \( h = OC = 30 \text{ mm} \)
- \( l = BM = 300 \text{ mm} \)

Accordingly:

\[ \Delta x_B = r \frac{\Delta \phi_1}{1.5} = 30 \frac{32.2}{1.5} \frac{\pi}{180} = 11.23. \]

The results for the numerical calculation are given in Table 2.

The attained function \( \theta_f(\phi_v) \) is presented in Figure 6c.

### Results of numerical calculation

<table>
<thead>
<tr>
<th>( \phi_1 ) [°]</th>
<th>0</th>
<th>32.2°</th>
<th>64.4°</th>
<th>96.6°</th>
<th>128.8°</th>
<th>161°</th>
<th>193.2°</th>
<th>216°</th>
</tr>
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<tbody>
<tr>
<td>( x_B ) [mm]</td>
<td>0</td>
<td>11.23</td>
<td>22.47</td>
<td>33.71</td>
<td>44.95</td>
<td>56.19</td>
<td>67.43</td>
<td>78.67</td>
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<tr>
<td>( S_3 ) [mm]</td>
<td>170</td>
<td>165.45</td>
<td>153.03</td>
<td>136.61</td>
<td>123.11</td>
<td>113.39</td>
<td>150.56</td>
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<tr>
<td>( \phi_3 ) [°]</td>
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<td>1.64</td>
<td>1.71</td>
<td>1.71</td>
<td>1.71</td>
<td>1.71</td>
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<td>1.71</td>
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<tr>
<td>( x_M ) [mm]</td>
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<td>25.11</td>
<td>21.70</td>
<td>29.81</td>
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<tr>
<td>( \theta_f ) [°]</td>
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<td>16.26</td>
<td>12.91</td>
<td>7.61</td>
<td>-2.88</td>
<td>-8.54</td>
<td>-18.76</td>
</tr>
</tbody>
</table>

### 4. Conclusions

The driving lever of the mechanism will have a maximum rotation angle of 270°, having its initial position along the base of the car, achieving symmetrical steering for left and right.

For achieving the requirements of the integral steering, the mechanisms with one contour must have a second drive to the base joint.

The linkage mechanisms within the steering box can be arranged horizontal or vertical. A constrictive solution is presented in Figure 7.
Acknowledgements

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References