JET FINSLER-LIKE GEOMETRY FOR THE $x$-DEPENDENT CONFORMAL DEFORMATION OF AN ONE-PARAMETER FAMILY OF BERWALD-MOÓR METRICS OF ORDER FOUR

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Abstract

The aim of this paper is to develop on the 1-jet space $J^1(\mathbb{R}, M^4)$ the Finsler-like geometry (in the sense of distinguished (d-) connection, d-torsions, d-curvatures and some gravitational-like and electromagnetic-like geometrical models) for the $x$-conformal deformed rheonomic Berwald-Moór metric of order four.

2000 Mathematics Subject Classification: 53C60, 53C80, 83C22.

Key words: $x$-conformal deformed rheonomic Berwald-Moór metric of order four, canonical nonlinear connection, Cartan canonical connection, d-torsions and d-curvatures, geometrical Einstein equations.

1 Introduction

The geometric-physical Berwald-Moór structure ([6], [13], [12]) was intensively investigated by P.K. Rashevski [18] and further fundamented and developed by D.G. Pavlov, G.I. Garas’ko and S.S. Kokarev ([15], [16], [9], [17]). At the same time, the physical studies of Asanov [1] or Garas’ko and Pavlov [9] emphasize the importance of the Finsler geometry characterized by the total equality in rights of all non-isotropic directions, in the theory of space-time structure, gravitation and electromagnetism. For such a reason, one underlines the important role played by the Berwald-Moór metric

$$F : TM \rightarrow \mathbb{R}, \quad F(y) = \sqrt[4]{y^1y^2...y^n},$$

whose tangent Finslerian geometry is studied by the geometers Matsumoto and Shimada [10] or Balan [3]. In such a perspective, according to the recent geometric-physical ideas proposed by Garas’ko in [8] and [7], we consider that a Finsler-like geometric-physical study for $x$-dependent conformal deformations of the jet Berwald-Moór structure is required. Note that, based on the works of Matsumoto and Shimada, Balan and Nicola [5] already presented the equation of motion in the $x$-conformally deformed 4-dimensional Berwald-Moór framework on tangent spaces.

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In such a geometrical and physical context, this paper investigates on the 1-jet space $J^1(\mathbb{R}, M^4)$ the Finsler-like geometry (together with a theoretical-geometric gravitational field-like theory) of the $x$-conformal deformed rheonomic Berwald-Moór metric of order four\footnote{We assume that $y_1 y_2 y_3 y_4 > 0$. This is the domain where we can $y$-differentiate the function $F(t, x, y)$.}

\[ F(t, x, y) = e^{\sigma(x)} \left( \sqrt{h_{11}(t)} \left[ y_1^1 y_2^2 y_3^3 y_4^4 \right] \right)^{1/4}, \]

where $\sigma(x)$ is a smooth non-constant function on $M^4$, $h_{11}(t)$ is the dual of a Riemannian metric $h_{11}(t)$ on $\mathbb{R}$, and $(t, x, y) = (t, x^1, x^2, x^3, x^4, y_1^1, y_2^2, y_3^3, y_4^4)$ are the coordinates of the 1-jet space $J^1(\mathbb{R}, M^4)$, which transform by the rules (the Einstein convention of summation is assumed everywhere):

\[ \tilde{t} = \tilde{t}(t), \quad \tilde{x}^i = \tilde{x}^i(x^i), \quad \tilde{y}_i^1 = \frac{\partial \tilde{x}^i}{\partial x^j} \frac{dt}{dt} \cdot y_j^1, \]

where $i, j = 1, 4$, rank $(\partial \tilde{x}^i/\partial x^j) = 4$ and $\frac{dt}{dt} \neq 0$. It is important to note that, based on the geometrical ideas promoted by Miron and Anastasiei in the classical Lagrangian geometry of tangent bundles [11], together with those used by Asanov in the geometry of 1-jet spaces [2], the differential geometry (in the sense of $d$-connections, $d$-torsions, $d$-curvatures, gravitational and electromagnetic geometrical theories) produced by a jet rheonomic Lagrangian function $L : J^1(\mathbb{R}, M^n) \to \mathbb{R}$ is now completely done in the monograph [4]. In what follows, we apply the general geometrical results from [4] to the $x$-conformal deformed rheonomic Berwald-Moór metric of order four (1).

\section{The canonical nonlinear connection}

Let us rewrite the $x$-conformal deformed rheonomic Berwald-Moór metric of order four (1) in the form

\[ F^*(t, x, y) = e^{\sigma(x)} F(t, y), \]

where

\[ F(t, y) = \sqrt{h_{11}(t)} \cdot \left[ y_1^1 y_2^2 y_3^3 y_4^4 \right]^{1/4} \]

can be regarded as a $t$-parameter family of Berwald-Moór metrics of order four. Hereinafter, using the notation $G_{1111} := y_1^1 y_2^2 y_3^3 y_4^4$, the fundamental metrical $d$-tensor produced by the metric (1) is given by the formula

\[ g_{ij}(t, x, y) = \frac{h_{11}(t)}{2} \frac{\partial^2 F^2}{\partial y_i^1 \partial y_j^1} = e^{2\sigma(x)} g_{ij}(y), \]

\[ \text{(2)} \]
where\(^3\) (see [4])

\[
g_{ij} = \frac{h_{11}(t)}{2} \frac{\partial^2 F^2}{\partial y_1^i \partial y_1^j} = \frac{(1 - 2\delta_{ij}) G^{1/2}_{1111}}{8 y_1^i y_1^j} \text{ (no sum by } i \text{ or } j). \tag{4}
\]

Moreover, the matrix \(^*g = (g^*_{ij})\) admits the inverse \(^*g^{-1} = (g^{*jk})\), whose entries are

\[
g^{*jk} = 2e^{-2\sigma(x)}(1 - 2\delta^{jk})G^{-1/2}_{1111} y_1^j y_1^k \text{ (no sum by } j \text{ or } k). \tag{5}
\]

Let us consider that the Christoffel symbol of the Riemannian metric \(h_{11}(t)\) on \(\mathbb{R}\) is

\[
K_{11}^1 = \frac{h^{11}}{2} \frac{dh_{11}}{dt},
\]

where \(h^{11} = 1/h_{11} > 0\). Then, using a general formula from [4] and taking into account that we have

\[
\frac{\partial G_{1111}}{\partial y_1^i} = \frac{G_{1111}}{y_1^i},
\]

we find the following geometrical result:

**Proposition 1.** For the \(x\)-conformal deformed rheonomic Berwald-Moór metric of order four (1), the energy action functional

\[
*E(t, x(t)) = \int_a^b \sqrt{h_{11}(t)} \ dt,
\]

where \(y = dx/dt\), produces on the 1-jet space \(J^1(\mathbb{R}, M^4)\) the canonical nonlinear connection

\[
*\Gamma = (M^{(i)}_{(1)1} = -K_{11}^1 y_1^i, \ N^{(i)}_{(1)j} = 4\sigma_i y_1^j \delta^i_j), \tag{6}
\]

where \(\sigma_i = \partial \sigma / \partial x^i\).

**Proof.** For the energy action functional \(\mathbf{E}\), the associated Euler-Lagrange equations can be written in the equivalent form (see [4])

\[
\frac{d^2 x^i}{dt^2} + 2H^{(i)}_{(1)1} \left( t, x^k, y_1^k \right) + 2G^{(i)}_{(1)1} \left( t, x^k, y_1^k \right) = 0, \tag{7}
\]

where the local components

\[
H^{(i)}_{(1)1} \overset{\text{def}}{=} -\frac{1}{2} K_{11}^1(t) y_1^i,
\]

and

\[
G^{(i)}_{(1)1} \overset{\text{def}}{=} \frac{h_{11}^{y_1^i y_1^j}}{4} \left[ \frac{\partial^2 F^2}{\partial x^r \partial y_1^i} y_1^r - \frac{\partial F^2}{\partial x^r} \frac{\partial^2 F^2}{\partial t \partial y_1^i} + \frac{\partial F^2}{\partial y_1^i} \right] + \frac{\partial F^2}{\partial y_1^i} K_{11}^1(t) + 2h^{11} K_{11}^1 \delta_{pp} y_1^i
\]

\[
= 2\sigma_i \left( y_1^i \right)^2
\]

\(^3\)Throughout this paper the Latin letters \(i, j, k, m, r, \ldots\) take values in the set \(\{1, 2, 3, 4\}\).
represent, from a geometrical point of view, a spray on the 1-jet space $J^1(\mathbb{R}, M^4)$. The canonical nonlinear connection associated to this spray has the components (see [4])

$$M^{(i)}_{(1)1} \equiv 2H^{(i)}_{(1)1} = -k^1_{11}y^i_1, \quad N^{(i)(j)}_{(1)j} \equiv \frac{\partial G^{(i)(1)}_{(1)1}}{\partial y^j_1} = 4\sigma_i y^i_1 \delta^i_j.$$

$$\square$$

3 The Cartan canonical $^\ast$Γ-linear connection. Its d-torsions and d-curvatures

The nonlinear connection (6) produces the dual adapted bases of d-vector fields (no sum by $i$)

$$\left\{ \frac{\delta}{\delta t} = \frac{\partial}{\partial t} + k^1_{11} y^p_1 \frac{\partial}{\partial y^1_1} ; \frac{\delta}{\delta x^i} = \frac{\partial}{\partial x^i} - 4\sigma_i y^i_1 \frac{\partial}{\partial y^1_1} ; \frac{\partial}{\partial y^i_1} \right\} \subset \mathcal{X}(E) \quad (8)$$

and d-covector fields (no sum by $i$)

$$\left\{ dt ; dx^i ; \delta y^i_1 = dy^i_1 - k^1_{11} y^i_1 dt + 4\sigma_i y^i_1 dx^i \right\} \subset \mathcal{X}^*(E), \quad (9)$$

where $E = J^1(\mathbb{R}, M^4)$. The naturalness of the geometrical adapted bases (8) and (9) is coming from the fact that, via a transformation of coordinates (2), their elements transform as the classical tensors. Therefore, the description of all subsequent geometrical objects on the 1-jet space $J^1(\mathbb{R}, M^4)$ (e.g., the Cartan canonical linear connection, its torsion and curvature etc.) will be done in local adapted components. Consequently, by direct computations, we obtain the following geometrical result:

**Proposition 2.** The Cartan canonical $^\ast$Γ-linear connection, produced by the $x$-conformal deformed rheonomic Berwald-Moór metric of order four (1), has the following adapted local components (no sum by $i$, $j$ or $k$):

$$C^G = \left( \kappa_{11}^1, \ G^k_{j1} = 0, \ L^i_{jk} = 4\delta_i^j \delta^k_k \sigma_i, \ C^{(1)(i)}_{jk} = C^i_{jk} \cdot \frac{y^i_1}{y^1_1 y^k_1} \right), \quad (10)$$

where

$$C^{i}_{jk} = \frac{2\delta^i_j + 2\delta^i_k + 2\delta_{jk} - 8\delta^i_j \delta_{jk} - 1}{8} =$$

$$= \begin{cases} 
  -\frac{1}{8}, & i \neq j \neq k \neq i \\
  \frac{1}{8}, & i = j \neq k \text{ or } i = k \neq j \text{ or } j = k \neq i \\
  -\frac{3}{8}, & i = j = k. 
\end{cases}$$
$x$-Conformal deformation of the quartic Berwald-Moór metric

Proof. The adapted components of the Cartan canonical connection are given by the formulas (see [4])

$$G_{j1}^k \overset{\text{def}}{=} \frac{g^{km} \delta g_{mj}}{2 \delta t} = 0, \quad L_{jk}^i \overset{\text{def}}{=} \frac{g^{im}}{2} \left( \frac{\delta g_{jm}}{\delta x^k} + \frac{\delta g_{km}}{\delta x^j} - \frac{\delta g_{jk}}{\delta x^m} \right) = 4 \delta_j^i \delta_k^l \sigma_l.$$

Using the derivative operators (8), the direct calculations lead us to the required results. Moreover, it is important to note that the vertical d-tensor $C^{j(1)}_{j(k)}$ also has the properties (see also [10] and [14]):

$$C^{i(1)}_{j(k)} = C^{i(1)}_{k(j)}, \quad C^{i(1)}_{j(m)} y_1^m = 0, \quad C^{m(1)}_{j(m)} = 0, \quad C^{m(1)}_{i(k)m} = 0,$$

with sum by $m$, where

$$C^{l(1)}_{i(k)j} \overset{\text{def}}{=} \frac{\delta C^{l(1)}_{i(k)j}}{\delta x^j} + C^{r(1)}_{i(k)} L_{rj}^l - C^{l(1)}_{r(k)} L_{ij}^r - C^{l(1)}_{i(r)} L_{ikj}^r.$$

\[\square\]

**Proposition 3.** The Cartan canonical connection of the $x$-conformal deformed rheonomic Berwald-Moór metric of order four (1) has two effective local torsion d-tensors:

$$R^{(r)}_{(1)ij} = 4 \left( \delta^r_i \sigma_{rj} - \delta^r_j \sigma_{ri} \right) y_1^r, \quad P^{(1)}_{(r)} \overset{\text{def}}{=} \frac{2 \delta^r_i + 2 \delta^r_j + 2 \delta_{ij} - 8 \delta^r_i \delta_{ij} - 1}{8} \cdot \frac{y_1^r}{y_1^i y_1^j},$$

where $\sigma_{pq} := \frac{\partial^2 \sigma}{\partial x^p \partial x^q}$.

Proof. Generally, an $h$-normal $\Gamma$-linear connection on the 1-jet space $J^1(\mathbb{R}, M^4)$ has eight effective local d-tensors of torsion (for more details, see [4]). For the Cartan canonical connection (10) these reduce only to two (the other six are zero):

$$R^{(r)}_{(1)ij} \overset{\text{def}}{=} \frac{\delta N^{(r)}_{(1)i}}{\delta x^j} - \frac{\delta N^{(r)}_{(1)j}}{\delta x^i}, \quad P^{(1)}_{(r)} \overset{\text{def}}{=} C^{(r)}_{(1)ij}.$$

\[\square\]

**Proposition 4.** The Cartan canonical connection of the $x$-conformal deformed rheonomic Berwald-Moór metric of order four (1) has three effective local curvature d-tensors:

$$R^{l}_{ijk} = \frac{\partial L_{ij}^k}{\partial x^k} - \frac{\partial L_{ik}^j}{\partial x^j} + L_{ij}^l L_{rk}^l - L_{ik}^l L_{rj}^l + C^{l(1)}_{i(r)k} R^{(r)}_{(1)jk}, \quad P^{l(1)}_{ij(k)} = -C^{l(1)}_{i(k)j},$$

$$S^{l(1)}_{i(j)(k)} \overset{\text{def}}{=} \frac{\partial C^{l(1)}_{i(j)k}}{\partial y_1^j} - \frac{\partial C^{l(1)}_{i(k)j}}{\partial y_1^j} + C^{l(1)}_{i(j)} C^{l(1)}_{r(k)} - C^{l(1)}_{i(k)} C^{l(1)}_{r(j)}.$$
Proof. Generally, an $h$-normal $\Gamma$-linear connection on the 1-jet space $J^1(\mathbb{R}, M^4)$ has five effective local d-tensors of curvature (for more details, see [4]). For the Cartan canonical connection (10) these reduce only to three (the other two are zero); these are $S^{d(1)(1)}_{i(j)(k)}$ and

$$R^{l}_{ijk} \overset{\text{def}}{=} \frac{\delta L^l_{ij}}{\delta x^k} - \frac{\delta L^l_{ik}}{\delta x^j} + L^r_{ij} L^l_{rk} - L^r_{ik} L^l_{rj} + C^{d(1)}_{i(r)} R^{(r)}_{(1)jk},$$

$$P^{(1)}_{ij(k)} \overset{\text{def}}{=} \frac{\partial L^{1}_{ij}}{\partial y^k} - C^{d(1)}_{i(k)j} + C^{d(1)}_{i(r)} P^{(r)(1)}_{(1)j(k)} = -C^{d(1)}_{i(k)ij},$$

where

$$P^{(r)(1)}_{(1)j(k)} \overset{\text{def}}{=} \frac{\partial N^{(r)}_{(1)j}}{\partial y^k} - L^r_{jk} = 0.$$

\[\square\]

4 From $x$-conformal deformations of the rheonomic Berwald-Moór metric of order four to field-like geometrical models

4.1 Gravitational-like geometrical model

The $x$-conformal deformed rheonomic Berwald-Moór metric (1) produces on the 1-jet space $J^1(\mathbb{R}, M^4)$ the adapted metrical d-tensor (sum by $i$ and $j$)

$$G = h_{11} dt \otimes dt + \delta y^i_j dx^i \otimes dx^j + h^{11} g_{ij} \delta y^i_1 \otimes \delta y^j_1, \quad (12)$$

where $g_{ij}$ is given by (3) and we have $\delta y^1_i = dy^1_i - \kappa^{11} y^1_i dt + 4x^i y^1_i dx^i$ (no sum by $i$). From a physical point of view, the metrical d-tensor (12) may be regarded as a “non-isotropic gravitational potential”. In our geometric-physical approach, one postulates that the non-isotropic gravitational potential $G$ is governed by the geometrical Einstein equations

$$\text{Ric} (CT^*) - \frac{\text{Sc} (CT^*)}{2} G = \kappa T, \quad (13)$$

where

- $\text{Ric} (CT^*)$ is the Ricci d-tensor associated to the Cartan canonical connection (10);

- $\text{Sc} (CT^*)$ is the scalar curvature;

- $\kappa$ is the Einstein constant and $T$ is the intrinsic stress-energy d-tensor of matter.

Therefore, using the adapted basis of vector fields (8), we can locally describe the global geometrical Einstein equations (13). Consequently, by some direct computations we find:
Lemma 1. The Ricci d-tensor of the Cartan canonical connection $C^\ast$ of the $x$-conformal deformed rheonomic Berwald-Moór metric of order four (1) has the following two effective local Ricci d-tensors (no sum by $i, j, k$ or $l$):

$$R_{ij} = \begin{cases} -2\sigma_{ij} - \sigma_{jk} \frac{y^k_1}{y^i_1} - \sigma_{jl} \frac{y^l_1}{y^i_1}, & i \neq j, \quad \{i, j, k, l\} = \{1, 2, 3, 4\} \\ 0, & i = j, \end{cases}$$

(14)

Proof. Generally, the Ricci d-tensor of a Cartan canonical connection $C^\Gamma$ of a Cartan canonical connection (10) these reduce only to the following two (the other four are zero):

$$R_{ij} \overset{\text{def}}{=} R_{ijm} = \frac{\partial L^m_{ij}}{\partial x^m} - \frac{\partial L^m_{im}}{\partial x^j} + L^r_{ij} L^m_{rm} - L^r_{im} L^m_{rj} + C^m_{i(r)} R^{(r)}_{(1)jm},$$

$$S^{(1)(1)}_{(i)(j)} \overset{\text{def}}{=} S^{m(1)(1)}_{i(j)(m)} = \frac{\partial C^{m(1)}_{i(j)}}{\partial y^m_1} - \frac{\partial C^{m(1)}_{i(m)}}{\partial y^j_1} + C^{r(1)}_{i(j)} C^{m(1)}_{r(m)} - C^{r(1)}_{i(m)} C^{m(1)}_{r(j)},$$

with sum by $r$ and $m$. $\square$

Lemma 2. The scalar curvature of the Cartan canonical connection $C^\ast$ of the $x$-conformal deformed rheonomic Berwald-Moór metric of order four (1) has the value

$$\text{Sc} (C^\ast) = -2e^{-2\sigma} G^{(1)}_{1111} (3h_{11} + 8Y_{11}),$$

where

$$Y_{11} = \sigma_{12} y_1^1 y_1^2 + \sigma_{13} y_1^1 y_1^3 + \sigma_{14} y_1^1 y_1^4 + \sigma_{23} y_1^2 y_1^3 + \sigma_{24} y_1^2 y_1^4 + \sigma_{34} y_1^3 y_1^4.$$

Proof. The scalar curvature of the Cartan canonical connection (10) is given by (for more details, see [4])

$$\text{Sc} (C^\ast) = g^{pq} R_{pq} + h_{11} g^{pq} S^{(1)(1)}_{(p)(q)} = -16e^{-2\sigma} G^{(1)}_{1111} Y_{11} - 6e^{-2\sigma} G^{(1)}_{1111} h_{11}. \square$$

The local description in the adapted basis of vector fields (8) of the global geometrical Einstein equations (13) is given by (for more details, see [4]):
Proposition 5. The geometrical Einstein equations produced by the $x$-conformal deformed rheonomic Berwald-Moór metric of order four (1) are locally described by:

$$
\begin{align*}
& e^{-2\sigma} G_{1111}^{-1/2} (3h_{11} + 8Y_{11}) h_{11} = \mathcal{K} T_{11} \\
& R_{ij} + e^{-2\sigma} G_{1111}^{-1/2} (3h_{11} + 8Y_{11}) \delta_{ij} = \mathcal{K} T_{ij} \\
& S_{(i)(j)}^{(1)} + e^{-2\sigma} G_{1111}^{-1/2} (3h_{11} + 8Y_{11}) h_{11} \delta_{ij} = \mathcal{K} T_{(i)(j)}^{(1)} \\
& 0 = T_{i1}, \quad 0 = T_{11}, \quad 0 = T_{(i)1}^{(1)} \\
& 0 = T_{1(i)}^{(1)}, \quad 0 = T_{i(j)}^{(1)}, \quad 0 = T_{(i)j}^{(1)}.
\end{align*}
$$

Corollary 1. The stress-energy d-tensor of matter $\mathcal{T}$ satisfies the following geometrical conservation laws (sum by $m$):

$$
\begin{align*}
& \mathcal{T}^{(1)}_{i/1} + \mathcal{T}^{m} = 0 \\
& \mathcal{T}^{(m)}_{i/1} + \mathcal{T}^{(m)}_{i|m} + \mathcal{T}^{(m)}_{(1)i} = 0
\end{align*}
$$

where (sum by $r$):

$$
\begin{align*}
& T_{i}^{(1)} = h_{11} T_{11} = \mathcal{K}^{-1} e^{-2\sigma} G_{1111}^{-1/2} (3h_{11} + 8Y_{11}), \quad T_{i}^{m} = g^{mr} T_{r} = 0, \\
& T_{(i)1}^{(1)} = h_{11} g^{mr} T_{(r)} = 0, \quad T_{i}^{1} = h_{11} T_{i1} = 0, \\
& T_{i}^{m} = g^{mr} T_{r} = E_{i}^{m} = \mathcal{K}^{-1} \left[ g^{mr} R_{ri} + e^{-2\sigma} G_{1111}^{-1/2} (3h_{11} + 8Y_{11}) \delta_{i}^{m} \right], \\
& T_{(i)1}^{(1)} = h_{11} g^{mr} T_{(r)} = 0, \quad T_{i}^{(1)} = h_{11} T_{i1}^{(1)} = 0, \quad T_{i}^{(m)} = g^{mr} T_{(r)} = 0, \\
& T_{(i)1}^{(m)} = h_{11} g^{mr} T_{(r)} = \frac{e^{-2\sigma} G_{1111}^{-1/2}}{\mathcal{K}} \left[ \frac{h_{11} y_{i}^{m}}{2} \delta_{i}^{m} + (h_{11} + 8Y_{11}) \delta_{i}^{m} \right], \text{and we also have}
\end{align*}
$$

(summation by $m$ and $r$, but no sum by $i$)

$$
\begin{align*}
& T_{i/1}^{1} = \frac{\delta T_{i}^{1}}{\delta t}, \quad T_{i|m}^{(1)} = \frac{\delta T_{i}^{m}}{\delta x^{m}} + T_{i}^{r} L_{i|m}^{r}, \\
& T_{(i)1}^{(1)} = \frac{\delta T_{(i)1}^{(1)}}{\delta y_{i}^{m}} + T_{(i)1}^{(r)} C_{(r)}^{m} = \frac{\delta T_{(i)1}^{(m)}}{\delta y_{i}^{m}}, \\
& T_{i/1}^{1} = \frac{\delta T_{i}^{1}}{\delta t} + T_{i}^{1} K_{i1}^{1} - T_{i}^{r} G_{i1}^{r} = \frac{\delta T_{i}^{1}}{\delta t} + T_{i}^{1} K_{i1}^{1},
\end{align*}
$$
Proof. The local Einstein equations (15), together with some direct computations, lead us to what we were looking for.

4.2 Electromagnetic-like geometrical model

In book [4], a geometrical theory for electromagnetism was also created, using only a given Lagrangian function $L$ on the 1-jet space $J^1(\mathbb{R}, M^4)$. In the background of the jet relativistic rheonomic Lagrange geometry from [4], one works with the electromagnetic distinguished 2-form (sum by $i$ and $j$)

$$F = F_{(i)j}^1 \delta y_1^j \wedge dx^i,$$

where (sum by $m$ and $r$)

$$F_{(i)j}^1 = \frac{h^{11}}{2} \left[ * g_{jm}^1 N_{(1)i}^m - * g_{jm}^1 N_{(1)j}^m + \left( * g_{ir}^1 L_{jm}^r - * g_{jr}^1 L_{im}^r \right) y_1^m \right].$$

This is characterized by some natural geometrical Maxwell equations (for more details, see Miron and Anastasiei [11] and Balan and Neagu [4]).

By a direct calculation, we observe that the $x$-conformal deformed rheonomic Berwald-Moór metric of order four (1) produces null electromagnetic components:

$$F_{(i)j}^1 = 0.$$

Consequently, our $x$-conformal deformed jet Berwald-Moór geometrical electromagnetic theory is trivial. Probably, this fact suggests that the $x$-conformal deformed rheonomic Berwald-Moór structure (1) has rather strong gravitational connotations than electromagnetic ones.

As a conclusion, it is possible for the new Voicu-Siparov approach to electromagnetism in spaces with anisotropic metrics (this electromagnetic approach is different from the electromagnetic theory exposed above, and it is developed by Voicu and Siparov in the paper [19]) to give other interesting electromagnetic-geometrical results.
References


